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LEVELING
CIRCULAR CURVES
STADIA AND PLANE-TABLE SURVEYING
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CONTENTS

LEVELING

THE DETERMINATION OF DIFFERENCES OF ELEVATION

DEFINITIONS AND METHODS

1. Definitions.—The process of determining the relative elevations of a series of points is called **leveling.**

A surface that is at every point perpendicular to the direction of gravity, as indicated by a plumb-line at that point, is a **level surface,** and a line lying in such a surface is a **level line.**

The surface of water in a quiescent state is a level surface, and any level surface is parallel to a water surface covering the same area. A level surface is not a plane surface and a level line is not a straight line, since plumb-lines at different points on the earth's surface are never parallel, but always converge toward the center of the earth. A plane surface that is perpendicular to the plumb-line or vertical line at any point is for that point a horizontal plane surface, but since no two plumb-lines can be parallel, no two planes that are respectively horizontal at different points on the earth's surface can be parallel. But for every point on the earth's surface a plane surface or a straight line is considered to be horizontal when it is perpendicular to the direction of gravity at that point.

A plane surface that is horizontal at any point on the earth's surface is tangent to a level surface at that point,

and since the radius of curvature of the earth's surface is very great, the plane surface will very nearly coincide with the level surface for a considerable distance in every direction from the point. Hence, any reasonably short straight line that is horizontal at any point on the earth's surface may for all ordinary purposes be considered to be a level line, and is commonly so considered.

The distance of a point vertically above a given level surface is called its **elevation**, or its **height**. The level surface above which the elevation of a point is measured may be an actual surface, as the surface of a body of water, or merely an imaginary surface assumed as a *surface of reference*. (See Art. **40.**) The relative elevations of a series of points can be determined by comparing their respective elevations above a surface of reference.

2. Methods of Leveling.—There are three general methods of determining elevations, differing essentially with regard to the principles involved and the instruments and processes employed. They are: (1) *gravity leveling*, commonly called *spirit leveling*, and also designated as *direct leveling*; (2) *trigonometric leveling*, also called *indirect leveling*; and (3) *barometric leveling*.

3. Gravity Leveling.—In this method of leveling, the elevations of any number of points are determined by their respective distances vertically above or below a level line or series of level lines whose elevations above the surface of reference are known. Each level line is prolonged horizontally from a point of known elevation, so as to pass directly over or under the point whose elevation is required, and the distance of the point vertically below or above the level line is measured. The elevation of each required point being determined in this way, the difference in the elevations of any two points is found by subtracting the elevation of the lower from the elevation of the higher. The level lines are usually mere visual lines, that is, lines of sight, and are prolonged horizontally by means of some device that directs

the line of sight in a direction at right angles to the direction of gravity at the point of observation.

The name **direct leveling** is sometimes applied to this method because the elevation of each point is determined by measuring directly its distance vertically above or below the horizontal line of sight. But since the direction of gravity is the essential basis of this method of leveling, the name **gravity leveling** is believed to be a proper one. The device most commonly, and almost universally, employed for determining when a line is horizontal is the spirit level, from which fact this method of leveling is commonly called **spirit leveling.** Other devices can be, and sometimes are, employed for this purpose, though they all depend on the action of gravity for determining a horizontal direction. The name gravity leveling, therefore, appears to be preferable.

4. Trigonometric Leveling.—This method of leveling is based on the solution of a right triangle in a vertical plane. The line joining the two points whose difference of elevation is to be determined is the hypotenuse of the triangle; its base is the horizontal line extending from either point to intersect the vertical line extending from the other point, which vertical line is the altitude of the triangle. The two acute angles of the triangle are thus at the two points whose difference of elevation is required. One acute angle and an adjacent side of the triangle are measured, and from these values is computed the altitude of the triangle, which is the difference of elevation required. As the difference of elevation between the two points is thus determined indirectly by calculation from the measurements taken instead of by direct measurement, this method of leveling is sometimes called **indirect leveling.** But since the necessary calculations are based on the principles of trigonometry, the name trigonometric leveling is used here.

5. Barometric Leveling.—In barometric leveling the relative elevations of different points are determined by calculations based on the difference in the intensities of pressure due to the weight of the atmosphere at the points.

The intensities of pressure are indicated by barometer readings, which should be taken simultaneously when possible. The barometer readings being proportional to the weight of atmosphere above the points of observation, it is possible by this means to calculate with fair approximation the relative elevations of the points where the observations were taken.

GRAVITY LEVELING

LEVELING INSTRUMENTS

6. General Principles.—Gravity leveling, called also spirit leveling and direct leveling, depends on three principles: first, that the surface of a liquid in repose is level; second, that a vertical line is perpendicular to a level surface; and third, that a bubble of air confined in a vessel otherwise filled with liquid will rise to the highest point of the inner surface of the vessel. Depending directly on these principles are a great variety of instruments for determining elevations. The spirit level is a common application of the third principle.

7. The Engineers' Level.—The most highly developed form of the spirit level is the **engineers' level.** This instrument is used more extensively than any other for determining elevations, and by means of it results of exceeding accuracy are obtained. This instrument consists essentially of a telescope having a very accurate spirit level attached, mounted on a tripod and controlled by leveling screws in such a manner that the line of sight can be made truly horizontal, that is, truly tangent to a level line at the instrument. There are two general classes of engineers' levels, namely: the *Y level*, also written *wye level*, and the *dumpy level.* The former is much the more popular with American engineers because of the facility with which it can be adjusted, while the latter is favored in Europe.

THE WYE LEVEL

8. Description.—An engineers' **Y level** is shown in Fig. 1. The telescope *A B*, having attached to it an accurate and sensitive spirit level *E F*, rests in the **Y**-shaped supports *Y* and *Y'*, in which it is held firmly by semicircular clasps, commonly called **clips**; these are hinged at one end, and passing over the telescope are held at the other end by small pins. The **Y**-shaped supports called **Y's,** also written **wyes,** are distinguishing features of this form of level, from

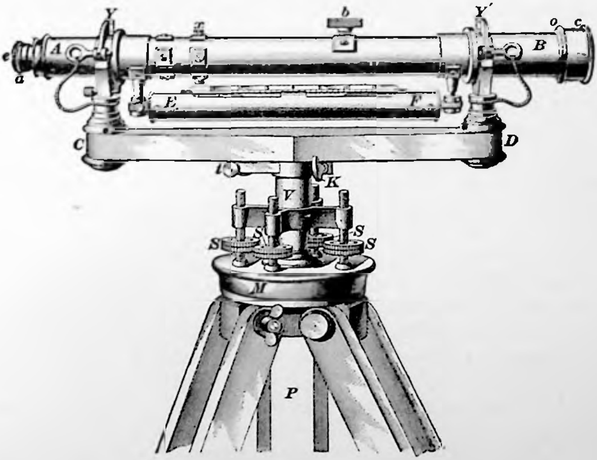

FIG. 1

which fact the instrument derives the name wye level. The lower ends of the wyes pass through the ends of the horizontal bar *C D*, sometimes called the **level bar,** and are adjustable vertically by means of the capstan-pattern nuts shown at *C* and *D*, which bear against the upper and lower surfaces of the bar. The bar *C D* is attached rigidly to the center or spindle, which turns in the socket *V*, permitting the telescope to be revolved in a horizontal plane. The

spindle can be clamped by the screw K and the telescope then revolved slowly by means of the tangent screw t, which operates against a short projecting arm having a spring bearing against its opposite side. The position of the socket V is controlled by the four leveling screws S, which, together with the lower leveling plate M, and the tripod P, are substantially the same as in a transit, except that a level does not commonly have a shifting center.

9. **The telescope** is in every respect similar to that of the transit, as described in *Transit Surveying*, Part 1, except that it is longer, and having no horizontal axis, it cannot be revolved in a vertical plane. The object glass, or objective, is at o; it is shown covered by a cap c, which is to protect it when the instrument is not in use. The objective is focused by sliding it in or out by means of the milled thumb wheel b, which is here shown on top of the telescope, though it is commonly placed on the right-hand side. The cross-hairs are either spider webs or platinum wires, and are in all respects the same as the cross-hairs of a transit. They are adjusted by the capstan-headed screws shown at x. The outer end of the eyepiece is shown at e; it is adjusted by rotating the milled ring a, which is attached to a hollow screw that encircles the eyepiece and carries it in or out by the rotary motion of the screw.

10. **The spirit level** EF attached to the telescope is similar to that attached to the telescope of a transit, as described in *Transit Surveying*, Part 1, but in a leveling instrument, it is usually more accurate and sensitive. As this is the part of the instrument on which its accuracy chiefly depends, the spirit level will be described quite fully. It consists of a hermetically sealed glass tube, curved slightly in a manner corresponding to the short upper arc of a large vertical circle, having the upper portion of its inner surface on a longitudinal section ground

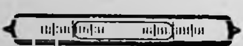

FIG. 2

truly to the arc, and so nearly filled with alcohol, or a mixture of alcohol and ether, as to leave only a small bubble of air. Alcohol is used extensively for the levels of surveying instruments, but is rather slow acting. Ether, though more sensitive and quick acting, is affected too greatly by changes of temperature to be used in surveying instruments. A mixture of alcohol and ether gives excellent results. A top view of the glass tube of a spirit level is shown in Fig. 2, in which the bubble is represented as truly centered. This tube, commonly called a **level tube, bubble tube,** or **level vial,** is protected by a metal case having a long, narrow longitudinal opening or slit in its upper side, through which the glass tube and bubble can be seen, and in which the glass tube is fastened securely but not rigidly. Each end of the metal case is attached to a stud projecting from the under side of the telescope tube, one end being adjustable vertically by means of capstan-pattern nuts, and the other end adjustable laterally by means of capstan-headed screws, as shown in Fig. 1. In some instruments the bubble scale is marked on the top of the glass, as in Fig. 2, while in others it is on a small strip of silvered brass attached to the upper side of the level tube, as shown in Fig. 1. Some instrument makers prefer one method and some the other. Since the air bubble rises to the highest point of the inner surface of the level tube in which it is confined, and since the upper portion of the inner surface of the tube is ground truly to the arc of a circle in the plane of its longitudinal section, it follows that a tangent to this arc at the center of the bubble is a horizontal line. A line tangent to the inner upper surface of the bubble tube *at its center* is called the **axis of the bubble,** or **axis of the level tube.** When the bubble is in the center of the tube, this line will be tangent to the center of the bubble, and, consequently, will be a horizontal line. Hence, *the axis of the level tube is horizontal when the bubble is in the center of the tube.*

11. Axis of Revolution; Essential Condition.—The level has but one axis of revolution; this is the vertical

axis of the instrument. When a level is in adjustment and is leveled up, the line of sight revolves on this axis in a horizontal plane. As in the transit, the line of sight, when in adjustment, is also called the line of collimation; it is the line determined by the intersection of the cross-hairs and the optical center of the object glass. In the use of the level the only essential requisite to accurate work is that this line of sight shall be truly horizontal, and that this condition shall be indicated by the bubble being in the center of its tube. The only adjustment of the level essential to accuracy, therefore, is that of making the line of sight parallel to the axis of the level tube. There are two methods of effecting this. One method depends on the accuracy of workmanship in the instrument, and consists of the two separate operations described in the following articles as the first and second adjustments of the level. The other method is independent of the workmanship of the instrument, and is known as the *direct method*, or *peg method*, of adjustment, which is described further on. By this method the level tube is adjusted so that its axis lies in a horizontal plane when the line of sight is horizontal. This adjustment is always the most reliable. Although the adjustments 1, 2, and 3 described in following articles are usually made, engineers often check the accuracy of the first two adjustments by this independent adjustment.

12. Adjustments.—There are three adjustments of the Y level, as follows:

1 To make the line of sight, or line of collimation, parallel to the axis of the collars, or rings, on the telescope by which it rests in the wyes.

2. To make the axis of the level tube bubble parallel to the axis of the collars, and, consequently, parallel to the line of collimation.

3. To make the axis of the level tube perpendicular to the vertical axis of the instrument, so that when the instrument is leveled up the bubble will remain centered while the telescope is revolved horizontally.

The first two adjustments are for the purpose of causing the line of sight to be parallel to the axis of the level bubble. This is done by making both of these parallel to an axis through the centers of the accurately turned collars, or rings, on the telescope tube by which the telescope rests in the wyes. Neither of these two adjustments is of any especial value without the other, but they are really the two parts of one adjustment, which depends on the principle of geometry that two lines that are each parallel to a third line are parallel to each other. These two adjustments have together been called the *indirect* method of adjusting the line of sight parallel to the axis of the bubble. They can be applied only to the wye level, and the construction of the instrument permitting this adjustment is the distinguishing and characteristic feature of this form of level. The *direct* method of performing this adjustment is always employed for the dumpy level, and is therefore explained in connection with that instrument. But it can also be applied to the wye level. According to a statement made in a preceding article, the line of sight must be parallel to the axis of the level bubble in order that the line of sight will be horizontal at the instrument when the level bubble is centered. Hence, the adjustment represented by the first two adjustments of the wye level is necessary to the accuracy of the instrument.

The third adjustment does not affect the accuracy of the work, but does affect the rapidity with which it can be performed. When this adjustment is perfect, and the instrument is leveled up accurately, the bubble will remain centered in whatever direction the telescope is turned. But if this adjustment is not perfect, the bubble will have to be recentered by the leveling screws every time the telescope is turned and pointed in a new direction.

The adjustments of the level should be tested frequently, at least once a day, when in use, as any defect in them will detract either from the value of the work performed or from the rapidity with which it can be performed, or from both.

13. First Adjustment.—*To make the line of sight parallel to the axis of the collars:*

This is commonly called the adjustment for collimation, and depends for its accuracy on the collars being truly circular and on the principal axis of the objective lens coinciding with the axis of the collars when the telescope is revolved in the wyes. This may or may not be the case, according to the degree of accuracy attained in the manufacture of the instrument.

Plant the tripod firmly; choose some distant and clearly defined point, the more distant the better, so long as it is distinctly visible and sharply defined. Remove the pins from the clips, clamp the spindle, and by means of the tangent screw and leveling screws bring the intersection of the cross-hairs to coincide exactly with the point sighted. Revolve or turn the telescope in the wyes through one-half a revolution, that is, until it is bottom side up. If the intersection of the cross-hairs is still on the point of sight, it shows that the line of sight coincides with the axis of the collars. But if, when the telescope is turned bottom side up, the line of sight defined by the intersection of the cross-hairs is no longer on the point, move the cross-hairs by means of the capstan-headed adjusting screws so as to correct one-half the apparent error, being careful to move them in the opposite direction to which it would appear they should be moved. The apparent error shown by reversing the telescope is double the real error, as is illustrated in

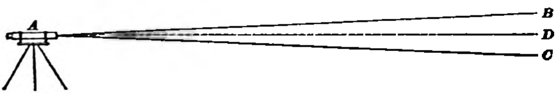

FIG. 3

Fig. 3. For if $A B$ is the original line of sight with the telescope in its natural position, then, when the telescope is turned upside down, the line of sight will be $A C$, and $B C$ will be the apparent error, whereas $B D$, $= C D$, is the real error.

Suppose that with the instrument at A the line of sight given by the intersection of the cross-hairs is directed to the point B, and that when the telescope has been revolved or turned upside down in the wyes, the line of sight strikes the point C; then the distance $B\,C$ will be double the real error, and the true line of sight will be at D, half way between B and C. Sometimes both the horizontal and the vertical cross-hairs are out of adjustment, in which case they should be moved alternately until their intersection will coincide with the same point throughout a complete revolution of the telescope.

14. Second Adjustment.—*To make the axis of the level tube parallel to the axis of the collars:*

When this and the first adjustment have been made, the axis of the level tube will be parallel to the line of sight. This adjustment depends for its accuracy on the two collars by which the telescope rests in the wyes having exactly the same diameter. The adjustment is in two parts, the first part being for the purpose of enabling the second part, or principal adjustment, to be performed accurately.

First.—Level up the instrument, remove the pins from the wyes, and open the clips; place the telescope over a pair of leveling screws and clamp the spindle. Bring the bubble exactly to the middle of the tube by means of the leveling screws, and revolve the telescope in the wyes in either direction through about an eighth of a revolution, so that the bubble tube will not be directly below the telescope tube but will stand out at an angle. If the bubble runs toward either end, it shows that the longitudinal axis of the bubble tube and the line of collimation, or longitudinal axis, of the telescope do not lie in the same plane. To correct the error, bring the bubble nearly to the center by means of the capstan-headed adjusting screws at one end of the level tube, which regulate its lateral movement, and repeat the operation until the bubble will remain centered during the partial revolution of the telescope.

This adjustment is necessary because when reversing the telescope end for end in the wyes in the following and principal adjustment, we cannot be certain of placing the level tube in the same vertical plane as before, and if the longitudinal axis of the level tube is not in the same plane as the axis of the telescope rings, the position of the bubble will be affected by any variation of the level tube from the plane of its former position. Consequently, it would be almost impossible to adjust the level tube without first correcting any lateral error that may exist.

Second.—Center the bubble accurately, take the telescope out of the wyes, turn it end for end, and replace it in the wyes very carefully so as not to disturb their position. If the bubble remains in the center of the tube, the adjustment is perfect. If the bubble runs to one end, bring it half way back by means of the capstan-pattern adjusting nuts at one end of the level tube, by which it can be raised or lowered, and then bring it to the middle of the tube by means of the leveling screws. Repeat the operation until the bubble will remain truly centered when the telescope is reversed in the wyes.

15. Third Adjustment.—*To make the axis of the level tube perpendicular to the instrument's vertical axis:*

When this adjustment is perfect and the instrument is leveled up, the bubble will remain centered while the telescope is revolved horizontally. This is sometimes called the **bar adjustment.**

Level up the instrument, using both pairs of leveling screws. Having centered the bubble carefully with the telescope over one pair of leveling screws, reverse the telescope or turn it end for end over the same pair of leveling screws. If the bubble runs toward one end, bring it half way back by means of the capstan-pattern nuts at the end of the level bar, then center it perfectly with the leveling screws. Repeat the operation over each pair of leveling screws alternately until the bubble will remain perfectly centered throughout an entire horizontal revolution of the telescope.

16. An Important Test.—Since in leveling, it is convenient to be able to sight with any portion of the horizontal cross-hair, without always being particular to sight with the exact intersection of the cross-hairs, it is essential for the horizontal cross-hair to be truly parallel to the telescope's plane of revolution about the vertical axis of the instrument, so that when the instrument is leveled up this cross-hair will be truly horizontal. In order to test this, sight upon any sharply defined point, focusing the telescope perfectly and bringing the point exactly in range with the horizontal cross-wire near either end, that is, near the right-hand or left-hand edge of the field of view. Then, revolve the telescope slowly on its vertical axis and notice if the point sighted is cut exactly the same by the cross-hair throughout its entire length. If any deviation is discernible, it should be corrected by carefully rotating the cross-hairs in a direction opposite to that in which it appears they should be rotated, until the horizontal cross-hair will cut the point exactly the same throughout its length when the telescope is revolved slowly on the vertical axis of the instrument. This test should always be made before performing the first adjustment, and it is well to again apply the test after this adjustment has been made.

In some leveling instruments this correction for the deviation of the cross-hair can be accomplished by merely loosening the pins that fasten the clips and turning the telescope in the wyes until the cross-hair is in the desired position, then securing it in that position by tightening the pins in the clips. Some leveling instruments, however, have a device by which the telescope is always secured in exactly the same position with respect to the wyes when the clips are closed and fastened. The horizontal cross-hair is placed truly parallel to the telescope's plane of rotation by the instrument maker, and in such an instrument will require no attention with regard to this condition except an occasional test as above, to see that the position of the cross-hair with respect to the axis of revolution has not become disturbed. If the horizontal cross-hair is found to deviate

from the point sighted when the telescope is turned slowly on its vertical axis, loosen the capstan-headed screws that control the cross-hairs, and by the pressure of the hand against the heads of the screws, or by tapping them lightly, rotate the cross-hairs very carefully in the direction opposite to that in which it appears they should be rotated until the horizontal cross-hair cuts the point exactly the same throughout its length. Then tighten the screws sufficiently to bring them to a firm bearing without straining.

If the horizontal cross-hair is parallel to the telescope's horizontal plane of revolution, it will be exactly horizontal when the instrument is leveled up accurately, and since the horizontal and vertical cross-hairs are at right angles to each other, the vertical cross-hair will also be exactly vertical. This condition can, therefore, be tested by sighting at a plumb-line suspended at a suitable distance and observing if the vertical cross-hair coincides exactly with the plumb-line. The vertical edge of a building may be sighted at instead of the plumb-line, provided it is known to be truly vertical.

THE DUMPY LEVEL

17. Description. — An engineers' **dumpy level** of American make is shown in Fig. 4. In its general construction it is similar to the wye level. The essential difference is that in the dumpy level the telescope $A B$ is attached rigidly to the horizontal level bar $C D$, and the level tube $E F$ is attached to the level bar and is adjustable at one end and in a vertical direction only, while the other end is attached permanently by a hinge. Since the telescope cannot be revolved in its supports, there is no necessity for a lateral adjustment of the level tube. The object glass, or objective, is at o; it is not shown in the view, being covered by the sunshade d, which projects around and beyond it. The objective is focused by the milled thumb wheel b, by which it is moved out or in, the same as in a wye level. The cross-hairs are controlled by the four capstan-headed adjusting screws shown at x, the same as in the wye level. In an

erecting telescope, the cross-hairs must be at a sufficient distance from the eye end to permit of four lenses being used in the eyepiece, as in the figure. The telescopes of dumpy levels are quite commonly of the inverting type, however, and in such a telescope the eyepiece contains only two lenses, and is consequently much shorter, so that the capstan-headed adjusting screws for the cross-hairs are very

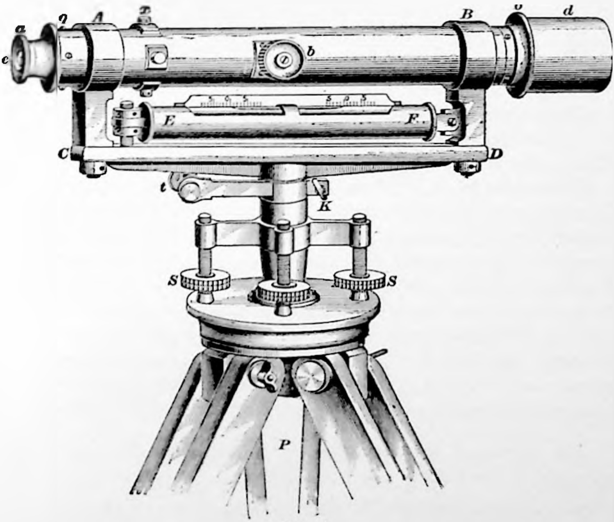

FIG. 4

near the eye end of the telescope, approximately in the position of the small screws shown at q in the figure near the eye end of the telescope tube. For the level shown in the figure, the clamp to the center is at K and the tangent screw at t; however, these are quite commonly omitted from dumpy levels. The center, leveling screws, and tripod are the same as in a wye level and will require no special description.

18. Adjustments.—There are two adjustments of the dumpy level, namely:

1. To make the axis of the level tube perpendicular to the vertical axis of rotation, so that the bubble will stand in the center of its scale when the telescope is revolved.

2. To make the line of sight parallel to the axis of the level tube, so that the line of sight will be horizontal when the level bubble stands in the center of its scale.

19. First Adjustment.—*To make the axis of the level tube perpendicular to the vertical axis of rotation:*

This adjustment having been made, the bubble will stand in the center of its scale when the telescope is revolved. This is the same as the third adjustment of the wye level.

Plant the tripod firmly and level up the instrument, using both pairs of leveling screws. With the telescope over one pair of leveling screws, center the bubble accurately, then reverse the telescope end for end over the same pair of leveling screws. If the bubble runs toward either end, bring it half way back by means of the capstan nuts at one end of the level tube, then center it with the leveling screws. Repeat the operation over each pair of leveling screws alternately until the bubble will remain centered perfectly throughout a complete revolution of the telescope.

20. Second Adjustment.—*To make the line of sight parallel to the axis of the level tube:*

When this condition is attained the line of sight will be horizontal when the level bubble is centered. This method performs directly what is accomplished indirectly by the first and second adjustments of the wye level. It is sometimes called the **direct method** of adjusting the line of sight parallel to the axis of the level tube, but is very commonly known as the **peg method** of adjustment.

Drive two pegs in the ground several hundred feet apart and set up the instrument either midway between them or at the same distance from each. The distances need not

be measured with great exactness. With the bubble centered accurately, sight on the rod* and note the reading on each peg. The difference in the readings of the rod on the two pegs is the true difference in the elevations of the pegs, whether the instrument is in adjustment or not. Next, set up the instrument over one peg with its center at a distance from the peg horizontally equal to about one-half the length of the telescope; bring the level bubble to the center of the tube, and with the leveling rod measure the exact height of the intersection of the cross-hairs above the peg. To determine this height on the rod, hold the graduated face of the rod about a half inch from the eye end of the telescope, and by looking into the object end of the telescope bring the point of a pencil in the center of the small field of view on the face of the rod. Or it can be determined closely by measuring to the middle of one of the horizontal adjusting screws to the cross-hairs. Set the target at this height, plus or minus the difference in the elevations of the pegs, according as the rod reading on the distant peg was more or less than on the peg over which the instrument is set; then direct the telescope toward the rod held on the distant peg and adjust the cross-hairs so that the horizontal cross-hair will exactly bisect the target when the level bubble stands in the middle of its scale.

If the ground is sufficiently level, it is more convenient to drive both pegs to the same elevation, as determined by the level when set up between them. In order to do this, drive one peg and read the rod held on that peg; then, with the target clamped at this reading, drive the other peg until the cross-hairs just bisect the target. In this case, after ascertaining the height of the cross-hairs above the peg over which the level is afterwards set, the target is clamped at that height.

Instead of setting up directly over one peg, it may sometimes be more convenient to set up near it, say from 10 to 20 feet from it, and determine the elevation of the cross-hairs

* Leveling rods are described in Arts. **29** to **31**, inclusive.

above it by sighting at the rod held on the peg. After sighting at the rod held on the distant peg and nearly completing the adjustment, sight again at the rod held on the nearest peg and repeat the operation until the adjustment is found to be perfect.

PROPERTIES OF LEVELING INSTRUMENTS

21. Sensitiveness. — The terms **sensitiveness, delicacy,** and **sensibility,** as applied to a level bubble, are employed synonymously to denote the quick-acting properties or degree of activity of the bubble as relating to the rapidity and amount of its movement. It is measured by the amount of its longitudinal movement for a given amount of angular movement of the telescope in a vertical plane, or, what is the same thing, by the vertical angle through which the telescope must move in order to cause a given amount of longitudinal movement in the bubble.

The sensitiveness of a level bubble depends directly on the radius of curvature of the level tube, and also to some extent on the length of the air bubble. The greater the radius of curvature of the bubble, or the less its rate of curvature, the more quickly and accurately will it act, while a long air bubble will settle more quickly and accurately than a short one. The movement of the bubble is measured by the graduated scale that is either marked on the top of the glass level tube or placed on a strip of silvered brass directly over it. Bubble scales are graduated in the same manner on each half length of the scale, the graduations being symmetrical with respect to the center and commonly reading in both directions from zero at the center. The most common value of each division is 0.1 of an inch, though other values are used.

22. Angular Value of Bubble Scale.—The sensitiveness of a level bubble is indicated by the angular value of a division on the bubble scale, that is, by the angle through which the line of sight must move in order to cause the

bubble to move over one division of the scale. The sensitiveness of the bubble is inversely as the size of this angle; the smaller the angle, the more sensitive the bubble. The value of this angle can be determined as follows:

On level ground measure off a distance of from about 200 to 500 feet and preferably a multiple of 100 feet. Set up the level at one end of this measured line, and direct the telescope toward a leveling rod held vertically at the other end. The level should be in such position that the point marking the end of the line will be directly under the center of the instrument, as shown by the plumb-line. By means of the leveling screws, center the bubble accurately, being sure that both ends of the bubble read exactly the same, and read the height of the horizontal cross-hair on the rod. Record this height and the reading of the center of the bubble, which, of course, is zero if the bubble is centered accurately. Then, by means of the leveling screws, bring the bubble near one end of its tube, and read both ends of the bubble; also read the height of the cross-hairs on the rod, recording all readings. From the readings of the ends of the bubble find the reading of its center. If the bubble scale reads from the middle outward toward both ends, as is usual, one-half the difference of the readings of the ends of the bubble will be the reading of its center—that is, will be the distance of the center of the bubble from the center of the scale, expressed in divisions of the scale. Fractional parts of a division should be expressed as decimals. It is necessary to read both ends of the bubble in order to determine accurately the reading of its center. The center reading of the bubble in the two positions having thus been obtained, the angular value of one division of the bubble scale can be determined from the following considerations:

In Fig. 5, let $b\,b'$, as shown in full lines, represent the level tube when centered, and let r be the point where the line of sight, which for this position is parallel to the bubble axis, strikes the rod. The line of sight $c\,r$ is parallel to the bubble axis and so very close to it that it may be considered to coincide with it, and, consequently, to be tangent to the

level tube at its center c, which is the highest point of the tube in this position. Now suppose that the instrument is tipped slightly so that the level tube $b\,b'$ will take the position indicated by the dotted lines. The line of sight, which is tangent to the level tube at its center, will now be the inclined line $c\,r'$, and the bubble will have moved so that its

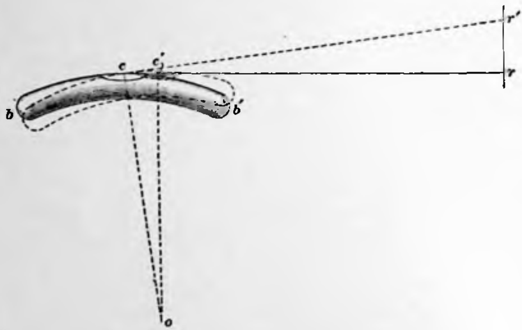

Fig. 5

center will be at c', the highest point of the level tube in its new position. Though the center of the bubble c' will now be slightly above the horizontal line $c\,r$, it will be so very near to it (much closer than indicated in the figure, which is exaggerated) that this line may still be considered tangent to the bubble at its center c'.

Let h denote the distance $r\,r'$, or the difference in the readings of the rod in the two positions; let c_n denote the distance $c\,c'$ between the two corresponding positions of the center of the bubble, expressed in divisions of the bubble scale, and let d denote the length of sight, or distance of the rod from the instrument. Since the angle $r\,c\,r'$ is very small, we may without appreciable error consider any part of it to be proportional to the corresponding part of its tangent. Consequently, the length h_1 on the rod corresponding to a movement of the bubble over one division of the scale

may be taken as equal to $\frac{h}{e_n}$. Let a_1 denote the corresponding angle, or the angular value of one division of the bubble scale; then

$$a_1 = \frac{rcr'}{e_n}$$

and

$$\tan a_1 = \frac{h_1}{d} = \frac{h}{e_n d}, \qquad (1)$$

which is the value of the tangent of the angle through which the line of sight moves when the bubble moves over one division of its scale.

Let n_s denote the number of seconds in a_1, or the angular value of one division of the bubble scale expressed in seconds; then, since the angle is very small,

$$n_s = \frac{\tan a_1}{\tan 1''} = \frac{h}{e_n d \tan 1''} \qquad (2)$$

It is expeditious to use logarithms in obtaining the value of n_s. Log tan 1″ = $\bar{6}.68557$.

In determining the value of a_1, the observation should be repeated several times, observing the movement of the bubble in different parts of the tube. After observing with the bubble in two positions, as just described, center the bubble again accurately and observe the height of the cross-hair on the rod; it should be the same as before. Then, bring the bubble near the end of the tube opposite to its former position, read both ends of the bubble, observe the height of the cross-hair on the rod, and determine the center reading of the bubble as before. It will be well to observe next the readings of the bubble in two positions on opposite sides of the center, then in two positions on the same side of the center, etc., and the corresponding readings of the rod in each case. If when the bubble is read in the two positions its center is on *opposite* sides of the center of the tube, e_n will be the *sum* of the two center readings of the bubble, but if its center is on the *same* side of the center of the tube in both readings, e_n will be the *difference* of the two center readings.

For each pair of observations, the value of h_1, or the length on the rod corresponding to the movement of the bubble over one division of its scale, should be found. These should not vary materially, and their average may be taken as the true value of h_1 to be substituted in formulas **1** or **2.** It is more expeditious to average the values of h_1 and then calculate the value of a_1 from this average value of h_1 than to complete the calculations for each pair of observations and average the results. The value of h_1 may also be observed directly by noting the reading of the rod with the center of the level bubble at one of the division marks of its scale; then by means of the leveling screws tip the instrument just sufficiently to cause the bubble to move over one division of its scale, so that its center will be exactly at the next division mark, and again note the reading of the rod; the difference between the two readings of the rod will be the value of h_1. In observations for determining the angular value of the bubble scale, the leveling rod should always be read to thousandths of a foot.

23. Radius of Curvature of Level Tube. — The radius of curvature of the level tube can be determined if

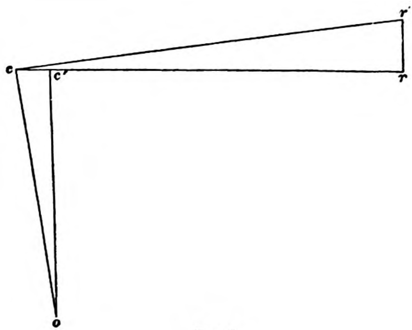

FIG. 6

desired. Since, when the level tube is in the position shown by the dotted lines in Fig. 5, the line of sight cr' is tangent

to the level tube at its center, and the line $c\,o$, perpendicular to this tangent, is a radius of the level tube at its center. Also, since $c'\,r$ is tangent to the center of the bubble, the line $c'\,o$, perpendicular to $c'\,r$, is a radius of the level tube at the center of the bubble. These two tangents and radii are shown in Fig. 6, the bubble being omitted for clearness. Although a line joining c and c' would really be a chord, yet, since the angle $c\,o\,c' = r\,c\,r'$ is very small, $o\,c\,c'$ and $c\,r\,r'$ may be considered to be similar triangles, and we can write the proportion $o\,c' : c\,r = c\,c' : r\,r'$. If we let b represent the length of one division of the bubble scale, expressed as the fractional part of a foot, the distance $c\,c'$ will be equal to $e_n\,b$, and if we denote the radius $c'\,o$ by r, and the distances $c\,r$ and $r\,r'$ by d and h, respectively, as before, and substitute these values in this proportion, it becomes

$$\frac{r}{d} = \frac{e_n\,b}{h}$$

from which

$$r = \frac{e_n\,b\,d}{h} \qquad (3)$$

The determination of the radius of the level tube, however, though of some interest, is not a matter of practical value to the engineer.

EXAMPLE.—The reading on a rod held at a distance of 100 feet from the instrument is 5.078 feet. After causing the level bubble to move over four divisions of the scale, the reading is 5.042 feet. What is the angular value in seconds of one division of the bubble scale?

SOLUTION.—The difference in the readings of the rod in the two positions is $5.078 - 5.042 = .036$ foot. To apply formula **2**, we have $h = .036$, $e_n = 4$, and $d = 100$. Thus

$$n_s = \frac{.036}{4 \times 100 \tan 1''}$$

By the use of logarithms this equation is solved in the following manner:

$$\begin{aligned} \log 4 &= 0.60206 \\ \log 100 &= 2.00000 \\ \log \tan 1'' &= \bar{6}.68557 \\ \hline & \bar{3}.28763 \end{aligned}$$

$$\log .036 = \bar{2}.55630$$
$$\log (4\,d \tan 1'') = \bar{3}.28763$$
$$\log n_s = 1.26867$$

The number corresponding to the logarithm 1.26867 is 18.564; therefore, $n_s = 18.6$ sec. Ans.

24. Magnifying Power of Telescope.—The **magnifying power** of a telescope is the measure of its capacity to enlarge the apparent size of an object. In a general way this is indicated by the apparent nearness of an object viewed through the telescope as compared with its position as viewed with the naked eye; the higher the magnifying power of the telescope, the nearer will the object appear to be when viewed through it. It is commonly expressed, however, by the number of times greater any linear dimension of an object appears when viewed through the telescope than when viewed with the naked eye, and is commonly spoken of as the number of **diameters** of magnifying power.

The magnifying power of a telescope can be determined by dividing the focal length of the object glass by the focal length of the eyepiece, considering the latter as a single lens. The latter focal length is difficult to determine, however, and the same result can be obtained by dividing the diameter of the clear aperture of the object glass by the diameter of the small disk of light at the eye end. This small disk of light can be seen at the small opening in the eye end when the telescope is focused on a very distant object and then pointed toward the sky, and the eye is drawn back a few inches from the eye end. Or if, when the telescope is in the same position, a card is held close to the end of the eyepiece, a small circle of light will be thrown on the card, and if the card is held at the proper distance from the eyepiece—that is, at its exact focus—the circle of light will be sharply defined. The diameter of the clear aperture of the object glass should be the same as its diameter between the opposite inner edges of the metal rim that encircles it; but this is not always the case, as in some telescopes a portion of the light is cut off by a diaphragm

inserted within the telescope tube. This can be ascertained by noticing whether the point of a pencil placed at the outer edge of the object glass is visible in the small disk of light just mentioned. If not, move it toward the center of the object glass until it is just visible; this point will be at the edge of the true or working aperture. Twice the distance of this point from the inner edge of the metal rim subtracted from the diameter of the object glass between the opposite inner edges of the metal rim, will be the diameter of the true clear aperture of the object glass. Denoting this diameter by D, and the diameter of the small disk of light at the eye end by d, the magnifying power m will be given by the formula

$$m = \frac{D}{d} \qquad (4)$$

The chief difficulty in determining the magnifying power of a telescope is in accurately measuring the diameter of the small disk of light and the diameter of the true or working aperture of the object glass. This can be accomplished in the following manner:*

The size of the clear aperture of the object glass is regulated by placing over it, and concentric with it, a metal diaphragm having a perfectly circular opening of known diameter, which should be about ¼ inch less than the diameter of the object glass, then placing a piece of "solio" or other sensitized paper in the exact focus of the eyepiece and protecting it from all light except what passes through the telescope. With the telescope turned toward a clear sky, the paper should be exposed about an hour, when on removal it will show a dark-colored dot with sharply defined edges, whose diameter can be measured accurately if a magnifying glass is used.

The magnifying power of a telescope can also be determined approximately in the following manner: Cut out a white card exactly 0.1 foot in width and attach it to a leveling

* Described in a communication from Mr. William Nelson in *Engineering News*, March 7, 1901.

rod so as to just cover one of the tenth divisions; set up the rod at a distance of, say, 25 feet, direct the telescope toward the rod, and focus it perfectly. Then, by observing the rod with both eyes, but with one eye looking through the telescope, note the number of divisions on the rod, as viewed with the naked eye, that appear to be covered by the white card, as viewed through the telescope. This will be, approximately, the number of diameters magnifying power of the telescope. It is well to repeat the observation with the other eye looking through the telescope.

25. Definition.—If the magnifying power of a telescope were the only condition affecting the distinctness with which an object can be viewed through it, then it would be desirable to have the magnifying power as high as possible. But the magnifying power of a telescope must in a general way be proportional to the focal length and size of its objective. If this is well proportioned, the object will show clear and sharply defined, but if the magnifying power is too high proportionately, the object viewed will appear dim and indistinct. The term **definition** is employed to indicate the clearness and sharpness of outline with which objects can be seen through a telescope. Since the essential conditions of lightness and portability limit the size and weight of the telescope in a surveying instrument, it is evident that the same conditions limit the power of the telescope that can have good definition. In a general way and to some extent, power and definition are opposed; that is to say, for the same size, a reasonably low-power telescope will have better definition than a high-power telescope, the excellence of the optical construction being the same in each case. It is well to notice here that for telescopes of the same length the inverting telescope gives considerably higher magnifying power, better definition, better light, and a much more brilliant image than the erecting telescope.

26. Length of Telescope.—The lengths of telescopes on engineers' levels vary commonly from 12 to 22 inches, according to the degree of accuracy and magnifying power

required, the most common length being 18 inches. A well-constructed erecting telescope of this length may have a magnifying power of 30 diameters, and an inverting telescope of the same length may have a much higher power, say 40 diameters. A power of 30 diameters is sufficiently powerful for most purposes, and where greater magnifying power is desired it is better to use an inverting telescope than to increase the length of the telescope, especially where lightness and portability are desirable conditions.

27. Care of the Level. — The level should not be exposed to the burning rays of the sun, to rapid changes of temperature, to unequal temperatures on its different parts, or to dust, and should not be used in rainy weather when possible to avoid it. Changes of temperature disturb the adjustments, dust is injurious to the bearings and the lenses, while moisture obscures the lenses and is otherwise injurious to the instrument. When it is impossible to avoid working in the rain, wipe the lenses frequently and carefully with a soft linen cloth, and after returning to the office or camp, wipe very carefully and thoroughly, finishing with a piece of dry chamois skin, and place in a moderately warm, dry place, so that every particle of moisture will be removed. When carrying a level on its tripod in open country, the spindle should always be clamped slightly to prevent the wearing of the centers by swinging, and the instrument should be carried with the object end of the telescope down. When working in a wooded country where underbrush is dense, the level should be carried with the spindle unclamped, so that the telescope will turn freely on the spindle and yield readily to any pressure. A blow that would inflict no injury upon an unclamped instrument might seriously damage one while clamped rigidly.

EXAMPLES FOR PRACTICE

1. A leveling rod is held 300 feet from the instrument; the reading is 7.824 feet, and after causing the level bubble to move over two divisions of the scale, the reading is 7.856 feet. What is the angular value in seconds of one division of the bubble scale? Ans. 11 sec.

2. The reading on a rod held at a distance of 400 feet from the instrument is 10.072 feet. After causing the level bubble to move over three divisions of the scale, the reading is 10.165 feet. What is the angular value in seconds of one division of the bubble scale.

Ans. 16 sec.

3. If the diameter of the clear aperture of the object glass of a telescope is 1.25 inches, and the diameter of the small disk of light at the eye end is .05 inch, what is the magnifying power of the telescope?

Ans. 25

4. If the diameter of the clear aperture of the object glass of a telescope is 1.5 inches, and the diameter of the small disk of light at the eye end is .05 inch, what is the magnifying power of the telescope?

Ans. 30

LEVELING RODS

28. The Principle of Direct Leveling.—When an adjusted level is set up and leveled up, its line of sight is horizontal, and, being at right angles to the vertical axis of the instrument, rotates about this axis in a horizontal plane, called the *plane of the instrument*. The elevation of this plane is the elevation of the instrument, and every line lying in the plane is a level line having the same elevation as the instrument. This elevation may be assumed arbitrarily or may be determined from the known elevation of some other point. In the latter case it is necessary to measure the vertical distance that the plane of the instrument, as defined by the line of sight, is above or below the point of known elevation. This vertical distance from the horizontal line of sight up or down to the point of known elevation can be measured by whatever instrument is most convenient, but a wooden rod, called a **leveling rod,** is usually employed for this purpose.

29. Different Kinds of Leveling Rods.—In general, a leveling rod is a graduated wooden rod. Two classes of leveling rods are in common use, namely: (1) Rods on which the graduations are sufficiently large and distinct to be read directly through the telescope of the instrument without any other aid; such rods are called **self-reading**

rods, or **speaking rods**. (2) Rods on which the graduations are small and not sufficiently distinct to be read with the instrument at any considerable distance, the readings

FIG. 7

being given by a sliding metal plate called a target that is marked conspicuously for sighting; such rods are called **target rods**. There are several different kinds of rods belonging to both these classes, differing in constructive details, but not in principle. One kind of rod in each of these classes will be described, this being in each case a kind that is well known and extensively used.

30. The Self-Reading Rod.—The form of self-reading rod in most general use is called a **Philadelphia rod.** A rod of this kind is shown in Figs. 7 and 8. It is in two sections, held together with brass clamps, shown at *A* and *B*, one section sliding over the other. The lower end of the front or lower section is covered by a brass plate bent so as to form a protecting shoe. The Philadelphia rod is made in different patterns, varying somewhat in length and in the graduations, but all embodying substantially the same construction. One pattern measures 7 feet when closed and slides out to 13 feet, another measures

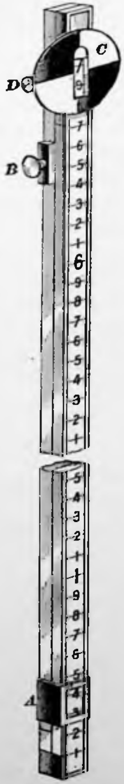

FIG. 8

the same closed, but slides out to only 12 feet, and yet another measures 6.5 feet closed and slides out to 12 feet.

Both sections are graduated on the front face for reading directly as a self-reading rod, and the rear section is also graduated on the rear face, reading downward, so as to give the reading of the target when the rod is extended. All Philadelphia rods are graduated to feet and tenths, and some are graduated to hundredths of a foot, as shown in Fig. 7. The feet are marked in large red figures, half above and half below the division marks; tenths of a foot are marked in black figures from 1 to 9 in a similar manner; when the face of the rod is graduated to hundredths of a foot, it is marked by lines .01 of a foot in width, alternating white and black, and extending about one-third the way across the face of the rod. Since the graduations and figures on this rod are plain enough to be read from the instrument, it is essentially a self-reading rod, but it also has a target, shown in each figure at *C*, which is either circular or elliptical, and is divided into quarters, alternating red and white. The dividing lines of the colors are so arranged that one is horizontal and the other vertical when the rod is in a vertical position. The target is fastened to a collar that slides up and down the rod, and is fitted with a screw *D* that clamps it at any desired point. The index point of the target, or point from which it reads, is always the horizontal dividing line of the colors. There is an opening in the face of the target rather more than 0.1 foot in length, through which the graduations on the face of the rod can be read. On one side of and within this opening a scale is marked along the edge of the target, which is beveled or chamfered to a thin edge, so that the scale will be close to the graduated face of the rod.

When the face of the rod is divided only to tenths or half-tenths of a foot, this scale is made just 0.1 foot long and is divided into 10 or 20 equal parts—that is, to hundredths or half-hundredths of a foot—so that by means of the target hundredths or half-hundredths can be read on the rod and thousandths can be estimated with some degree of accuracy.

In some Philadelphia rods in which the face of the rod is divided to hundredths of a foot, the scale on the target is

0.1 foot long and is divided into 20 equal parts, reading directly to half-hundredths—that is, to the two-hundredth part of a foot—and giving thousandths approximately by estimation, as just described, while in other rods of this class the scale on the target is a vernier scale reading to thousandths of a foot, as described in the following article. The arrangement of the target vernier differs slightly in different rods; in some cases it reads upwards from zero at the line dividing the colors, and in others it is entirely below this line, as shown in Fig. 10.


Fig. 9

31. The Target Rod.—The form of target rod that is used most extensively in this country is shown in Fig. 9. It is known as the **New York rod.** This rod is quite similar to the Philadelphia rod in its general form and construction, being in two parts that slide on each other by means of a tongue and groove, but is somewhat lighter and differs in the character of its graduations, which, instead of being painted as on the Philadelphia rod, are stamped and blackened much the same as the graduations on an ordinary scale or rule. The tenths figures are also stamped and blackened; the feet figures are larger, slightly depressed into the surface of the rod, and painted red. All rods of this kind are graduated to hundredths of a foot and read to thousandths by a vernier; in the regular pattern the graduations on the face of the rod always extend to 6.5 feet, and below this the rod can be read to thousandths of a foot by means of the scale on the target, which is a vernier scale. A space on the vernier equal to 9 hundredths of a foot is divided into 10 equal parts, and, consequently, each division on the vernier is equal to nine-tenths of a hundredth division on the rod, or one-thousandth of a

foot shorter than each hundredth division of the rod. The rod is read to hundredths of a foot by the first division mark on its face below the line dividing the colors on the target, and to thousandths of a foot by that division mark on the vernier that coincides with some division mark on the rod. Fig. 10 represents a portion of a New York rod, showing the target and target vernier. In the position shown, the target reads 4 feet, 3 tenths, 9 hundredths, and 6 thousandths, or 4.396 feet. Above 6.5 feet the target is set and clamped at 6.5 feet and the long rod slides out to the required reading, up to a reading of 12 feet; the rod is then read by means of graduations reading downward on the side of the rear part of the rod, while a vernier on the side of the front part reads to thousandths of a foot, as shown in Fig. 9. Some rods of this kind

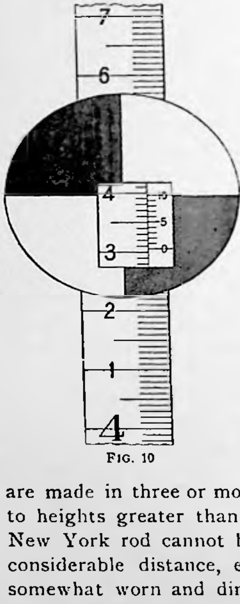

FIG. 10

are made in three or more sliding pieces and give readings to heights greater than 12 feet. The graduations on the New York rod cannot be read with the telescope at any considerable distance, especially after they have become somewhat worn and dim, but the target must usually be set for every reading of this rod. For this reason it is not as expeditious to use on some kinds of work as the Philadelphia rod, but it is usually preferred where a high degree of accuracy is required.

32. With either of the rods just described, readings under 7.0 feet, or 6.5 feet, as the case may be, are taken with the **short rod**—that is, with the two sections of the rod closed—and if the rod is at such a distance that it cannot be

read directly from the instrument with accuracy, the target is moved up or down until its horizontal color line coincides with the horizontal cross-hair of the telescope. When readings of more than 7.0 (or 6.5) feet are taken, the target is clamped at 7.0 (or 6.5) feet, the clamp, which is at *B* on the Philadelphia rod and at *A* on the New York rod, is loosened and the sliding rear section is raised upwards until the horizontal line of the target coincides with the horizontal cross-hair of the telescope. The rod is then clamped, and can be read to thousandths by means of a vernier which, on the Philadelphia rod, is attached to the collar at *B* and, on the New York rod, is on the side, as shown in Fig. 9. When thus extended the rod is called a **long rod,** or **high rod.**

33. In **setting the target** when using the short rod, the leveler should read the rod as closely as he can with the telescope, calling the reading to the rodman, who sets the target at the reading called and holds the rod up for a check-reading. In most cases the leveler's reading will be the correct one. More mistakes are made in reading the number of feet than in reading the smaller divisions. The leveler by first calling the reading to the rodman will be certain to prevent such an error, as it would at once be detected in the check-reading. The rod should always be held truly vertical, or plumb, when it is being sighted at. An experienced rodman can hold a rod practically plumb, however, by balancing it as nearly as possible, and for all ordinary work his care is considered sufficient, though it is difficult for him to plumb the rod accurately in a heavy wind.

34. The **rod can be plumbed** in the direction across the line of sight by the leveler observing if it coincides with the vertical cross-hair of the instrument. If it does not, he indicates the direction in which it should be plumbed by raising his right hand over his head and moving it to the right, or raising his left hand and moving it to the left, as the case may be. Whether or not the rod is plumb in the

direction of the line of sight cannot be distinguished from the instrument, but the correct reading for the plumb-rod can be taken in the following manner: At the direction of the leveler, who waves his hand slowly above his head as a signal, the rodman slowly waves or tips the rod backward and forward in the direction of the line of sight, and the leveler takes the shortest reading of the rod as the correct reading. If the target is used, it must be set down until the index line or division line of the colors just reaches to, but does not rise above, the cross-hair of the telescope when the rod is waved.

35. Different devices are also employed for indicating when the rod is plumb. As stated above, the rod can be plumbed in a direction laterally to the line of sight by means of the vertical cross-hair of the instrument. On some rods a special form of target is used for indicating when the rod is plumb in the direction of the line of sight. The face of such a target, or the portion sighted at, instead of being composed of a single plane surface, as shown in Fig. 10, consists of two plane surfaces, one of which is either a few inches behind the other or makes a considerable angle with it, so arranged that when the rod is plumb—that is, at right angles to the line of sight—the line dividing the colors will appear straight and continuous, and when the rod is not at right angles to the line of sight, this line will appear either broken or crooked, according to the construction of the target. For work requiring great accuracy, such as bridge foundations, a rod level, which fits closely to the angle of the rod and carries two small spirit levels, is used to plumb it accurately.

36. The Rodman.—The man who carries the rod and holds it on the points whose elevations are to be taken is called a **rodman.** A good rodman is essential to accurate and rapid leveling. A man who is inattentive to the work, or averse to rapid movement, is not suitable for a rodman. In most localities, a line of levels of any considerable length

will have enough rough places in it—that is to say, places where abrupt and considerable changes in elevation occur—to retard progress, however diligent the level party may be. The laziness or carelessness of an individual should never be allowed to delay the progress of the party.

THE OPERATIONS OF LEVELING

37. Example in Direct Leveling. — The general principles and methods of direct leveling are illustrated in Fig. 11.

Let *A* be the starting point whose elevation is either known or assumed to be 20.00 feet. The instrument is set at *B*, leveled up, sighted to a rod held at *A*, and the target set. This sight being directed back toward the starting point, the rod reading is called a **backsight.** The reading of the rod, 8.42 feet, is the distance of the line of sight above the point *A*, as measured by the rod, and when added to the elevation of the point *A* gives the elevation of the line of sight, which is called the **height of instrument** and in the notes designated by the letters *H. I.* Thus, $20.00 + 8.42 = 28.42$, $= H.\ I.$ The instrument is now turned in the opposite direction and another point *C* is chosen, which must be below the line of sight. This point is called a **turning point,** and in the notes is designated by the letters *T. P.* A peg is driven at *C*, or a point of rock or other permanent object on which the rod can be held is taken for the turning point. The first rod reading on a turning point is a **foresight;** it is taken for the purpose of determining the elevation of the point, and when subtracted from the height of instrument, the remainder will be the elevation of the point. Suppose that the foresight rod reading on the point *C* is 1.20 feet. Since the height of instrument at *B* is 28.42, we have $28.42 - 1.20 = 27.22$ feet, which is the elevation of the point *C*. The rodman, remaining at the turning point *C*, is careful to keep the rod on the same point, while the leveler carries the instrument

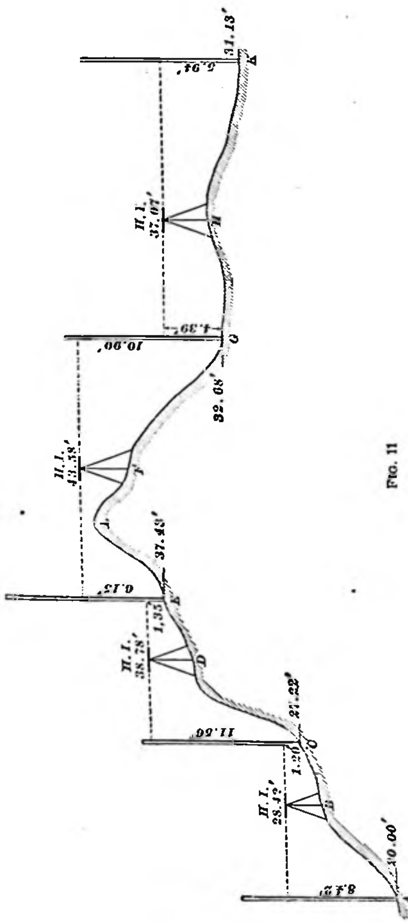

FIG. 11

forward and sets it up at D, which should be at such a height above C that when the instrument is leveled up the line of sight will cut the rod near the top. Suppose that the backsight to C gives a reading of 11.56 feet; this, added to 27.22 feet, the elevation of C, gives 38.78 feet for the height of the instrument at D. The rodman then goes forward to E, a point where the foresight reading is found to be 1.35 feet; this reading subtracted from 38.78, the $H. I.$ at D, gives 37.43 feet for the elevation of E. The level is then carried forward and set up at F, in a position high enough so that the line of sight will clear the hill at L. Suppose the backsight to be

6.15 feet, which added to 37.43 feet, the elevation of *E*, gives 43.58 feet as the *H. I.* at *F*. The rod held at *G* is found to give a foresight of 10.90 feet, which subtracted from 43.58, the *H. I.* at *F*, gives 32.68 feet for the elevation at *G*. Again moving the level to *H*, the backsight to *G* of 4.39 feet added to 32.68 feet, the elevation of *G*, gives 37.07 feet as the *H. I.* at *H*. Holding the rod at *K*, a foresight of 5.94 feet subtracted from 37.07 gives 31.13 feet for the elevation of the point *K*. The elevation of the starting point *A* is 20.00 feet; the elevation of the point *K*, as thus found, is 31.13 feet, and the difference in the elevations of *A* and *K* is 31.13 − 20.00 = 11.13 feet; that is, the point *K* is 11.13 feet higher than the point *A*.

The points *A*, *C*, *E*, *G*, and *K*, whose elevations with reference to one another have been determined, may or may not be in a straight line, that is, in the same vertical plane; it will make no difference in the determination of their elevations. Such a series of elevations, however, is called a **line of levels.** It is evident that the elevations that can be determined by a line of levels need not be restricted to the turning points. At each setting of the instrument, foresight readings can be taken on any number of different points for the purpose of determining their elevations, before taking a foresight on the turning point preparatory to moving the instrument. Points whose elevations are thus determined are called **intermediate points.** The intermediate points on which rod readings are taken are usually the regular stations of the survey.

38. Turning Points. — These are the points mentioned in the preceding article, on which foresights are taken, and then, after the instrument has been moved forward, backsights are also taken. As each turning point is thus sighted to by the instrument in two adjacent positions, the turning points, therefore, serve as connecting points between the different settings of the instrument and render the line of levels continuous. Since the foresight on a turning point is to be subtracted from the preceding height of instrument in order to obtain the elevation of the turning point, it is considered as a minus (−) reading, and as each

backsight is to be added to the elevation of the preceding turning point in order to obtain the height of instrument, it is considered as a plus (+) reading. The rodman should make a peg of hard wood, about 9 inches in length and 1 inch in diameter, sharpened at one end and rounded at the other end, on which the rod is held for the turning point. For driving the peg he should have a light hatchet, which is carried conveniently in a leather scabbard or merely in a belt. The position for a turning point having been determined, the rodman drives the peg firmly in the ground and holds the rod on it. After the instrument is moved forward, set up, and a backsight taken, the peg can usually be pulled up and carried in the pocket until required for another turning point. An iron pin, having a ring in it for carrying, is sometimes used by the rodman for turning points. Each pair of turning points should be taken at about equal distances from the instrument, in order to equalize, and thus eliminate, any small errors due to imperfect adjustment.

39. Bench Marks.—A permanent point whose elevation is determined in running a line of levels, and recorded for reference, is called a **bench mark**. Bench marks should be established at frequent intervals, say from 1,000 to 2,000 feet apart, depending on the character of the surface and purpose for which the survey is made. Any well-defined and easily identified point on a permanent object, such as the door sill or water-table of a building, the projecting root of a tree, or a point on a large rock, will serve for a bench mark. Where trees are used, the point for the bench mark is cut on a projecting root. A tree with a large exposed root is chosen, the bench mark is cut on the root in the form of a pyramid, a tack is driven into the apex, and the rod held on it. The tree is blazed smooth and the letters *B. M.*, together with the elevation of

FIG. 12

the bench mark, are written in the blaze with red chalk. A bench mark of this kind is shown in Fig. 12; the point A is the bench mark whose elevation is determined and written in the blaze at B. The elevation of each bench mark is recorded in the field notes, together with a sufficient description to unmistakably identify it. For this purpose the abbreviation $B. M.$ is always used instead of the words *bench mark*.

The rod is always read more closely on bench marks and turning points than on the intermediate points or stations. This is because an error in the rod reading on a turning point will affect all subsequent elevations in the line of levels, and an error in the rod reading on a bench mark will affect all later levels that may be referred to the bench mark as a starting point, whereas an error in the rod reading on an intermediate point will affect the elevation of that point only. Consequently, the rod is usually read on an intermediate point only as closely as it is desired to determine the elevation of that point. For grading earth roadways and work of a similar character, the rod is usually read to the nearest tenth of a foot on intermediate points and to the nearest hundredth of a foot on turning points and bench marks, while for work requiring a higher degree of accuracy, such as the surface of a finished pavement, the rod is usually read to the nearest hundredth of a foot on intermediate points and to the nearest thousandth of a foot on turning points and bench marks.

40. Surface of Reference: Reference Base, or Datum.—It has been stated that the starting point A, Fig. 11, is either known or assumed to have an elevation of 20.00 feet, and that from this the elevations of the other points C, E, G, and K are determined. This elevation was assigned to the starting point merely for the purpose of illustration; an elevation of 25.00, 50.00, 76.48, or any other number of feet, could as well have been taken. This elevation, whatever it is, is merely the distance of the starting point above an imaginary level surface, chosen as a surface of reference, whose elevation is assumed to be zero and to which are referred the elevations of all the other points for

which the levels are taken. The elevation of the starting point above any level surface being known or assumed, the elevations of all the other points connected by a system of levels can be stated or compared conveniently by means of their elevations above the same level surface, or above a line or series of lines lying in this surface, as determined by the method already described. Such an imaginary surface considered as a surface of reference is commonly called the **reference base, or datum**; though really a level surface, and not a plane surface, is not uncommonly spoken of as the **datum plane, plane of reference,** or **plane of zero elevation.** If, instead of the surface, the line or series of lines lying in the surface directly below the line of levels is considered as the base or reference, it is called the **datum line, base line,** or **line of zero elevation.**

The actual position of the surface of reference is a matter of no importance, except that, in order to avoid negative elevations, it should be lower than any point in the line of levels; that is, the elevation of the starting point should be assumed high enough so that every point whose elevation is taken in the line of levels will be above the surface of zero elevation. Any convenient elevation that will fulfil this condition may be assumed for the starting point. It is customary, however, to assume the elevation at some multiple of 100 feet; the estimated amount of variation in the elevations of the surface over which the levels are to be run determines what multiple of 100 feet is assumed. In localities near the seashore, if the elevations are to be established permanently, or are for work of a permanent character, as, for example, those that relate to public improvements in seaside cities and towns, the elevation of mean low tide is commonly taken as the base of reference. This practice is gradually being extended inland; it is to be highly commended, since its tendency is to establish a uniform base of reference over a considerable area.

41. Signals and Calls.—When taking levels, it is necessary for the leveler and rodman to be in almost constant communication with each other. As a means of communication,

certain convenient signals are employed, though when the rod is read by the rodman, the readings must usually be called out to the leveler. It is important that the leveler and rodman understand perfectly the signals and calls, in order to avoid mistakes. When a target reading is taken, the target is set by the rodman in the proper position on the rod, according to signals given by the leveler. An upward movement or raising of the hand is the signal for raising the target; a downward movement or lowering of the hand is the signal for lowering it; a circle described by the hand is the signal for clamping the target, and a wave with both hands indicates that the target is set properly, or *all right*.

The rodman should then read the position of the target on the rod and call out the reading; he should call first the number of feet, or, if the reading is less than 1 foot, he should call the first figure *naught* (not *ought*); then, after pausing a moment, call the decimal part of the reading. Thus, if the rod is being read to hundredths only, the number 8.40 is read and called *eight, four, naught;* if 8.04, it is read and called *eight, naught, four.* If the rod is being read to thousandths, the number 8.401 is read and called *eight, four, naught, one;* if 8.410, it is read and called *eight, four, one, naught.*

When a self-reading rod is used, the rod is read on intermediate points directly by the leveler, whose signal for *all right* is usually a single outward wave of the hand. For such readings, the rodman should use the long rod—that is, keep the rod extended at ful' length, unless the country is exceedingly level.

The distinctness of a cal, is in no way proportional to the amount of noise it makes. A few days' practice will enable a rodman with moderate effort to call a reading so as to be heard distinctly at a distance of 500 feet. Should a high wind be blowing, the sights will be shorter, owing to the vibration of the instrument, so that the rodman's effort in calling will not usually have to be greatly increased by reason of the wind. The rod reading should always be recorded promptly by the leveler when called out by the rodman; after recording it the leveler should call it back to

the rodman, who should either answer *all right* or signal to that effect by a wave of the hand. When moving the instrument, it is a safe practice for the leveler to check the reading as he passes the rodman. In general, however, the leveler relies entirely on the accuracy of the rodman's readings. If he cannot be trusted, his place is usually supplied by one who can. As a check on the level notes, rodmen are sometimes required to keep notes of the rod readings on turning points; this affords a check of considerable value and is good practice for the rodman.

In taking levels on preliminary railroad surveys, self-reading rods are quite generally used, and it is not an uncommon practice for the leveler to read the rod through the telescope directly on the turning points, as well as on the intermediate stations, without the readings being checked by the target. The rodman has still plenty of opportunity for the exercise of judgment in the judicious selection of the positions for the turning points, as the rate of progress depends largely on the care shown in the selection of the turning points.

42. Length of Sights. — The most advantageous lengths of sights will depend on the optical properties of the instrument, the character of the surface of the country, the sensitiveness of the level bubble, and the degree of accuracy required. In the interest of accuracy, it is always best to have the sights of moderate length, avoiding either extremely long or short sights. For reasonably accurate work the sights should not usually exceed about 400 feet nor be less than about 100 feet. Where the country is level and time is of great importance, while only an ordinary degree of accuracy is required, quite long sights may be taken, though little if anything is to be gained by taking sights longer than 800 feet. Where the surface rises or falls rapidly, short sights become necessary, but extremely short sights can always be avoided by setting the instrument to one side of the line or by following a somewhat zigzag course.

43. Backsights and Foresights Should Balance.— The most valuable and reliable safeguard against errors in

leveling is obtained by equal backsights and foresights on turning points. They should usually be equal in pairs—that is to say, each pair of sights on turning points, one backsight and one foresight, should be of approximately equal lengths. Should any inequality of length occur in one pair of sights, it should be balanced up in the next pair, or as soon as possible. For example, should the foresight in one pair of sights be longer than the backsight, then in the next pair of sights the backsight should be made correspondingly longer than the foresight. The sights should be balanced as perfectly as possible between bench marks. It is not necessary to measure the lengths of the sights accurately; they can be determined closely enough by counting steps in walking. A man of ordinary stature, when walking naturally, will average about 40 steps in each 100 feet of distance, usually a somewhat less number on smooth and level ground, and a greater number where the ground is rough or sloping, either ascending or descending.

44. Rate of Progress.—The rate of progress in leveling will depend largely on the character of the country. Since the line of sight of a leveling instrument is always horizontal, it is evident that in running a line of levels, the more rise and fall encountered, the greater will be the amount of work required. Consequently, leveling progresses more slowly in rough and hilly country than in smooth and level country. Where the country is rough and broken, and the surface of the ground rises or falls rapidly, the sights are necessarily short, turning points are near together, and the instrument must be moved and set up often, so that the rate of progress will be limited chiefly by the dexterity of the leveler in setting up and leveling the instrument and the judgment of the rodman in selecting advantageous turning points. On the other hand, where the surface of the ground is comparatively level, long sights can be taken and the turning points can be at considerable distances apart, not uncommonly about 1,000 feet, so that the rate of progress will be limited chiefly by the ability of the rodman to move rapidly from point to point, to understand

and respond to the signals of the leveler in setting the target on long sights, and to call out the readings so as to be understood.

In a railroad survey, where the line is run by a transit party and the levels taken by a level party, the rate of progress is usually limited by the progress of the transit party, which must always precede the level party. If the country is open, so as to permit long sights to be taken with the transit and the chaining to be done with reasonable rapidity, but is rather hilly, so that short sights must be taken with the level, the level party will not usually be able to keep up with the transit party. But if the country is open and reasonably level, so that long sights can be taken with both instruments, both parties can usually make about the same rate of progress. If, however, the country is covered thickly with timber or underbrush, the rate of progress will be limited entirely by the progress of the transit party, and the level party will either work quite leisurely or have more or less idle time. A day's work for such parties will usually vary from 2 to 8 miles, according to the conditions encountered.

45. Check-Levels.—Before the elevations obtained by a line of levels are finally adopted as a basis for construction work, their accuracy should be verified by a second line of levels over the most important and permanent points whose elevations were taken by the former line. Levels for this purpose are called **check-levels,** or **test levels.** In running check-levels, the most common practice is to take the elevations of bench marks, important and permanent points only, with of course such turning points as may be necessary in order to cover the distance, but omitting nearly all the intermediate points or stations. Where variations of any considerable amount are found in the elevations as given by the two sets of levels, the levels should be run again over that portion of the line in which the variation occurs, in order to determine which of the former elevations is correct. Where the true elevation of a bench mark is found to differ from the elevation marked on it, the marking should be corrected. When running check-levels, the adjustments of the

instrument should be tested frequently, and the rodman should carry a rod level to insure the plumbing of the rod.

46. Water Checks.—When a line of levels follows along or near the shore of a body of water, the surface of the water can be used as a check on the levels. If the water has no current, as in the case of an ordinary lake or pond, its level for an ordinary space of time will remain unchanged and a close check will be given. In the case of a stream, the check is not so close, since the elevation of the water surface differs along different parts of the stream, but an approximate and reliable check is given by the fact that we know the elevation of the surface of the water must be less at a point farther down stream and greater at a point farther up stream, if no material change takes place in the condition of the stream between the times that the levels are taken. The sea, whose mean or general elevation is constant, is the base for all barometric leveling, and, as has been stated, at seaports mean low tide is taken as the base for direct leveling.

FORMS FOR LEVEL FIELD NOTES

47. Different Methods.—Forms and methods of keeping level field notes differ somewhat, according to the individual preferences of engineers. All the different methods, however, are based on one or the other of two conditions, namely, the rise or fall between the consecutive points on which rod readings are taken and the elevation of the line of sight above the base of reference.

48. The **method by rise and fall** is based on the first-mentioned condition and was formerly used extensively in this country. It consists in determining the difference in the rod readings on each of two consecutive stations, which difference is the rise or fall between them, and if a rise adding it to, or if a fall subtracting it from, the elevation of the first station, in order to determine the elevation of the next station. This method requires either two subtractions or a subtraction and an addition in determining the elevation of each station. Though formerly used

extensively, this method is now seldom if ever employed, and consequently will not be described in detail here.

49. The **method by height of instrument** is more expeditious than the method just described, and is now employed very generally in this country. The forms in which the notes for this method are recorded by different engineers differ slightly in detail, but all are based on the same condition, namely, the elevation of the line of sight above datum, commonly called the height of instrument. This is of course constant for each setting of the instrument; the elevation of each point on which the rod is read in any setting is determined by subtracting the rod reading from the height of instrument. The elevation of a turning point having been determined in this manner, and the instrument moved forward and set up in a new position, the new height of instrument is determined by adding the rod reading of the backsight on the turning point to the elevation of the turning point. The heights of instrument are thus connected and the line of levels made continuous. This method permits the simplest possible form of field notes and requires the least amount of computation.

Three different forms for keeping level notes according to this method will now be illustrated and explained. It will be noticed that in Forms Nos. 1 and 3 it is assumed that the rod is read on the stations to the nearest tenth of a foot only, and that the elevations of the surface are likewise carried to the nearest tenth of a foot only. This method is commonly used in leveling for grading and in preliminary work. In work requiring greater accuracy, such as paving, trackwork, building foundations, etc., the rod is read to the nearest hundredth of a foot, and the elevation of the respective stations carried to the second decimal place, as in Form No. 2. On turning points and bench marks the rod should be read more closely than is required for the intermediate stations. On the former it should in all cases be read as close as to the nearest hundredth of a foot, and in work requiring a high degree of accuracy it should be read to the nearest thousandth, as recorded in Form No. 2.

50. Form No. 1 shows what is probably the most common form of keeping field notes by the method of height of instrument. In the first column are recorded the stations, in the second the backsights, in the third the heights of instrument, in the fourth the foresights, and in the fifth the elevations, as is shown by the headings of the columns.

LEVEL NOTES: FORM NO. 1

Station	*Back-sight*	*Height of Instru-ment*	*Fore-sight*	*Eleva-tion*			*Remarks*
B. M.	*2.17*	*102.17*		*100.00*			*On root of white oak stump 60 ft. to left of Sta. 0*
0			*4.8*	*97.4*			
1			*6.2*	*96.0*			
2			*9.1*	*93.1*			
2 + 50			*8.2*	*94.0*			
3			*7.6*	*94.6*			
4			*7.4*	*94.8*			
T. P.	*4.58*		*8.54*	*93.63*			
4 + 60		*98.21*	*11.9*	*86.3*			*Spring Brook*
5			*0.5*	*97.7*			
T. P.	*10.32*		*2.67*	*95.54*			
5 + 75		*105.86*	*2.4*	*103.5*			
6			*2.1*	*103.8*			
7			*6.4*	*99.5*			
8			*7.7*	*98.2*			
9			*6.5*	*99.4*			
10			*8.7*	*97.2*			
T. P.	*2.44*		*10.17*	*95.69*			
11		*98.13*	*2.4*	*95.7*			
12			*7.2*	*90.9*			
13			*8.8*	*89.3*			
T. P.			*11.29*	*86.84*			

These are usually abbreviated to *Sta.* for station, *B. S.* for backsight, *H. I.* for height of instrument, *F. S.* for foresight, and *El.* for elevation, and occupy the first five columns of the left-hand page, usually leaving two columns that can be used for other purposes. In the column for

stations, bench marks are designated by the letters *B. M.* and turning points by the letters *T. P.* The right-hand page is for remarks, descriptions of bench marks, etc.

51. Form No. 2 shows another form for keeping level notes that is in quite general use among engineers. The notes shown in this form are for the same line of levels as

LEVEL NOTES: FORM NO. 2

Station	*Rod Reading*	*Height of Instrument*	*Elevation*				*Remarks*
B. M.	*+2.172*	*102.172*	*100.000*				*On root of white oak stump 60 ft. to left of Sta. 0*
0	*−4.75*		*97.42*				
1	*−6.14*		*96.03*				
2	*−9.03*		*93.14*				
2 + 50	*−8.19*		*93.98*				
3	*−7.58*		*94.59*				
4	*−7.36*		*94.81*				
T. P.	*−8.543*		*93.629*				
	+4.583	*98.212*					
4 + 60	*−11.93*		*86.28*				*Spring Brook*
5	*−0.47*		*97.74*				
T. P.	*−2.674*		*95.538*				
	+10.324	*105.862*					
5 + 75	*−2.39*		*103.47*				
6	*−2.04*		*103.82*				
7	*−6.38*		*99.48*				
8	*−7.67*		*98.19*				
9	*−6.43*		*99.43*				
10	*−8.70*		*97.16*				
T. P.	*−10.171*		*95.691*				
	+2.443	*98.134*					
11	*−2.45*		*95.68*				
12	*−7.20*		*90.93*				
13	*−8.85*		*89.28*				
T. P.	*−11.292*		*86.842*				

those given in Form No. 1, but are arranged in a different manner. The distinguishing feature of this form of level notes is a single column for all rod readings. The backsights being additive and the foresights subtractive readings,

they are distinguished by the signs + and −, respectively. The turning points are further designated by the abbreviation *T. P.* marked on the line of the first, or foresight, reading on each turning point.

52. Form No. 3.—This form of level notes is a modification of Form No. 1. It is the same as the last-mentioned form in every respect except that the foresights on turning

LEVEL NOTES: FORM NO. 3

Sta.	*B. S.+*	*H. I.*	*F. S.−*	*I. S.−*	*El.*		*Remarks*
B. M.	*2.17*	*102.17*			*100.00*		*On root of white*
0				*4.8*	*97.4*		*oak stump 60 ft.*
1				*6.2*	*96.0*		*to left of Sta. 0*
2				*9.1*	*93.1*		
2 + 50				*8.2*	*94.0*		
3				*7.6*	*94.6*		
4				*7.4*	*94.8*		
T. P.	*4.58*		*8.54*		*93.63*		
4 + 60		*98.21*		*11.9*	*86.3*		*Spring Brook*
5				*0.5*	*97.7*		
T. P.	*10.32*		*2.67*		*95.54*		
5 + 75		*105.86*		*2.4*	*103.5*		
6				*2.1*	*103.8*		
7				*6.4*	*99.5*		
8				*7.7*	*98.2*		
9				*6.5*	*99.4*		
10				*8.7*	*97.2*		
T. P.	*2.44*		*10.17*		*95.69*		
11		*98.13*		*2.4*	*95.7*		
12				*7.2*	*90.9*		
13				*8.8*	*89.3*		*100.00*
T. P.			*11.29*		*86.84*√		*19.51*
	19.51		*32.67*				*119.51*
							32.67
							86.84√

points, designated as foresights, and the foresights on intermediate points, designated as intermediate sights, are written in different columns, the former being headed *F. S.*− and

the latter *I. S.* —, or simply, *Rod.* This permits both the backsights and foresights on turning points to be added conveniently for the purpose of checking the notes, in the manner shown on this form and described in the following article.

53. How to Check Level Notes. — The method of checking level notes that is in general use affords a reliable check on the elevations of turning points and heights of instrument, which is a sufficient check on the line of levels as a whole, since all other elevations are deduced from these. The method depends on the fact that all the backsights are additive or + quantities and all foresights are subtractive or − quantities. Since this is the case, if the sum of all the backsights in a line of levels, or any portion of it, is added to the elevation of the starting point, which is usually a bench mark, and from the sum thus obtained the sum of all the foresights *on* turning points in the same is subtracted, the remainder is the last height of instrument or the elevation of the last turning point, according as the last sight included is a backsight or a foresight. The manner in which this method of checking level notes is applied is shown in the form of notes given in the preceding article. These notes are checked as follows: The elevation of the bench mark at Station 0 is 100.00 feet, to which all backsights or + readings are to be added, and from this sum all foresights or − readings are to be subtracted. The sum of the backsights or + readings, as determined by adding the column of backsights, is 19.51 feet, which added to the elevation of the bench mark at Station 0 gives a sum of 119.51 feet. The sum of the − readings or foresights is 32.67 feet, which subtracted from the sum just obtained gives a remainder of 86.84 feet, which is the elevation of the turning point last taken. The leveler should check each page of level notes, placing a check-mark √ at the last height of instrument or elevation checked. This should preferably be done as soon as the page is filled, but if there is not time for this during the field

work, then each day's work should be entirely checked the same night.

Example.—The backsight rod reading on a bench mark is 5.28 feet and the foresight reading on a turning point is 3.25 feet. If the elevation of the bench mark is 142.00 feet, what is (*a*) the height of instrument and (*b*) the elevation of the turning point?

Solution.—(*a*) The height of instrument is equal to the elevation of the bench mark plus the rod reading on the bench mark or 142.00 + 5.28 = 147.28 ft. Ans.

(*b*) The elevation of the turning point is equal to the height of instrument minus the foresight on the turning point, or 147.28 − 3.25 = 144.03 ft. Ans.

EXAMPLES FOR PRACTICE

1\. The backsight rod reading on a bench mark is 7.36 feet and the foresight reading on a turning point is 2.84 feet. If the elevation of the bench mark is 200.00 feet, what is (*a*) the height of instrument and (*b*) the elevation of the turning point?

Ans. (*a*) 207.36 ft. (*b*) 204.52 ft.

2\. If, when the instrument is moved forward and set up in a new position, the backsight on the turning point mentioned in the preceding example is 11.32 feet, what is the height of instrument?

Ans. 215.84 ft.

3\. The height of instrument is 125.00 feet and the foresight on a turning point is 4.33 feet. After the instrument has been moved forward to a new position, the backsight on the turning point is 8.53 feet. What is the elevation of the succeeding station, on which the rod reading is 9.20 feet? Ans. 120.00 ft.

4\. The height of instrument is 233.06 feet and the foresight on a turning point is 6.32 feet. After the instrument has been moved forward and set up in a new position, the backsight on the turning point is 9.58 feet. What are the elevations, written to the nearest tenth of a foot, of the three succeeding stations, on which the rod readings are 5.2 feet, 6.3 feet, and 7.5 feet, respectively?

Ans. 231.1 ft. 230.0 ft. 228.8 ft.

CONDITIONS RELATING TO THE ACCURACY OF LEVELING

EFFECT OF CURVATURE AND REFRACTION

54. Curvature of the Earth's Surface. — Since a level surface is a curved surface, corresponding in a large and general way to the surface of the earth, and since the line of sight given by an instrument is necessarily a straight line, except as affected by atmospheric refraction (see next article), it is evident that the line of sight is not a level line, but is tangent to a level line at the instrument. Consequently, the difference in elevation between the cross-wires of the instrument and the point on which the rod is held is not exactly equal to the reading of the rod, because the point where the line of sight cuts the rod is not at exactly the same elevation as the cross-wires of the instrument. This point is higher than the cross-wires by an amount equal to the divergence of the line of sight from the curved level line to which it is tangent, so that the rod reading exceeds the true reading by the same amount. This error, due to the curvature of the earth's surface, can be computed in the following manner:

In Fig. 13, let $OA_1 = OD$, represent the radius of the earth; AB represent the apparent line of sight given by a leveling instrument at A, and AD represent the corresponding level line lying in the earth's surface; then BD is the error due to curvature. In order to show the conditions clearly, the figure is drawn very much out of proportion, since OA is about 50,000 times AB for a sight of 400 feet, which is a common length of sight for a level. In geometry it is shown that:

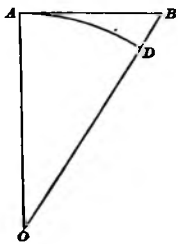

FIG. 13

If from a point without a circle a tangent to the circle and a secant are drawn, the tangent is a mean proportional between the secant and its external segment.

Thus, in Fig. 14, $A\ B$ is a mean proportional between $B\ D'$ and $B\ D$, or $\overline{A\ B}^2 = B\ D' \times B\ D$. But if $D\ D'$ is a diameter, it is equal to twice the radius $O\ D$. Hence, in Fig. 14, we have $\overline{A\ B}^2 = B\ D \times (B\ D + 2\ O\ D)$. But as $B\ D$ is exceedingly small compared with the diameter of the earth $2\ O\ D$, it may be dropped from the quantity within the parenthesis without appreciable error, so that if we denote the length of sight by d, the radius of the earth by r, and the error due to curvature by e_c, all expressed in the same unit, then by substituting d for $A\ B$, r for $O\ D$, and e_c for $B\ D$ in the preceding expression and omitting the quantity $B\ D$ within the parenthesis, it becomes $d^2 = 2\ r\ e_c$, from which

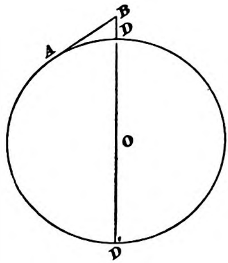

FIG. 14

$$e_c = \frac{d^2}{2\ r} \qquad (5)$$

55. Atmospheric Refraction. — It is a well-established law of physics that a ray of light in passing from a rarer to a denser medium is refracted or bent in a direction toward the denser medium, that is, so that its path will be concave on the side toward the denser medium. As the atmosphere is most dense at the surface of the earth and becomes rarer as the distance from the earth's surface increases, it follows that a ray of light in passing from a higher to a lower elevation by an inclined path will be bent, or refracted, toward the surface of the earth, that is, so that its path will curve in the same general direction as the earth's surface. By referring now to Fig. 15, it is evident that if the line of sight, which is

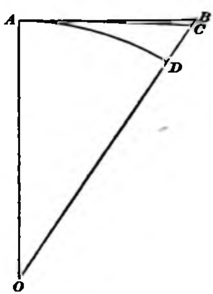

FIG. 15

apparently the straight line BA, is tangent to a level line at A, the point B at its opposite extremity is higher, that is, farther from the center of the earth O, than the point A, so that a ray of light, in passing from B to the instrument at A, is bent or refracted toward the earth. Consequently, the path of the ray, or the line of sight, which we have considered to be the straight line BA, is really the curved line CA, and the point observed, which is apparently the point B, is really the point C. Near the earth's surface and within the limits of all observations with a leveling instrument, the path CA of the ray of light may be considered to be the arc of a circle whose radius is seven times the radius of the earth. Hence, the amount of error e_r due to atmospheric refraction, represented by BC in Fig. 15, is given by substituting $7r$ for r in formula **5**, which then becomes

$$e_r = \frac{d^2}{14r} = .071\frac{d^2}{r} \qquad \textbf{(6)}$$

The numerical coefficient .071 in this formula is called the coefficient of refraction; if we denote this by m, the formula becomes

$$e_r = m\frac{d^2}{r} \qquad \textbf{(7)}$$

The coefficient of refraction m is commonly assumed to have a mean value of .070, though it has been found to have somewhat higher values for lower elevations.*

56. Combined Effect of Curvature and Refraction.—From an inspection of Fig. 15 it will be evident that the effect of refraction is to lessen the error due to curvature. Consequently, the combined or resultant effect of

* On the United States Coast Survey in New England the coefficient of refraction was found to have mean values of .071 between primary stations, .075 for small elevations, and .078 for a sea horizon.—Johnson's *Theory and Practice of Surveying*.

both the curvature and the refraction is equal to the error due to the curvature minus that due to the refraction; or if e is the resultant error, then $e = e_c - e_r$, and from formulas **5** and **6**, we have

$$e = \frac{d^2}{2\,r} - \frac{d^2}{14\,r} = \frac{3\,d^2}{7\,r} \qquad \textbf{(8)}$$

As used in this formula, the coefficient of refraction has a value of $\frac{1}{14}$, $= 0.0714$. By writing this formula in form for logarithmic computation, using a seven-place logarithmic table, it reduces to

$$\log e = 2 \log d - 7.6883680 \qquad \textbf{(9)}$$

If it is desired to use other values of this coefficient, then from formulas **5** and **7** we have also the expression

$$e = \frac{d^2}{2\,r} - m\,\frac{d^2}{r} = (.5 - m)\,\frac{d^2}{r}, \qquad \textbf{(10)}$$

in which any value of m can be substituted.

Table I, which gives the values of the corrections e in fractions of a foot for distances d expressed in feet, was calculated by formula **8**, taking $r = 20{,}911{,}790$ feet, as given by Bowditch. The correct value of the mean radius of the earth, however, is probably more nearly 20,890,590 feet ($\log r = 7.3199507$), and by substituting this value for r and a value of .0719 for m in formula **10**, results very nearly the same as given by formula **8** will be obtained.

When it is necessary to allow for the error due to curvature and refraction Table I may be used. By means of the formulas that have been stated the table can easily be extended for any length of sight desired. But it is to be borne in mind that it is wholly unnecessary to make any allowance for curvature and refraction in ordinary leveling if the backsights and foresights are balanced. In a single sight of ordinary length, the error due to curvature and refraction is so small as to be of no consequence. It is

TABLE I

CORRECTION FOR THE COMBINED EFFECT OF CURVATURE AND REFRACTION

d	e	d	e	d	e	d	e
300	.002	1,600	.052	2,900	.172	4,200	.362
400	.003	1,700	.059	3,000	.184	4,300	.379
500	.005	1,800	.066	3,100	.197	4,400	.397
600	.007	1,900	.074	3,200	.210	4,500	.415
700	.010	2,000	.082	3,300	.223	4,600	.434
800	.013	2,100	.090	3,400	.237	4,700	.453
900	.017	2,200	.099	3,500	.251	4,800	.472
1,000	.020	2,300	.108	3,600	.266	4,900	.492
1,100	.025	2,400	.118	3,700	.281	5,000	.512
1,200	.030	2,500	.128	3,800	.296	5,100	.533
1,300	.035	2,600	.139	3,900	.312	5,200	.554
1,400	.040	2,700	.149	4,000	.328	5,280	.571
1,500	.046	2,800	.161	4,100	.345	10,560	2.285

only when a large number of such errors are cumulative that they become of sufficient amount to appreciably affect the results, and such a condition can always be avoided by making each pair of sights, one backsight and one foresight, of equal length, or by making the backsights and foresights balance. This eliminates the errors due to curvature and refraction, and by the exercise of sufficient care can be made to also eliminate those due to imperfect adjustment by balancing the errors of opposite character. The value of the principle of equal backsights and foresights in direct leveling, thus causing the errors of opposite character to be compensating, cannot be too strongly emphasized.

EXAMPLE.—What is the error due to the combined effect of curvature and refraction in a sight of 1,300 feet, taking the radius of the earth as 20,890,590 feet and the coefficient of refraction as .0719?

SOLUTION.—Substituting known values in formula **10**, we have

$$e = (.5 - .0719)\frac{1,300^2}{20,890,590} = .035 \text{ ft., nearly. Ans.}$$

This result is the same as that given in Table I.

EXAMPLES FOR PRACTICE

NOTE.—In the following examples, compare the results obtained with those obtained in Table I.

1. Taking the radius of the earth as 20,890,590 feet and the coefficient of refraction as .0719, find the error due to the combined effect of curvature and refraction (*a*) in a sight of 700 feet and (*b*) in a sight of 800 feet.

Ans. { (*a*) .010 ft.
(*b*) .013 ft., closely

2. Taking the radius of the earth and the coefficient of refraction the same as in the preceding example, find the error due to the combined effect of curvature and refraction (*a*) in a sight of 900 feet and (*b*) in a sight of 1,000 feet.

Ans. { (*a*) .017 ft., nearly
(*b*) .020 ft., closely

3. Taking the radius of the earth and the coefficient of refraction the same as in example 1, find the error due to the combined effect of curvature and refraction (*a*) in a sight of 1,500 feet and (*b*) in a sight of 2,500 feet.

Ans. { (*a*) .046 ft., closely
(*b*) .128 ft., closely

4. Taking the radius of the earth as 20,911,790 feet and the coefficient of refraction as .0714, compute the error due to the combined effect of curvature and refraction (*a*) in a sight of 1,500 feet, and (*b*) in a sight of 2,500 feet.

Ans. { (*a*) .046 ft., closely
(*b*) .128 ft., closely

THE ERROR AND ACCURACY OF SPIRIT LEVELING

57. Sources of Error.—In leveling, the principal sources of error are defects of adjustment, which are usually the fault of the leveler, but are sometimes inherent in the instrument, and failure of the rodman to hold the rod truly vertical and to read the target correctly. The error due to curvature, resulting from unequal backsights and foresights on turning points, may become considerable when cumulative, that is, when the longer sights are all in the same direction, even though the instrument is in perfect adjustment. The most effective precaution against errors due to imperfect adjustment and those due to curvature and refraction is to have the backsights and foresights

on turning points of as nearly the same length as possible, that is, to have each pair of sights on turning points, one forward and one back, of as nearly the same length as possible.

The rays of the sun shining directly on the object glass render the field of view indistinct and the sighting of the telescope uncertain. To prevent this, most instruments are provided with a sunshade, which fits the end of the telescope and projects over the object glass. If the sunshade is lacking, the leveler can hold his hat so as to shade the object glass.

Wind is also a source of error; it sometimes causes the instrument to vibrate, thus preventing the accurate setting of the target; it frequently exerts sufficient pressure against the instrument to cause the bubble to run toward one end of its scale, and as the pressure is fluctuating this renders the accurate centering of the bubble almost impossible. When this happens, the leveler should wait for a lull in the wind, during which, if his rodman is alert, he can usually get a reasonably accurate sight. At a second lull, he can check the target and feel safe regarding the sight.

58. Personal Equation.—There is also what is known as the individual error, sometimes called the **personal equation,** which is recognized as a defect of vision peculiar to the individual. By reason of this peculiarity of vision, two persons may observe the reading of a rod or set the target, on the same sight and under precisely the same conditions, and obtain somewhat different readings. But as this personal equation, or error, is constant for the same person and affects all his observations in the same manner, it does not materially detract from the accuracy of work. Haste is also a fruitful source of error and is little if any aid to progress. Rapid and accurate work can be performed without haste, but work done in a hurry is not usually performed either rapidly or accurately.

59. Required Degree of Accuracy.—It has been found from experience that small errors occur much more frequently than large ones, and that those of an accidental character tend to balance each other. A line of levels 20 miles long, in which the rod readings are taken to the nearest hundredths with a self-reading rod, will give nearly the same results on intermediate points, and nearly the same difference of elevation between the points at its extremities, as a line of precise levels taken with exact target readings. That painful degree of exactness termed hair-splitting is of no advantage in ordinary engineering work; it represents very little actual gain in the accuracy of the results, and a very considerable increase in the cost of the work, while the required degree of accuracy can usually be obtained without such exactness. The degree of accuracy required in each case should be ascertained and the levels taken sufficiently close to obtain the accuracy required; greater exactness is an unnecessary waste of time.

It is a reasonably well-established principle that the total or final value of the accidental errors of direct leveling is proportional to the square root of the length of the circuit or distance covered by the line of levels. Hence, the degree of accuracy to be obtained by, and, consequently, the error of closure permissible in, direct leveling, may be expressed by the general formula

$$E = c\sqrt{L}, \qquad \textbf{(11)}$$

in which E is the permissible error of closure, L is the length of the circuit, and c is a numerical coefficient whose value depends on the character of the survey and on the units in which the values of E and L are expressed. In practice, this formula has been given the following forms, which may be taken as representative of the respective degrees of accuracy required in the various surveys named:

		(a)	(b)
1.	Chicago Sanitary District..........	$E_m = 3\sqrt{K}$	$E_f = .012\sqrt{M}$
2.	Missouri River Commission........	$E_m = 3\sqrt{2K}$	$E_f = .018\sqrt{M}$
3.	Mississippi River Commission (1891)	$E_m = 3\sqrt{2K}$	$E_f = .018\sqrt{M}$
4.	Same (previous to 1891)............	$E_m = 5\sqrt{K}$	$E_f = .021\sqrt{M}$
5.	United States Coast Survey........	$E_m = 5\sqrt{2K}$	$E_f = .029\sqrt{M}$
6.	United States Lake Survey.........	$E_m = 10\sqrt{K}$	$E_f = .042\sqrt{M}$
7.	United States Geological Survey...		$E_f = .05\sqrt{M}$
8.	Good average work of ordinary character............................		$E_f = .05\sqrt{M}$
9.	Preliminary railroad surveys.......		$E_f = .1\sqrt{M}$

In these formulas E_m denotes the permissible error of closure expressed in millimeters and E_f denotes the same value when expressed in feet, while K denotes the length of the circuit expressed in kilometers and M denotes the same value when expressed in miles. When the formula expresses the error of closure in millimeters and the length of circuit in kilometers, it can be transformed so as to express the former in feet and the latter in miles by multiplying the numerical coefficient by .00416. Thus, if $E_m = 3\sqrt{K}$, then $E_f = .00416 \times 3\sqrt{M} = .012\sqrt{M}$; or if $E_m = 3\sqrt{2K}$, then $E_f = .00416 \times 3 \times 1.414\sqrt{M} = .018\sqrt{M}$, nearly. The length of the circuit, as substituted for M in the formula, may be expressed in feet if the numerical coefficient in formula (b) is multiplied by .01376.

EXAMPLE.—Suppose it is desired to determine the error permissible in making the preliminary survey for a railroad 100 miles long.

SOLUTION.—By substituting a value of 100 for M in the equation for item 9 above, we have

$$E_f = .1\sqrt{100} = 1.0 \text{ ft.}$$

EXAMPLES FOR PRACTICE

1. What would be the permissible error of closure in the preliminary survey for a railroad 150 miles long? Ans. 1.22 ft.

2. In good average work of the ordinary character, what would be the permissible error of closure in a circuit of which the length is 18 miles? Ans. .212 ft.

3. In a United States Geological Survey, what is the permissible error of closure in a circuit 50 miles long? Ans. .354 ft.

4. In a United States Lake Survey, what is the permissible error of closure in a circuit of 30 miles? Ans. .230 ft.

PROFILES AND GRADE LINES

60. Profiles.—The relative elevations of the different points whose elevations are determined by a line of levels can be represented graphically by a vertical section of the line of survey. Such a vertical section is called a **profile.** In it all changes of elevation indicated by the levels are clearly outlined. In a profile the vertical measurements are usually represented to a larger scale than the horizontal measurements, thus exaggerating the irregularities of surface and rendering them more distinct. For railroad work, profiles are commonly constructed to a horizontal scale of 400 feet to the inch and a vertical scale of 20 feet to the inch. For municipal work, it is common to use a horizontal scale of 40 feet to the inch and a vertical scale of 4 feet to the inch. Other scales are also used according to the character and requirements of the work.

A profile can be constructed on plain paper in the following manner: A horizontal line is first drawn near the lower edge of the paper. On this line the stations of the survey are laid off to the horizontal scale, and the number of each station, or of every tenth station, as may be most desirable, is written directly below the point representing it. From each station on the horizontal line a vertical line is drawn upward in pencil. Some elevation is now assumed for the horizontal line. This elevation may be zero, that is, the horizontal line may be assumed to represent the base of reference; or any convenient elevation may be assigned to it, so long as the elevation assigned to it is lower than the lowest point in the elevation of the surface. On each vertical line representing a station, the elevation of the station that it represents is laid off at the proper distance above the horizontal line, according to the vertical scale. In case an

elevation is taken at a plus, between two regular stations, the position of the plus is located on the horizontal line, a vertical line is drawn upward from the point thus located, and the elevation of the plus is laid off on the vertical line, the same as for a regular station. After having thus located on the vertical lines the elevations of all points of the surface whose elevations were taken in the line of levels, the surface line is drawn through the points thus located. It is usually best to draw the surface line in pencil from point to point by means of a straightedge, then to ink in this surface line freehand, following the pencil line as closely as possible. This will give a slightly irregular surface line that will represent the actual surface conditions better than a perfectly straight line drawn from point to point. After the surface line is drawn, the vertical lines representing the stations are usually inked in, the lower end of each vertical line terminating in the horizontal line and its upper end terminating in the surface line.

Fig. 16 represents a profile of the level notes given in Art. **50**, constructed in the manner just described to a

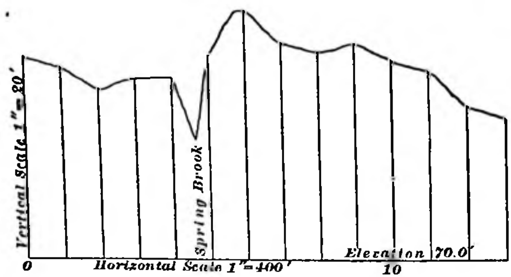

Fig. 16

horizontal scale of 400 feet to the inch and to a vertical scale of 20 feet to the inch. The elevation of the horizontal line is assumed to be 70.0 feet. The elevation of Station 0 is 97.4 feet, and this station is therefore $97.4 - 70.0 = 27.4$ feet

above the elevation represented by the horizontal line. This distance is laid off to the vertical scale on the vertical line representing the position of Station 0, thus locating a point in the surface at Station 0. The elevation of Station 1 is 96.0 feet, and, consequently, the height of this station above the horizontal line is equal to $96.0 - 70.0 = 26.0$ feet. This distance is laid off to the vertical scale on the vertical line representing the position of Station 1, thus locating a point in the surface at Station 1. The elevation of Station 2 is 93.1 feet, and, consequently, the height of this station above the horizontal line is equal to $93.1 - 70.0 = 23.1$ feet. This distance is laid off to the vertical scale on the vertical line representing the position of Station 2, thus locating a point in the surface at Station 2. The elevations of the remaining stations are laid off on the corresponding vertical lines in the same manner, and the surface line is drawn through the points thus located in the manner just described.

61. Profile Paper.—In order to facilitate the construction of profiles, paper prepared especially for the purpose is commonly used; this has horizontal and vertical lines in pale green, blue, or orange, so spaced as to represent certain distances to the horizontal and vertical scales. Such paper is called **profile paper.** In the most common form of profile paper the horizontal lines are $\frac{1}{20}$, and the vertical lines $\frac{1}{4}$, inch apart. Consequently, with the scale commonly used for the profiles in railroad work, each space between horizontal lines represents 1 foot to the vertical scale and each space between vertical lines represents 100 feet to the horizontal scale. Other values can of course be assumed for the spaces, according to the character and requirements of the work.

Every fifth horizontal line and every tenth vertical line is heavy, while every fifth one of the heavy horizontal lines is extra heavy. By the aid of the system of light and heavy lines, distances and elevations are quickly and correctly estimated and the work of platting greatly facilitated. A profile is usually drawn on strips of such paper wide enough

to accommodate the rise and fall of the surface line, with what margin may be desirable, and long enough to contain the entire line of levels.

A small piece of profile paper is represented in Fig. 17, on which are platted the level notes given in Art. **50.** The elevation of some horizontal line is assumed, and this elevation, which is, of course, referred to the datum line, is the

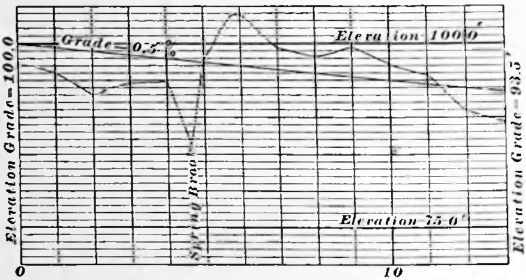

FIG. 17

base from which the other elevations are estimated. Every tenth station number is written at the bottom of the sheet under the heavy vertical lines. The elevations of all points along the line whose elevations have been taken are located consecutively on the profile and the surface line drawn through them freehand in pencil, then inked in black. Thus, the elevation of Station 0 is 97.4 feet, which, as located on the profile, is 2.6 spaces below the extra-heavy horizontal line whose elevation is assumed to be 100.0 feet; for Station 1 the elevation is 96.0 feet, which is located at the horizontal line whose elevation is 96.0 feet, which is the fourth line below the extra-heavy line; the elevation of Station 2 is 93.1 feet, which is located .1 space above the horizontal line whose elevation is 93.0 feet, etc.

62. Grade Lines. — In important engineering work, before beginning the actual construction, it is customary to decide on the position that some prominent line of the

completed work or structure is to have and adopt it as a line of reference that determines the elevations of the different parts of the structure during the progress of the work. This imaginary line is called a **grade line**; its position is always shown on the profile. Thus, the grade line for a street is a line representing on the profile what is to be the surface of the street along its center line. Likewise the grade line for a railroad during construction is a line representing on the profile what is to be the surface of the roadbed when completed. Such a grade line will usually be straight and have a uniform slope for a considerable distance, and, as drawn on the profile, will necessarily be below the surface line in some places and above it in other places. In order to bring the surface of the street, or roadbed, as the case may be, to the position indicated by the grade line, it is necessary to excavate the material where the grade line is below the surface and fill in material where the grade line is above the surface. This process is called **grading.** When the surface of the street, or roadbed, as the case may be, is graded true to the position indicated by the grade line, it is said to be **at grade,** and this position, or the surface of the street or roadbed in this position, is spoken of as the **grade.**

The principal purpose for which a profile is constructed is to enable the engineer to establish the grade line. For a railroad or highway, the grade line should, when possible, be located in such position that the excavation and embankment will be nearly the same in amount, or the embankment somewhat exceed the excavation. The position of the grade line having been determined, it is drawn on the profile in red ink.

63. Rate of Grade.—The rate of rise or fall along the grade line is called the **rate of grade,** or **gradient.** The rate of grade may be expressed as the amount of rise or fall in a unit of horizontal length, in a hundred units of horizontal length, or in any given horizontal distance, as in 1 mile. It may be expressed as a decimal or as a vulgar fraction;

when expressed as the latter, the rise or fall is written as the numerator and the horizontal distance in which the given rise or fall occurs is written as the denominator, of the fraction, both being expressed in the same unit. When the rate of grade, expressed as a vulgar fraction, is reduced to the form of a decimal fraction, it will express the amount of rise or fall per horizontal unit and will correspond to the tangent* of the slope. The rate of grade is very commonly expressed by the amount of rise or fall per hundred units; when so expressed, it is usually spoken of as the *rate per cent.* The **rate per cent.** of a grade is, therefore, the number of units of rise or fall in each hundred units of horizontal distance; as commonly used and expressed in this country, it is the number of feet rise or fall in each hundred feet of horizontal distance. The rate of grade will be thus expressed here.

In writing the rate of grade, it is customary to indicate a rising grade by a + sign and a falling grade by a – sign. Thus, a grade line that rises 1 foot in each 100 feet of its length is called an ascending grade of one per cent. and on the profile is written + 1.00 per hundred or + 1.00%. A grade that falls .5 of a foot in each 100 feet of its length is called a descending grade of five-tenths per cent. and is written on the profile – 0.50 per hundred, or – 0.50%. The sign % is frequently omitted, however. The rate of grade is written along the grade line on the profile, and the elevation of grade is written at the extremities of the grade line, and at each point where the rate of grade changes. It is a common practice to enclose in a small circle the point on the profile where the rate of grade changes.

All computations relating to rates of grade may be based on the following simple rules:

Rule I.—*The rate per cent. of a grade is equal to its total rise or fall in any horizontal distance divided by the horizontal distance and multiplied by 100.*

* This is often spoken of as the *sine* of the slope; the angle of the slope is usually so small, however, that it makes little difference which term is used.

Rule II.—*The total rise or fall of a grade line in any given horizontal distance is equal to the rate per cent. of grade multiplied by the horizontal distance and divided by 100.*

Rule III.—*The horizontal distance in which a grade line having a given rate per cent. will rise or fall a certain amount is equal to the amount of rise or fall divided by the rate per cent. and multiplied by 100.*

EXAMPLE.—The total rise of a certain grade is 66 feet in a horizontal distance of 1 mile. (*a*) What is the rate of grade? (*b*) If the elevation of the grade at Station 2 is 150.00 feet, what is the elevation of the grade at Station 13?

SOLUTION.—(*a*) In 1 mile there are 5,280 feet; therefore, applying rule I, we have for the rate of grade

$$\frac{66 \times 100}{5{,}280} = 1.25\% \quad \text{Ans.}$$

(*b*) The horizontal distance between Station 2 and Station 13 is 1,100 feet. The total rise of the grade line between these two stations is, according to rule II,

$$\frac{1{,}100 \times 1.25}{100} = 13.75 \text{ ft.}$$

The elevation of the grade at Station 2 is 150.00 feet; therefore, the elevation of the grade at Station 13 is $150.00 + 13.75 = 163.75$ ft. Ans.

64. Cut and Fill.—This does not belong properly to the subject of leveling, but since it is associated very closely with the subjects of profiles and grade lines, it is treated here. The grade line, by its distance below or above the surface line at the different points along the profile, shows the depths of excavation or embankment in the proposed work at the different points. The vertical distance of the grade line below the surface line at any point, as shown on the profile, will be the depth of excavation, or cutting, at that point, necessary to bring the surface of the roadbed to the established grade. Likewise the vertical distance of the grade line above the surface line at any point will represent the depth of embankment, or filling, at that point necessary to bring the surface of the roadbed to the

established grade. The depth of excavation and depth of embankment are, for short, commonly spoken of as the **cut** and **fill,** respectively.

The depth of cutting and filling is usually calculated for each station in connection with the level notes and recorded in the notebook. At each station where the elevation of the surface exceeds the elevation of the grade, the difference will be the depth of cutting. At each station where the elevation of the grade exceeds the elevation of the surface, the difference will be the depth of filling.

65. Calculation of Cut and Fill. — The following notes are a repetition of those given in Form No. 2, Art. **51**, to which are added the elevation of grade for each station, with the corresponding cut or fill. This form of notes is convenient for this purpose, since in a notebook of the usual form it leaves three blank columns on the left-hand page. For Station 0 the elevation of grade is fixed at 100.00 feet; this is written opposite Station 0 in the column headed *Elevation of Grade.* The rate of grade is assumed to be — 0.5 per cent., that is, a descending grade of .50 foot per station. Hence the elevation of grade for Station 1 is equal to 100.00 — .50 = 99.50 feet; the elevation of grade for Station 2 is equal to 99.50 — .50 = 99.00 feet; the elevation of grade for Station 3 is equal to 99.00 — .50 = 98.50 feet, etc. The elevation of grade for each succeeding station is worked out in the same manner and is written in the column headed *Elevation of Grade.* The depth of filling at Station 0 is now determined by merely subtracting the elevation of the surface at that station from the corresponding elevation of grade; it is 100.00 — 97.42 = 2.58. This is written in the column headed *Fill.* The depth of filling for each succeeding station is found in the same manner as far as Station 4. At Station 5 the elevation of the surface exceeds the elevation of grade, so that the grading will here consist of cutting instead of filling. Hence, the elevation of the grade subtracted from the elevation of the surface gives the depth of cutting at this station; it is equal to 97.74 — 97.50 = .24.

This is written in the column headed *Cut*. The depth of cutting at each succeeding station is determined in the same manner as far as Station 11. At Station 12 the grade line is again above the surface line, and at this station there is a fill of 3.07 feet.

LEVEL NOTES, WITH GRADE ELEVATIONS, CUTS, AND FILLS

FORM NO. 2

Station	*Rod Reading*	*Height of Instrument*	*Elevation of Surface*	*Elevation of Grade*	*Cut*	*Fill*	*Remarks*
B. M.	*+2.172*	*102.172*	*100.000*				*On root of white*
0	*−4.75*		*97.42*	*100.00*		*2.58*	*oak stump 60 ft.*
1	*−6.14*		*96.03*	*99.50*		*3.47*	*to left of Sta. 0*
2	*−9.03*		*93.14*	*99.00*		*5.86*	
2 + 50	*−8.19*		*93.98*	*98.75*		*4.77*	
3	*−7.58*		*94.59*	*98.50*		*3.91*	
4	*−7.36*		*94.81*	*98.00*		*3.19*	
T. P.	*−8.543*		*93.629*				
	+4.583	*98.212*					
4 + 60	*−11.93*		*86.28*	*97.70*		*11.42*	*Spring Brook*
5	*−0.47*		*97.74*	*97.50*	*0.24*		
T. P.	*−2.674*		*95.538*				
	+10.324	*105.862*					
5 + 75	*−2.39*		*103.47*	*97.12*	*6.35*		
6	*−2.04*		*103.82*	*97.00*	*6.82*		
7	*−6.38*		*99.48*	*96.50*	*2.98*		
8	*−7.67*		*98.19*	*96.00*	*2.19*		
9	*−6.43*		*99.43*	*95.50*	*3.93*		
10	*−8.70*		*97.16*	*95.00*	*2.16*		
T. P.	*−10.171*		*95.691*				
	+2.443	*98.134*					
11	*−2.45*		*95.68*	*94.50*	*1.18*		
12	*−7.20*		*90.93*	*94.00*		*3.07*	
13	*−8.85*		*89.28*	*93.50*		*4.22*	
T. P.	*−11.292*		*86.842*				

66. When the level notes used are those of Form No. 1, Art. **50**, there will usually remain only two blank columns on the left-hand page of the notebook, though this will, of course, depend on the form of notebook used. In case only

two columns remain on the left-hand page, it is customary to write the elevations of grade in the first column following the elevation of the surface, and write both the cut and fill in the succeeding column, or last column of the page. In this case the cuts and fills will be distinguished either by placing the plus sign before each cut and the minus sign before each fill, or by writing the letter *C* before each cut and the letter *F* before each fill. This is shown in the following form, in which the rod readings on stations are taken to the nearest tenth of a foot only, and, consequently, the surface and grade elevations, and the cuts and fills, are carried only to the nearest tenth.

LEVEL NOTES, WITH GRADE ELEVATIONS, CUTS, AND FILLS
FORM No. 1

Station	*Back-sight*	*Height of Instrument*	*Fore-sight*	*Elevation of Surface*	*Elevation of Grade*	*Cut or Fill*	*Remarks*
B. M.	*2.17*	*102.17*		*100.00*			*On root of white*
0			*4.8*	*97.4*	*100.00*	*F 2.6*	*oak stump 60 ft.*
1			*6.2*	*96.0*	*99.5*	*F 3.5*	*to left of Sta. 0*
2			*9.1*	*93.1*	*99.0*	*F 5.9*	
2 + 50			*8.2*	*94.0*	*98.8*	*F 4.8*	
3			*7.6*	*94.6*	*98.5*	*F 3.9*	
4			*7.4*	*94.8*	*98.0*	*F 3.2*	
T. P.	*4.58*		*8.54*	*93.63*			
4 + 60		*98.21*	*11.9*	*86.3*	*97.7*	*F 11.4*	*Spring Brook*
5			*0.5*	*97.7*	*97.5*	*C 0.2*	
T. P.	*10.32*		*2.67*	*95.54*		*C 6.4*	
5 + 75		*105.86*	*2.4*	*103.5*	*97.1*	*C 6.8*	
6			*2.1*	*103.8*	*97.0*	*C 3.0*	
7			*6.4*	*99.5*	*96.5*	*C 2.2*	
8			*7.7*	*98.2*	*96.0*	*C 3.9*	
9			*6.5*	*99.4*	*95.5*	*C 2.2*	
10			*8.7*	*97.2*	*95.0*		
T. P.	*2.44*		*10.17*	*95.69*		*C 1.2*	
11		*98.13*	*2.4*	*95.7*	*94.5*	*F 3.1*	
12			*7.2*	*90.9*	*94.0*	*F 4.2*	
13			*8.8*	*89.3*	*93.5*		
T. P.			*11.29*	*86.84*			

EXAMPLES FOR PRACTICE

1. Between Stations 10 and 25 of a certain survey there is grade of +2.00 per cent. If the elevation of the grade at Station 10 is 48.00 feet, what is the elevation of the grade (*a*) at Station 15, (*b*) at Station 18 + 75, and (*c*) at Station 23 + 67 ?

Ans. { (*a*) 58.00 ft. (*b*) 65.50 ft. (*c*) 75.34 ft.

2. If the elevation of the grade at Station 0 is 150.10 feet and that of the grade at Station 15 + 80 is 67.15 feet, what is the rate of grade ?
Ans. −5.25 per cent.

3. What is the total rise of a +3.75-per-cent. grade in a distance of 2,640 feet ?
Ans. 99.00 ft.

4. In what horizontal distance will a grade of +4.2 per cent. effect a rise of 94.50 feet ?
Ans. 2,250 ft.

TRIGONOMETRIC LEVELING

67. Trigonometric leveling, called also **indirect leveling,** is the process of determining the relative elevations of different points, taking two at a time, by measuring the distance between them and also the vertical angle between the line joining them and a horizontal line. This determines for the two points one acute angle and an adjacent side of a right triangle in a vertical plane, from which can be determined the altitude of the triangle, which is the difference in elevation between the points. A transit having a vertical limb is used to measure the angle. When the sights are of considerable length, they must be corrected for the curvature of the earth's surface and for refraction. This is not necessary, however, when the sights are short. The following examples will serve as simple illustrations of the general process of indirect leveling.

68. Short Sights.—Let *DB*, Fig. 18, represent a perfectly vertical flagstaff whose height is to be determined. This can be found in the following manner:

A transit is set up at *A* and leveled up carefully, both the plate levels and the telescope level being centered accurately with the telescope axis clamped. The line of

sight will then be truly horizontal, and the vertical arc, if in perfect adjustment, will read zero. The telescope is then sighted toward the flagstaff, the horizontal line of sight striking it at the point C. By measurement it is found that the distance $A\,C$ of the center of the transit from the edge of the flagstaff is 180 feet,

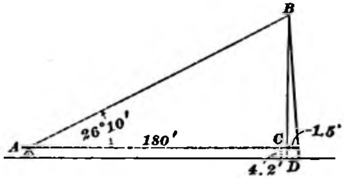

FIG. 18

the distance $C\,D$ or the height of the point C above the ground is 4.2 feet, and the diameter of the flagstaff at $C = 1.5$ feet. The telescope axis is then unclamped, the telescope sighted at the extreme top or apex B of the flagstaff, and the angle $C\,A\,B = 26°\ 10'$ is read on the vertical limb.

Denoting by h the vertical height $B\,C$, and by d the horizontal distance from A to a point vertically below B, we have from trigonometry $\tan A = \frac{h}{d}$, from which

$$h = d \tan A \qquad (12)$$

Tan $26°\ 10' = .49134$, and since $A\,C = 180$ feet and the diameter of the flagstaff at C is 1.5 feet, the horizontal distance d is equal to $180 + \frac{1}{2} \times 1.5 = 180.75$ feet. By substituting these values in the above formula, we get for the vertical height $B\,C$ the value $h = 180.75 \times .49134 = 88.81$ feet. Hence, the total height $B\,D$ of the flagstaff is $88.81 + 4.20 = 93.01$ feet.

69. Let it be required to determine the height $B\,D$, Fig. 19, or in other words the elevation of the inaccessible point B, which may be the summit of a mountain, above the point A, it being impossible to measure the entire distance $A\,D$. This can be determined in the following manner:

The transit is set up at A and leveled up carefully, both the plate levels and the telescope level being centered

accurately, and the angle CAB, $= a$, between a horizontal line and the line AB is measured. The transit is then moved to C and set up at the same height and the angle DCB, $= c$,

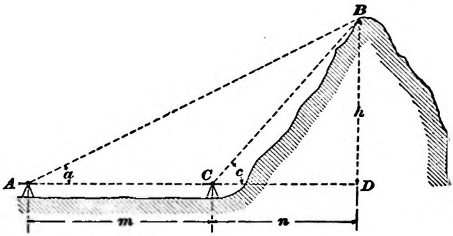

FIG. 19

between a horizontal line and the line CB is measured. The distance AC, $= m$, is also measured. Denoting the distance CD by n, and the height BD by h, we have, from trigonometry,

$$m + n = h \cot a, \text{ also } n = h \cot c$$

Subtracting the latter equation from the former, we have

$$m = (m + n) - n = h (\cot a - \cot c)$$

From which

$$h = \frac{m}{\cot a - \cot c} \qquad (13)$$

Having determined the value of h by this formula, the horizontal distance n can be determined easily from the expression $n = h \cot c$.

70. When determining differences of elevation by the foregoing method, however, it is usually inconvenient and sometimes impossible to set up the transit at C in such position that the height of instrument will be the same as when at A. It is not necessary that the heights of instrument be the same in both cases; the transit can be set up at any

other height, as at C', Fig. 20. The vertical angles a and c between horizontal lines and the lines $A\,B$ and $C'\,B$, respectively, and the horizontal distance m between instrument points, are measured as before. The vertical distance y between the heights of instrument at A and C' is also determined, calling it $+$ if C' is below A and $-$ if C' is above A. From trigonometry we know that the horizontal distance x, between the instrument point C' and the point C where the line of sight $C'\,B$ intersects the horizontal line $A\,D$ through the cross-wires of the instrument at A, is given by the expression

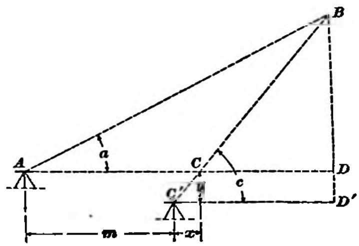

FIG. 20

$$x = y \cot c$$

But the distance $A\,C = m + x = m + y \cot c$, and by substituting this value for m all conditions become the same as in formula **13**, and we have

$$h = \frac{m + y \cot c}{\cot a - \cot c}, \qquad \textbf{(14)}$$

in which y must be given its proper sign, $+$ or $-$, according as C' is below or above A, and h is the elevation of the point B above the instrument at A.

71. Let it be required to determine the height $C\,D$, Fig. 21, or in other words the elevation above the point A of the inaccessible point C, it being impossible to measure the distance $A\,D$. A base line $A\,B$, as nearly perpendicular to $C\,A$ as may be convenient, is measured on the horizontal plain. Let this base line be 500 feet in length. The transit is then set up at A, the telescope sighted to B, and the instrument turned in azimuth until the point C is sighted.

Let the angle turned on the horizontal circle be 89° 15′ and the angle turned on the vertical circle be 29° 44′. The transit is then set up at B, the telescope sighted to A, and the instrument then turned in azimuth until the point C is

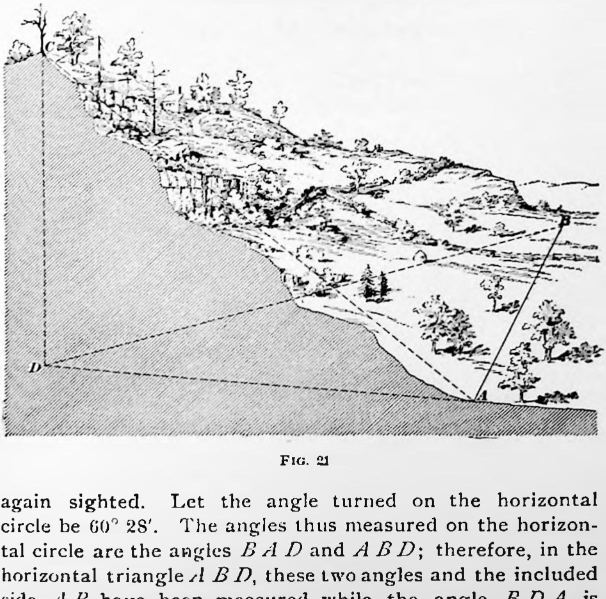

FIG. 21

again sighted. Let the angle turned on the horizontal circle be 60° 28′. The angles thus measured on the horizontal circle are the angles $B\,A\,D$ and $A\,B\,D$; therefore, in the horizontal triangle $A\,B\,D$, these two angles and the included side $A\,B$ have been measured while the angle $B\,D\,A$ is equal to

$$180° - (B\,A\,D + A\,B\,D) = 180° - (89°\ 15' + 60°\ 28')$$
$$= 30°\ 17'$$

In the triangle $A\,B\,D$, we know from trigonometry that

$$A\,D = \frac{A\,B \sin A\,B\,D}{\sin B\,D\,A} = \frac{500 \sin 60°\ 28'}{\sin 30°\ 17'}$$

Hence, using logarithms, $A\,D = 862.68$ feet.

Now, in the right triangle $A\,D\,C$, we know the side $A\,D$ and that the vertical angle $C\,A\,D$, which has been measured on the vertical circle of the transit, is 29° 44′. Then, from trigonometry,

$$CD = A\,D \tan C\,A\,D = 862.68 \times \tan 29° \, 44'$$

Hence, using logarithms, $C\,D = 492.73$ ft. Ans.

72. Long Sights.—When the distances between points whose relative elevations are to be determined by trigonometric leveling are great, correction for the curvature of the earth's surface and for refraction must be made. The following treatment, though not rigid, will give results that are close enough, probably, for work of the usual class.

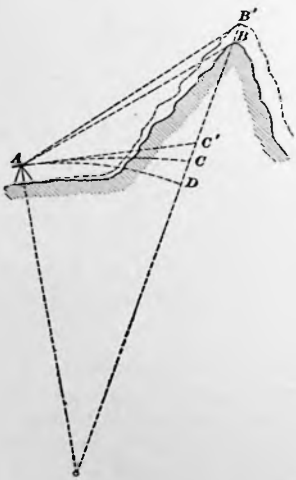

Fig. 22

Let it be required to determine the elevation of the summit B above the instrument point A, Fig. 22. For clearness, the vertical distances in this figure are greatly exaggerated. With the telescope level centered so that the telescope is sighted on a horizontal line, the line of sight is apparently the straight line $A\,C'$, but by reason of the atmospheric refraction it is really the curved line $A\,C$. Similarly when the telescope is directed toward the summit, the line of sight is apparently the straight line $A\,B'$, but is really the curved line $A\,B$. The lengths of the lines of sight $A\,B$ and $A\,C$, though differing considerably in the figure by reason of the greatly exaggerated vertical scale, are really

nearly the same, since the height BC is always small as compared with the horizontal distance AC. If we assume $AB = AC$, then errors due to atmospheric refraction are equal; that is, $BB' = CC'$. Hence, the height BC is equal to the apparent height $B'C'$ as computed by applying formula **12** to the right triangle $AB'C'$, or by any of the methods for short sights explained in the preceding articles. The height CD is the error on the horizontal sight due to the combined effect of curvature and refraction as computed by formula **9.** For the total height of the summit B above A we now have

$$BD = BC + CD = B'C' + CD$$

The determination of the relative elevations of different points will be treated further under the subject of stadia measurements.

EXAMPLE.—If, in Fig. 20, the angle $a = 17° 37'$, the angle $c = 31° 24'$, the horizontal distance m between the two positions of the instrument is 300 feet, and its position at C' is 2.5 feet higher than its position at A, what is the elevation of the point B above the horizontal line AD?

SOLUTION.—Substituting known values in formula **14,** and giving y the minus sign, since the point C' is above the point A, we have

$$h = \frac{300 - 2.5 \times \cot 31° 24'}{\cot 17° 37' - \cot 31° 24'} = \frac{300 - 4.09565}{3.14922 - 1.63826} = 195.84 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. If, in Fig. 18, the angle $A = 30° 15'$, the distance $AC = 100$ feet, the height $CD = 5$ feet, and the diameter of the staff at C is 2 feet, what is the height of the staff? Ans. 63.90 ft.

2. If, in Fig. 19, the angle $a = 25° 24'$, the angle $c = 30° 28'$, and the distance m between the two positions of the instrument is 400 feet, what is (*a*) the height BD and (*b*) the horizontal distance CD?

Ans. { (*a*) 985.03 ft. ; (*b*) 1674.5 ft.

3. If, in Fig. 20, the angle $a = 15° 37'$, the angle $c = 29° 16'$, the horizontal distance m between the two positions of the instrument is

200 feet, and its position at C' is 4.0 feet below its position at A, what is the elevation of the point B above the horizontal line $C' D'$?

Ans. 119.51 ft.

4. If, in Fig. 21, the angle $A B D = 65° 30'$, the angle $B A D = 87° 45'$, the angle $C A D = 35° 24'$, and the distance $A B = 400$ feet, what is the vertical distance between the points A and C?

Ans. 574.69 ft.

BAROMETRIC LEVELING

73. Principle of the Measurement.—The difference of elevation between two places can be determined approximately by their relative depths below the surface of the atmosphere as indicated by the comparative weights of the air at the two places. The weight or pressure of the air is determined by observing the height of a column of mercury, or other liquid, so arranged that it is balanced by the column of air above it having the same sectional area, or by observing the pressure exerted by the column of air on the external surface of a hermetically sealed metal box from which the air has been exhausted. In the latter case the pressure is measured by the deflection of the elastic sides of the box, as indicated on the dial face by the movement of a hand to which the deflection is communicated by a system of levers.

74. Barometers. — Instruments for registering the weight or pressure of the air are called **barometers.** Those that indicate the pressure by the height of a column of mercury are called **mercurial barometers,** while those that indicate it without the use of mercury by the deflections of the metal sides of a vacuum chamber are called **aneroid barometers.** The former are the more accurate and reliable, but not being of a portable character, are not suitable for surveying purposes. The latter, though less reliable, are by reason of their portability better suited to surveying purposes.

75. The Aneroid Barometer.—This instrument consists of a circular metal box having within it a vacuum

chamber in the form of a flat cylindrical box, exhausted of air and hermetically sealed. The circular sides of this vacuum box, corresponding to the ends of the cylinder, consist of thin metal plates corrugated in concentric grooves, except at the center, where they are reenforced by strong metal disks. One of the disks connects the lower plate firmly to the heavy plate forming the back of the outer

FIG. 23

metal box, while the other carries a small stout pillar that connects with a lever that is attached to a strong steel spring and communicates the movement of the upper plate of the vacuum box to the system of levers mentioned below. This corrugated form of the sides of the vacuum box permits greater freedom to, and tends to equalize, their movement under the varying atmospheric pressure. The

corrugated sides of the box are compressed or deflected inward as the external pressure due to the weight of the air increases, and the elastic resistance of the deflected metal causes them to again recover or move outward as the pressure diminishes. This motion is slight, but it is greatly multiplied by a system of levers by which it is transmitted to an index or pointer that moves over a circular scale on the outer face of the instrument, as illustrated in Fig. 23. Most aneroid barometers have two concentric scales, as shown in the figure; the inner scale corresponding to the heights of the mercury column is called the **mercury scale,** while the outer scale is an **altitude scale,** giving heights in feet.

76. The Altitude and Mercury Scales.—Some aneroid barometers read to altitudes above sea level only, while others read above and below sea level, substantially as the barometer shown in Fig. 23. It will be noticed that in this figure the zero of the altitude scale is opposite 31 inches of the mercury scale, but it must not be understood from this that the barometer always reads 31 inches at sea level, for 31 inches is not even its average reading at sea level. The altitude and mercury scales are arranged in this manner on many barometers in order that they may agree with standard altitude tables that have been prepared on this basis. In the barometer shown in the figure, the divisions of the inner or mercury scale correspond to fiftieths of an inch in the height of the mercury column and each tenth of an inch is numbered; the graduations of the scale extend from 27 to 33 inches. Each division of the outer scale corresponds to 10 feet of altitude, and the graduations extend from 2,000 feet below sea level to 4,000 feet above. By means of a movable vernier that revolves around the outer edge of the altitude scale, shown opposite the end of the pointer in the figure, this scale can be read to single feet of elevation. A thermometer is also commonly attached to the face of the barometer in order to show the temperature when the observations are taken.

Since the density of the air decreases as the altitude increases, the altitude scale is a decreasing scale in the older types of barometer, that is to say, corresponding divisions of the scale become smaller as the altitude increases. The use of the vernier was therefore impracticable on the older types of barometer having a varying altitude scale, but the later and more improved types of barometers have these scales so arranged that the mercury scale is a varying scale and the outer or altitude scale is a regular scale, which permits the use of a vernier for reading this scale to single feet.

For accurate work the positions of the altitude and mercury scales of a barometer should be fixed with respect to each other. Barometers are made in which the altitude scale is adjustable with respect to the mercury scale; so that the zero of the altitude scale can be set at the division of the mercury scale corresponding to the barometer reading for any elevation, in order that the difference of elevation may be read directly from the altitude scale. But owing to the varying relation between the altitude and mercury scales, this method gives only approximate results. It will be observed that in Fig. 23 the number of feet corresponding to 1 inch of mercury column is not constant. According to Airy's tables of barometric heights, computed for a barometer reading of 31 inches at sea level and a temperature of 50° F., 1 inch of mercury column at sea level corresponds to 893.6 feet of air column or altitude, while at an elevation of 10,000 feet above sea level 1 inch of mercury column corresponds to 1,300 feet of air column, or altitude. These figures show the varying relation between the two scales.

77. Compensation for Temperature. — It will be noticed that the barometer shown in the illustration is marked *compensated;* this means that it is compensated for temperature so as to read the same at different temperatures. This compensation depends on the following principle: The resistance of metal diminishes with any increase of temperature. Consequently, as the temperature increases,

the elastic resistance of the steel spring attached to the corrugated side of the vacuum box becomes less, so that it deflects more under the external pressure of the atmosphere, the increase of temperature having the same effect as increase of pressure. By the air not being completely exhausted from the vacuum box, the small amount that remains within the box, expanding with the increase of temperature and to some extent supporting the corrugated side of the box against the external pressure, tends to counterbalance the effect of the diminished resistance of the metal due to the increase of temperature. By a very nice adjustment of this condition, regulating the amount of air within the vacuum chamber, it is claimed that almost perfect compensation for changes of temperature is obtained. It is probable, however, that this compensation is never really perfect.

In some aneroid barometers the compensation for temperature is effected by combining steel and brass in the main lever of the instrument. The two metals are combined in such a manner that their different expansion and contraction under the changes of temperature will, to some extent, counteract the effect of the varying resistance to the deflection of the sides of the vacuum box. This compensation is based on the same principle as that in the balance wheel of a watch.

78. Care of the Aneroid Barometer.—All aneroid barometers are provided with leather cases, in order to protect them as much as possible from moisture, sudden changes of temperature, and from shocks and jars; the instrument should always be carried in its case and should not be removed from it. It should be borne in mind that the aneroid barometer is necessarily of fragile construction. In order that the very slight movements of the side of the vacuum chamber can be so multiplied and measured as to be indicated accurately by the index, exceedingly delicate mechanism is required, which is necessarily of a frail character and liable to get out of order. Great care should

therefore be exercised in the use and transportation of the instrument to protect it from shocks and jars, and from the direct rays of the sun, and also to prevent it from being affected by artificial heat, as the heat of the body in carrying, or of an artificially warmed room. In making observations the barometer should always be read in a horizontal position in the open air and should be tapped gently before reading; it should have as nearly as possible the temperature of the surrounding air, which should also be observed at the same time. When a barometer with an adjustable scale is used, before making the observations the zero of the altitude scale should be set to the reading of a standard mercurial barometer at the point to which the subsequent observations are to be referred, and this adjustment should remain unchanged while the observations are being taken.

79. Determining Elevations With the Barometer.— By means of the barometer differences of elevation can be determined with a reasonable degree of approximation, if proper care is exercised. It has been stated that the altitude scale of an aneroid barometer can be read to single feet of elevation. But from this it must not be inferred that elevations can be determined with any such degree of accuracy by means of a barometer. This instrument merely registers the atmospheric pressure at the time and place of observation, and by this means indicates the approximate difference in elevation between different places, when the other atmospheric conditions are the same. But owing to various causes, the pressure of the atmosphere changes almost constantly, and for the same place varies greatly, so that a change in the reading of the barometer does not necessarily indicate a change in elevation. Consequently, observations at the two stations whose difference of elevation is required should be made as nearly simultaneous as possible. For the best results two barometers should be used, one at each of the stations whose difference of elevation is to be determined. They should both be adjusted to the same standard mercurial barometer and should be compared

before and after the observations are taken. By this means the errors due to atmospheric changes can be largely eliminated. When only one barometer is used the observer should pass from one station to the other as rapidly as possible, and should repeat the observation in the reverse order, usually taking the mean of the two observations. Barometric observations should be made when the humidity of the air and the general atmospheric conditions are as nearly constant as possible; they should not be made in changeable or snowy weather.

80. Formula for Difference of Elevation. — The inner scale of the aneroid barometer is graduated to correspond with the scale of a standard mercurial barometer in such manner that when it is properly adjusted both instruments will read the same under the same pressures. The outer scale is graduated to read feet of altitude with some degree of approximation. All barometric computations for altitude are based on the mercurial barometer, aneroid barometers being adjusted to agree with the mercurial barometer at a temperature of 32° at the sea level in latitude 45°. Since they are based on varying conditions whose laws are not sufficiently understood to be stated by exact mathematical expressions, formulas for the barometric determination of altitudes vary somewhat with regard to the numerical constants used, and, consequently, in the results obtained. But all are derived in substantially the same way and have about the same general form, though some attempt to provide for the variation in a greater number of conditions than others. The assumed fundamental conditions on which they are chiefly based are (1) that the atmospheric strata throughout which the pressures are equal are horizontal, and (2) that the temperature of a vertical column of air of any height will have the same effect as the arithmetical mean of the temperature observed at its top and at its bottom. It is probable that these two conditions are never fulfilled perfectly, but they are always fulfilled with some degree of approximation. The more closely the conditions actually

existing at the time the observations are made correspond to these conditions, the more accurate will be the results obtained.

The weight of a given portion of air decreases from the surface of the earth upward to the assumed surface of the atmosphere. It has been found that as the heights to which the barometer is carried increase in arithmetical progression, the weights of the column of air above the barometer, and, consequently, its readings, decrease in geometrical progression. Consequently, for a constant temperature the difference in height of any two not very distant points on the earth's surface is in a constant ratio to the difference of the logarithms of the readings of the barometer at the two points—that is to say, it is equal to this latter difference multiplied by some constant coefficient. The value of this coefficient is determined by experiment for a constant temperature of 32° F., that is, for the freezing point, and in this country is so expressed that when multiplied by the difference between the logarithms of the readings of the barometer in inches, the product will be the difference in elevation in feet. Let k denote this coefficient; let h_1 and h' denote the reading of the barometer in inches at the lower and higher stations, respectively; and let z denote the difference in elevation of the two stations for a constant temperature of 32° F. The mathematical expression for the conditions just stated will then be

$$z = k\,(\log h_1 - \log h') \qquad \textbf{(15)}$$

This expression is for a constant temperature, but the warmer the air is, the lighter it is; so that a column of warm air of any height will weigh less than a column of air of the same height that is cooler. Consequently, the mercury in warm air falls less in ascending any height, and is relatively higher at the higher station than it would be if the air were cooler, so that if the temperature is above 32° F., the height given by the preceding approximate expression will be too small. The volume of a gas under a constant pressure varies directly as the

temperature. Hence, if we let c denote the coefficient of expansion of air for an increase of temperature of 1° F., the result given by the preceding expression must be increased by its product into c multiplied by the number of degrees F. that the mean temperature of the air is above 32°. Let t_1 and t' denote the temperature of the air at the lower and upper stations, respectively. Then, if the mean temperature is taken at $\frac{t_1 + t'}{2}$, since the above expression is for a constant temperature of 32° F., for observations taken at any temperatures the expression will be

$$z = k(\log h_1 - \log h')\left[1 + c\left(\frac{t_1 + t'}{2} - 32\right)\right] \qquad \textbf{(16)}$$

This is the general expression for the difference of elevation between two points as given by the barometer. The value of the coefficients k and c are as follows:

As is to be expected, the values obtained for k differ somewhat. Values of 60,159, 60,373.6, 60,384.3, 60,520, and probably other values, have been found for this coefficient at the freezing point or temperature of 32° F.

It is probable that much of the difference in these values is due to the attempt in some cases to provide for minor conditions that are not recognized in other cases, and it would be difficult to decide which of the values is nearest to the correct value. The value 60,384.3 will here be used for k, though there appears to be no special reason for believing it to be more accurate than the other values.

The coefficient of expansion of air is from about 0.002034 to 0.002039 for an increase of temperature of 1° F.; a mean of these would be 0.0020365 = $\frac{1}{491}$. It has been estimated that the effect of moisture in the air will change this fraction to $\frac{1}{450}$. By substituting this value for c and a value of 60,384.3 for k in the above expression, it becomes

$$z = 60{,}384.3\,(\log. h_1 - \log h')\left(1 + \frac{t_1 + t' - 64}{900}\right) \qquad \textbf{(17)}$$

This formula has been used by the Engineer Corps of the United States Army and will be used here in all calculations

for difference of elevation, as determined by the barometer, when no other formula or process is specified.

EXAMPLE.—Suppose the barometer at the lower station reads 26.25 inches with the temperature at 72° F. and at the upper station reads 24.95 inches with the temperature at 46° F. What is the difference in elevation?

SOLUTION.—Substituting known values in formula **17**, we have

$$z = 60{,}384.3\,(\log 26.25 - \log 24.95)\left(1 + \frac{72 + 46 - 64}{900}\right)$$

Or, $z = 60{,}384.3 \times .02206 \times 1.06 = 1{,}412$ ft. Ans.

BAROMETER TABLES

81. Different tables have been prepared for the purpose of facilitating the calculations of differences of elevations from the differences in the readings of the barometer. The best of these is the table prepared by Sir G. Airy, late Astronomer-Royal of England. This table is calculated for a barometer reading of 31 inches at sea level and for a mean temperature of 50° F. Very nearly the same results may be obtained by substituting in formula **17** a value of 50 for both t_1 and t', the respective barometer readings for h_1 and h', and using a value of 60,348 for the coefficient k.

Table II is Airy's table; it gives the reading of the barometer corresponding to each successive 50 feet of elevation above sea level, based on a reading of 31 inches at sea level and a mean temperature of 50° F. By means of this table the difference between the elevations of two points can be determined from the barometer readings in the following manner:

82. Case I.—*When the mean temperature is 50° F.:*

Find from the table the height corresponding to each barometer reading, interpolating for the height in case the corresponding barometer reading is not given in the table, but falls between two readings that are given. Subtract the height corresponding to the barometer reading of the lower station, as obtained from the table, from that corresponding to the barometer reading of the higher station; the remainder is the difference in elevation.

TABLE II

Arranged for Temperature of 60° F.

Height in Feet	Aneroid or Corrected Barometer in Inches	Height in Feet	Aneroid or Corrected Barometer in Inches	Height in Feet	Aneroid or Corrected Barometer in Inches	Height in Feet	Aneroid or Corrected Barometer in Inches
0	31.000	3,050	27.718	6,050	24.829	9,050	22.241
50	30.943	3,100	27.667	6,100	24.784	9,100	22.200
100	30.886	3,150	27.616	6,150	24.738	9,150	22.160
200	30.773	3,250	27.515	6,250	24.648	9,250	22.079
150	30.830	3,200	27.566	6,200	24.693	9,200	22.119
250	30.717	3,300	27.465	6,300	24.602	9,300	22.038
300	30.661	3,350	27.415	6,350	24.557	9,350	21.998
350	30.604	3,400	27.364	6,400	24.512	9,400	21.957
400	30.548	3,450	27.314	6,450	24.467	9,450	21.917
450	30.492	3,500	27.264	6,500	24.423	9,500	21.877
500	30.436	3,550	27.214	6,550	24.378	9,550	21.837
550	30.381	3,600	27.164	6,600	24.333	9,600	21.797
600	30.325	3,650	27.115	6,650	24.288	9,650	21.757
650	30.269	3,700	27.065	6,700	24.244	9,700	21.717
700	30.214	3,750	27.015	6,750	24.200	9,750	21.677
750	30.159	3,800	26.966	6,800	24.155	9,800	21.638
800	30.103	3,850	26.916	6,850	24.111	9,850	21.598
850	30.048	3,900	26.867	6,900	24.067	9,900	21.558
900	29.993	3,950	26.818	6,950	24.023	9,950	21.519
950	29.938	4,000	26.769	7,000	23.979	10,000	21.479
1,000	29.883	4,050	26.720	7,050	23.935	10,050	21.440
1,050	29.828	4,100	26.671	7,100	23.891	10,100	21.401
1,100	29.774	4,150	26.622	7,150	23.847	10,150	21.361
1,150	29.719	4,200	26.573	7,200	23.803	10,200	21.322
1,200	29.665	4,250	26.524	7,250	23.760	10,250	21.283
1,250	29.610	4,300	26.476	7,300	23.716	10,300	21.244
1,300	29.556	4,350	26.427	7,350	23.673	10,350	21.205
1,350	29.502	4,400	26.379	7,400	23.629	10,400	21.166
1,400	29.448	4,450	26.330	7,450	23.586	10,450	21.128
1,450	29.394	4,500	26.282	7,500	23.543	10,500	21.089
1,500	29.340	4,550	26.234	7,550	23.500	10,550	21.050
1,550	29.286	4,600	26.186	7,600	23.457	10,600	21.012
1,600	29.233	4,650	26.138	7,650	23.414	10,650	20.973
1,650	29.179	4,700	26.090	7,700	23.371	10,700	20.935
1,700	29.126	4,750	26.042	7,750	23.328	10,750	20.896
1,750	29.072	4,800	25.994	7,800	23.285	10,800	20.858
1,800	29.019	4,850	25.947	7,850	23.242	10,850	20.820
1,850	28.966	4,900	25.899	7,900	23.200	10,900	20.782
1,900	28.913	4,950	25.852	7,950	23.157	10,950	20.744
1,950	28.860	5,000	25.804	8,000	23.115	11,000	20.706
2,000	28.807	5,050	25.757	8,050	23.072	11,050	20.668
2,050	28.754	5,100	25.710	8,100	23.030	11,100	20.630
2,100	28.701	5,150	25.663	8,150	22.988	11,150	20.592
2,150	28.649	5,200	25.616	8,200	22.946	11,200	20.554
2,200	28.596	5,250	25.569	8,250	22.904	11,250	20.517
2,250	28.544	5,300	25.522	8,300	22.862	11,300	20.479
2,300	28.491	5,350	25.475	8,350	22.820	11,350	20.441
2,350	28.439	5,400	25.428	8,400	22.778	11,400	20.404
2,400	28.387	5,450	25.382	8,450	22.736	11,450	20.367
2,450	28.335	5,500	25.335	8,500	22.695	11,500	20.329
2,500	28.283	5,550	25.289	8,550	22.653	11,550	20.292
2,550	28.231	5,600	25.242	8,600	22.611	11,600	20.255
2,600	28.180	5,650	25.196	8,650	22.570	11,650	20.218
2,650	28.128	5,700	25.150	8,700	22.529	11,700	20.181
2,700	28.076	5,750	25.104	8,750	22.487	11,750	20.144
2,750	28.025	5,800	25.058	8,800	22.446	11,800	20.107
2,800	27.973	5,850	25.012	8,850	22.405	11,850	20.070
2,850	27.922	5,900	24.966	8,900	22.364	11,900	20.033
2,900	27.871	5,950	24.920	8,950	22.323	11,950	19.996
2,950	27.820	6,000	24.875	9,000	22.282	12,000	19.959
3,000	27.769						

83. Case II.—*When the mean temperature is above or below 50° F.:*

Proceed as in Case I and correct the result as follows: Add together the temperature of the upper and lower stations; if the sum is greater than 100° F., increase the difference in elevation by $\frac{1}{1000}$ part for every degree in excess of 100°; if the sum is less than 100°, diminish the result by $\frac{1}{1000}$ part for every degree less than 100°. This may be expressed by formula as follows:

$$z = (H' - H_1)\left(1 + \frac{t' + t_1 - 100}{1,000}\right) \qquad \textbf{(18)}$$

in which t' and t_1 denote the temperature at the upper and lower stations, respectively, and H' and H_1 denote the heights in feet corresponding to the barometer readings at the upper and lower stations, respectively, as taken from the table.

EXAMPLE.—Suppose as in the preceding article, that the barometer reads 26.25 inches at the lower station with the temperature at 72° F., and at the upper station reads 24.95 inches with the temperature at 46° F. What is the difference in elevation?

SOLUTION.—By reference to the table, we find that the elevation corresponding to a barometer reading of 26.25 inches is not given, but that elevations of 4,500 feet and 4,550 feet correspond to barometer readings of 26.282 inches and 26.234 inches, respectively. At this height above sea level, therefore, 50 feet difference in elevation corresponds to a difference of 26.282 − 26.234 = .048 inch in the reading of the barometer. The difference between a barometer reading of 26.25 inches and that corresponding to an elevation of 4,500 feet is equal to 26.282 − 26.25 = .032 inch, which corresponds to a difference of elevation equal to $\frac{.032}{.048} \times 50 = 33.3$ feet. Hence, a barometer reading of 26.25 inches corresponds to a tabular elevation of 4,500 + 33.3 = 4,533.3 feet. Likewise, elevations of 5,900 and 5,950 correspond to barometer readings of 24.966 and 24.920 inches, respectively, giving at this elevation a difference of 24.966 − 24.920 = .046 inch for 50 feet difference of elevation. For a difference of 24.966 − 24.95 = .016 inch in the barometer reading, the difference in elevation is equal to $\frac{.016}{.046} \times 50 = 17.4$ feet, and, consequently, a barometer reading of 24.95 inches corresponds to a

tabular elevation of 5,900 + 17.4 = 5,917.4 feet. By now applying formula **18**, we get for the difference in elevation

$$z = (5{,}917.4 - 4{,}533.3) \times \left(1 + \frac{72 + 46 - 100}{1{,}000}\right)$$

$$= 1{,}384.1 \times 1.018 = 1{,}409 \text{ feet, closely. Ans.}$$

Formula **17** gives 1,412 feet for the difference in elevation.

EXAMPLES FOR PRACTICE

NOTE.—In the following examples each answer is given to the nearest foot only. It is useless and misleading to calculate the results of barometric observations closer than this, since the instrument itself cannot be relied on to give results as close as to the nearest foot.

1. Reading of barometer h_1 at the lower station is 28.44 inches and the temperature t_1 is 60°. At the higher station the reading h' is 24.33 inches and the temperature t' is 40°. Required the difference in elevation between the two stations. Ans. 4,257 ft., closely

2. In the preceding example determine the difference in elevation by means of Table II. Ans. 4,254 ft., closely

3. Reading of barometer h_1 at the lower station is 29.52 inches and the temperature t_1 is 70°. At the higher station the reading h' is 27.15 inches and the temperature t' is 62°. Required the difference in elevation between the two stations. Ans. 2,361 ft., nearly

4. In the preceding example, determine the difference in elevation by means of formula **18** and Table II. Ans. 2,354 ft., nearly

CIRCULAR CURVES

THEORY OF CIRCULAR CURVES

DEFINITIONS AND FUNDAMENTAL PRINCIPLES

23.* **Tangents.**—The line of a railroad consists of a series of straight lines connected by curves. Each two adjacent lines are united by a curve having the radius best adapted to the conditions of the surface. The straight lines are called **tangents**, because they are tangent to the curves that unite them. In order to determine the curve by which two tangents are to be united, the angle between them must be known.

24. Intersection of Tangents.—Let AB and CD, Fig. 17, be two intersecting tangents. It is desired

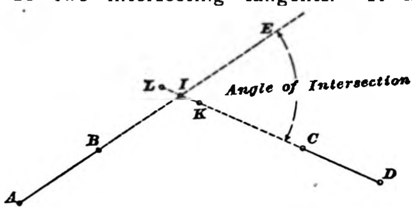

FIG. 17

to determine their point of intersection and the angle that they make with each other. A picket is first set up at B and another at A, or at some other point on

*On account of the omission, after this section was printed, of some matter that was transposed to another section, the article and figure numbers begin here with 23 and 17, respectively, instead of 1.

the line $A\,B$. The transit is set up at C, the telescope is backsighted to D, then reversed forward and sighted at a flag held in the line given by the transit and also about in range with the pickets at A and B, as nearly as the flagman can judge. With a little practice he can determine approximately the intersection I of the two lines. Two temporary hubs are then driven at K and L in the line $C\,D$, one on either side of the estimated position of the point of intersection I. These hubs are centered by driving a tack half its length in the top of each, exactly on the line $C\,D$, and a cord is stretched between the tacks. The instrument is then set up at B, the telescope is backsighted to A, reversed forward, and a flag lined in at I where the line of sight intersects the cord, which will be the intersection of the line $A\,B$ prolonged with $C\,D$ prolonged. A permanent hub is now driven at I and centered by driving a tack at the exact point where the prolongation of $A\,B$, as given by the instrument, crosses the cord connecting the tacks in the hubs at K and L. Having thus located the point I, the instrument should be set over it, and the angle $E\,I\,C$ should be measured.

The point I is the intersection of the tangents $A\,B$ and $C\,D$. Such a point is called a **point of intersection**; it is commonly designated by the letters P. I., and the guard stake indicating its position is marked P. I. The external angle $E\,I\,C$, formed by the intersecting tangents, is called the **intersection angle** or the **angle of intersection.**

25. Curves.—Railroad curves are usually circular and are divided into three general classes, namely: simple, compound, and reverse curves.

A **simple curve** is a curve having but one radius, as the curve $A\,B$, Fig. 18, whose radius is $A\,C$.

A **compound curve** is a continuous curve composed of two or more arcs of different radii, as the curve $C\,D\,E\,F$, Fig. 19, which is composed of the arcs $C\,D$, $D\,E$, and $E\,F$, whose respective radii are $G\,C$, $H\,D$, and $K\,E$. In the general class of compound curves may be included what are

known as **easement curves, transition curves,** and **spiral curves,** now used very generally on the more important

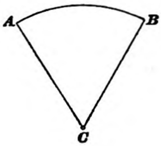

FIG. 18

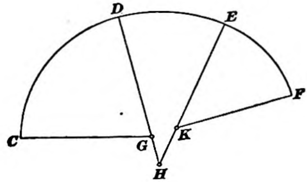

FIG. 19

railroads. These do not belong properly to the subject of Surveying.

A **reverse curve** is a continuous curve composed of the arcs of two circles of the same or different radii, the centers of which lie on opposite sides of the curve, as in Fig. 20. The two arcs composing the curve meet at a common point M, called the **point of reversal,** at which point they are tangent to a common line perpendicular to the line joining their centers. Reverse curves are becoming less common on railroads of standard gauge.

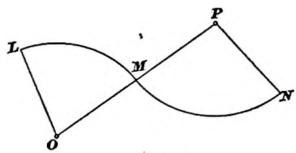

FIG. 20

26. Geometry of Circular Curve.—In studying the subject of curves, a thorough knowledge of the principles of geometry relating to the circle is very essential. The elementary parts of a circular curve and the related geometric values required in laying it out are illustrated in Fig. 21. $A B$ and $C D$ represent two tangents that are united by the curve $B G H K C$. The prolongations of these tangents intersect at E and form the angle of intersection $F E C$. The following principles of geometry are of especial importance as relating to curves, and are restated here for convenience of reference, the form of statement being modified to

suit present requirements by substituting the subtending chord for the arc:

1. A tangent to a circle is perpendicular to the radius at its tangent point. Thus, AE, Fig. 21, is perpendicular to BO at its tangent point B, and CD is perpendicular to CO at C.

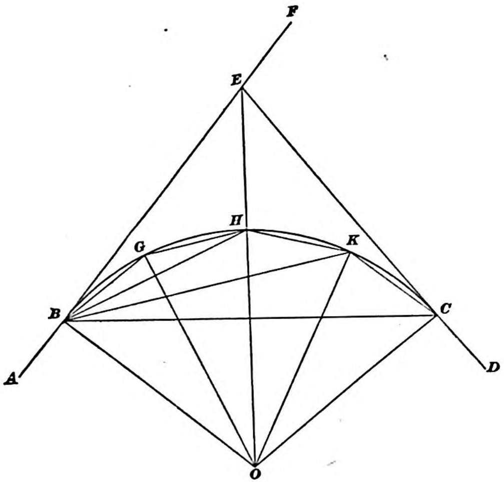

FIG. 21

2. Two tangents to a circle from any point without the circle are equal in length, and make equal angles with the chord joining their points of tangency. Thus BE and CE are equal, and the angles EBC and ECB are equal.

3. An angle not exceeding 90° formed by a chord and the tangent at one of its extremities is equal to one-half the central angle subtended by the chord. Thus, the angle $EBC = ECB =$ one-half BOC.

4. An angle not exceeding 90° having its vertex in the circumference of a circle and subtended by a chord of the

circle, is equal to one-half the central angle subtended by the chord. Thus, the angle GBH, whose vertex B is in the circumference, is subtended by the chord GH and is equal to one-half the central angle GOH, subtended by the same chord GH.

5. Equal chords of a circle subtend equal angles at its center and also in its circumference, if the angles lie in corresponding segments of the circle. Thus, if BG, GH, HK, and KC are equal, $BOG = GOH$, $GBH = HBK$, etc.

6. The angle of intersection between any two tangents of a circle is equal to the central angle subtended by the chord joining the two points of tangency. Thus, the angle $CEF = BOC$.

7. A radius that bisects any chord of a circle is perpendicular to the chord.

27. The Angular Unit.—The rate of divergence of the two lines forming an angle, from their common or angular point, determines the size of the angle. As we know from geometry, the unit of angular measurement is the degree, equal to $\frac{1}{360}$ part of a circle. In the calculations relating to curves, it is convenient to remember that two lines forming an angle of 1° with each other will, at a distance of 100 feet from the angular point, diverge by 1.745 feet, very closely.

In Fig. 22, the lines AB and AC, meeting at the point A, are supposed to form an angle of 1°, and the angle BAC is

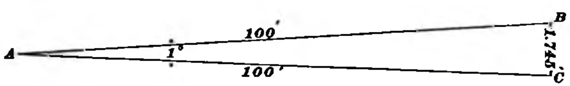

FIG. 22

measured by the arc BC, described with the radius AB, which is 100 feet in length. The chord BC, which is the straight line joining the extremities of that arc, is equal to twice the sine of one-half the angle BAC, multiplied by 100. The natural sine of 30 minutes, expressed to seven decimal

places, is .0087265,* and $2 \times .0087265 \times 100 = 1.7453$. Hence, the length of the chord BC, expressed to three decimal places, which is close enough for all practical purposes, is 1.745 feet.

28. Degree of Curvature.—The sharpness of a curve, or the rapidity with which it turns, may be designated by its radius; that is, by the radius of the circular arc composing it. In this country, however, this property of a curve is usually designated by the number of degrees contained in the central angle of the curve subtended by a chord of 100 feet. This is called the **degree of curvature,** or **degree of curve.** Thus, if the chord BG, Fig. 21, is 100 feet long, and the angle BOG is 1°, the curve is called a **one-degree curve**; but if with the same length of chord the angle BOG is 4°, the curve is called a **four-degree curve**; while if the angle BOG is 10°, the curve is called a **ten-degree curve**; etc. These are commonly written 1° curve, 4° curve, 10° curve, etc. That is to say, a chord of 100 feet on the curve, or, at the circumference of the circle of which the curve is an arc, will subtend an angle of 1° at the center of a 1° curve, an angle of 4° at the center of a 4° curve, or an angle of 10° at the center of a 10° curve, etc., formed between two radii.

It should be noticed, however, that the term *degree of curvature*, as used in this sense, is not a perfectly general term, since it applies only to curves of comparatively long radii. Evidently it can be applied only to curves having diameters not less than the designated length of chord (100 feet), and, consequently, the extreme limit of its application is reached in the case of a circle whose diameter is 100 feet. The degree of curvature of an arc of such a circle would be 180°. But, although this is the theoretical limit of application of the expression *degree of curvature*, the practical limit is reached in the designation of curves of much larger radii, such as will contain several chords of 100 feet

* The value of the sine is here stated to seven decimal places, because the value .00873, as given in a table of five decimal places, would give for the divergence of the lines in 100 feet the value $2 \times .00873 \times 100 = 1.746$, which is not as close as the value 1.745.

each in a semi-circumference. In sharper curves, or curves of smaller radii, it is customary to designate the rate of curvature by the length of the radius.

29. Deflection Angle.—The angle formed between any chord of a curve and a tangent to the curve at one extremity of the chord is the **deflection angle** for the chord; that is, it is the angle by which the chord is deflected from the tangent. According to the principle stated in Art. **26**, 3, this angle is equal to one-half the central angle subtended by the chord. Thus, $E\,B\,G$, Fig. 21, is the deflection angle by which the chord $B\,G$ is deflected from the tangent $B\,E$. According to the principle just referred to, it is equal to one-half the central angle $B\,O\,G$. Since it is customary to designate a curve by its degree of curvature, which is equal to the central angle subtended by a chord of 100 feet, it follows from what has just been stated *that the deflection angle for a chord of 100 feet is equal to one-half the degree of curvature.*

But, according to the principle stated in Art. **26**, 4, the angle $G\,B\,H$, which the chord $B\,H$ makes with the preceding chord $B\,G$, is equal to one-half the central angle $G\,O\,H$, subtended by the chord $G\,H$, so that if the chord $G\,H$ is equal to the chord $B\,G$, the central angle $G\,O\,H$ will be equal to $B\,O\,G$ and the angle $G\,B\,H$ equal to $E\,B\,G$. The same is true of the angles $H\,B\,K$ and $K\,B\,C$. Hence, it also follows from this principle that *when a series of chords is deflected from a common point on a curve to consecutive stations 100 feet apart along such curve, the common difference between the consecutive deflection angles is always equal to one-half the degree of curvature.*

But while the name deflection angle is applied to the angle between any chord of a curve and the tangent at one extremity of the chord, it is customary to restrict its application to an angle of this character that is subtended by a chord of 100 feet, and the term deflection angle is commonly understood to mean the deflection angle subtended by such a chord. When used in this sense it may be designated as

the **regular deflection angle.** Hence, in any curve, *the regular deflection angle is equal to one-half the degree of curvature.*

30. Relation Between Deflection Angle and Radius. In Fig. 23, let OL be perpendicular to the chord BG at its

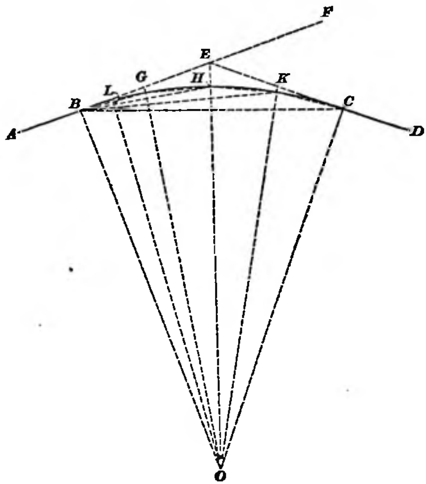

FIG. 23

center. In the right triangle BOL, we have

$$OB = \frac{BL}{\sin BOL};$$

but OB is the radius of the curve, BL is one-half the chord BG, and the angle BOL, being equal to one-half BOG, is equal to the deflection angle EBG. If we now denote the radius OB by R, the chord BG by c, and the deflection angle $EBG = BOL$ by D, the equation written above becomes

$$R = \frac{\frac{1}{2}c}{\sin D} \qquad \textbf{(9)}$$

This formula is perfectly general and applies to any length of chord. If we let D_{100} denote the deflection angle for a chord of 100 feet and substitute 100 for c, we shall have

$$R = \frac{50}{\sin D_{100}} \qquad \textbf{(10)}$$

31. Approximate Value of Radius.—When the degree of curve is not too great, its radius can be determined approximately and closely enough for most practical purposes by dividing the radius of a 1° curve by the degree of the given curve. The deflection angle of a 1° curve is ½° or 30 minutes. By using seven-place logarithmic tables and substituting the logarithmic sine of 30 minutes in formula **10**, we have

$$\begin{aligned} \log 50 &= 1.6989700 \\ \log \sin 0° 30' &= \bar{3}.9408419 \\ \hline \log R &= 3.7581281 \end{aligned}$$

Hence, $R = 5{,}729.65$ feet.

This is the correct value of the radius of a 1° curve to two decimal places, but in order to obtain it seven-place logarithmic tables must be used; it will not be given exactly by five-place tables. In practice, the radius of a 1° curve is commonly taken at 5,730 feet.

If we now refer to a table of natural sines, we shall see that for small angles the sines are very nearly proportional to the angles, so that the sine of 1°, the deflection angle of a 2° curve, is approximately and very closely equal to twice the sine of 0° 30', the deflection angle of a 1° curve, etc. It is therefore evident that the above formula will give values of the radius R that are very nearly inversely proportional to the deflection angles, and consequently, to the degrees of curve.

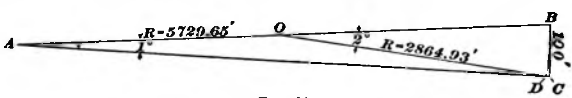

FIG. 24

In order to illustrate this, let $A\ B$ and $A\ C$, Fig. 24, be radii 5,729.65 feet in length, forming an angle of 1° at the

center A; then the arc BC subtended by these radii will be 100 feet in length. The curve BC is called a 1° curve. If, from the point O as a center, with a radius OB equal to 2,864.93 feet, we describe an arc BD 100 feet in length, the radii OB and OD will form an angle of 2° at the center O, and the curve BD is called a 2° curve. But, the radius of the 1° curve, divided by 2, or $5{,}729.65 \div 2 = 2{,}864.83$, is very nearly equal to 2,864.93, the radius of the 2° curve. And likewise, the radius of the 1° curve divided by 3, or $5{,}729.65 \div 3 = 1{,}909.88$, is very nearly equal to the true radius of a 3° curve, which is 1,910.08 feet, etc. Hence, if we assume the radius of a 1° curve to be 5,730 feet, and denote the degree of curve by D_c, the approximate value, in feet, of the radius of a curve of greater degree can be found by the formula

$$R = \frac{5{,}730}{D_c} \qquad \textbf{(11)}$$

The values of the radii given in the Table of Radii and Deflections were calculated by formula **10**, seven-place logarithmic tables being used; values quite close to these, however, can be obtained by means of five-place tables.

The results obtained by formula **11**, however, are sufficiently accurate for most practical purposes, for curves of from 1° to 10°. But for sharp curves, that is, for those exceeding about 10°, the radii should be found by means of formula **10**, especially if they are to be used as a basis for further calculation. The error increases as the degree of curve increases, as is shown by the following examples:

EXAMPLE 1.—What is the radius of a 4° curve?

SOLUTION.—Applying formula **10**, knowing that the deflection angle D_{100} of a 4° curve is 2°, we have

$$R = \frac{50}{.0349} = 1{,}432.66 \text{ ft.}$$

Applying formula **11**, we have

$$R = \frac{5{,}730}{4} = 1{,}432.5 \text{ ft.}$$

In this case the error is only .16 foot, and may be ignored in practical work.

EXAMPLE 2.—What is the radius of a 30° curve?

SOLUTION.—Applying formula **10**, we have

$$R = \frac{50}{.25882} = 193.18 \text{ ft.}$$

Applying formula **11**, we have

$$R = \frac{5,730}{30} = 191 \text{ ft.}$$

In this case the error is 2.18 feet.

EXAMPLES FOR PRACTICE

NOTE.—In the following exercises, compute the radius, (*a*) by formula **10** and (*b*) by formula **11**, and compare the results.

1. What is the radius of a 5° curve? Ans. $\begin{cases}(a) & 1,146.3 \text{ ft.} \\ (b) & 1,146.00 \text{ ft.}\end{cases}$

2. What is the radius of a 9° curve? Ans. $\begin{cases}(a) & 637.27 \text{ ft.} \\ (b) & 636.67 \text{ ft.}\end{cases}$

3. What is the radius of a 15° curve? Ans. $\begin{cases}(a) & 383.05 \text{ ft.} \\ (b) & 382.00 \text{ ft.}\end{cases}$

4. What is the radius of a 20° curve? Ans. $\begin{cases}(a) & 287.94 \text{ ft.} \\ (b) & 286.50 \text{ ft.}\end{cases}$

OTHER VALUES RELATING TO CURVES

32. Subchords.—On curves of short radii, that is, curves of about 20° and upwards, center stakes are usually driven at intervals of 25 feet, nominally, that is, at intervals of one-fourth the length of the arc subtended by a chord of 100 feet. Chords shorter than 100 feet are commonly called **subchords.** For curves of very long radii, the chord and arc may be assumed to be of the same length. But as the degree of curvature increases, the difference in length between the arc and chord also increases, and for curves above 20° the excess in the length of the arc over that of the chord becomes very considerable. If in Fig. 23, the chord BC is 100 feet long, the arc $BGHKC$ must have a length greater than 100 feet; and if the arcs BG, GH, HK, and KC are each equal to one-quarter the arc BHC, then the equal chords BG, GH, HK, and KC subtending

these equal arcs must each have a length greater than one-quarter BC, or, since $BC = 100$ feet, each of these chords must have a length greater than 25 feet. The actual length of the subchord can be calculated by means of formula **9**, which, when transposed to express the value of the chord, gives

$$c = 2R \sin D \qquad \textbf{(12)}$$

This formula applies to any length of chord when D is the deflection angle for the chord considered. But if the chords BG, GH, HK, and KC each subtend one-fourth of the arc BHC, they are equal to each other, and according to the principles stated in Art. **26,** 3 and 5, the angles EBG, GBH, HBK, and KBC are also equal to each other, and consequently, are each equal to one-fourth the deflection angle EBC. Hence, the length of one of the chords BG, GH, HK, or KC, subtending one-fourth of the arc BHC, is given by merely substituting in the above formula the sine of one-fourth the deflection angle EBC. We thus have the following important principle:

The deflection angle for any fractional part of an arc is equal to the corresponding fractional part of the deflection angle for the whole arc.

In applying this principle for the purpose of computing the deflection angles for subchords, it is customary to consider the deflection angle to be proportional to the length of the chord. Though this practice is not strictly correct, the resulting error is very slight in curves of large radius.

EXAMPLE 1.—Suppose the curve BHC, Fig. 23, to be a 20° curve, and the chord BC subtending it to be 100 feet in length. What is the length of the subchord BG subtending one-fourth the arc BHC?

SOLUTION.—Since the degree of curve is the central angle subtended by a chord of 100 feet, and the chord BC is 100 feet in length, the central angle BOC is 20°, and as the arc BG is one-fourth of the arc BHC, the central angle subtended by the chord BG is equal to $\frac{1}{4} \times 20° = 5°$. The deflection angle for the chord BG, being equal to one-half the central angle subtended by the same chord, is equal to $\frac{1}{2} \times 5° \, 00' = 2° \, 30'$. Or, also, since the deflection angle for the chord BC is $\frac{1}{2} \times 20° = 10°$, according to the principle just stated, the deflection angle for the chord BG is equal to $\frac{1}{4} \times 10° = 2° \, 30'$. Sin $2° \, 30' = .04362$.

By formula **10** the radius of a 20° curve is equal to $\frac{50}{\sin 10°} = \frac{50}{.17365}$ $= 287.94$ feet. Hence, by substituting these values in formula **12**, we get for the length of the subchord BG the value $c = 2 \times 287.94 \times .04362 = 25.12$ ft. Ans.

Consequently, in measuring each of the subchords BG, GH, HK, and KC, 25.12 feet should be used instead of 25 feet.

EXAMPLE 2.—What is the deflection angle for a chord of 15 feet in a 3° curve?

SOLUTION.—The deflection angle for a chord of 100 feet in a 3° curve is equal to one-half the degree of curve, or $1° 30' = 90'$. According to the principle stated above, therefore, the deflection angle for a chord of 15 feet in a 3° curve is equal to

$$\frac{15}{100} \times 90' = 13.5' \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The degree of a curve is 5° 30'; what is the deflection angle for a chord of 16.2 feet? Ans. 0° 26.7'

2. The degree of a curve is 7° 15'; what is the deflection angle for a chord of 38.4 feet? Ans. 1° 23½'

3. What is the length of the chord subtending one-fifth of the arc subtended by a chord of 100 feet in a 16° curve? Ans. 20.061 ft.

4. In a 10° curve, what is the length of the chord subtending .12 of the length of the arc subtended by 100 feet? Ans. 12.013 ft.

33. Required Degree of Curve.—In proceeding to unite two tangents by a curve when the angle of intersection between the tangents has been measured, the first matter to be decided is the degree of the curve that is to unite them. This will depend on the character of the work and the topographical conditions of the surface. In railroad work the requirements of the anticipated traffic naturally impose a limit on the sharpness of curve or degree of curvature allowed. On the other hand, while it is always desirable to connect the tangents by as light and easy a curve as possible, the conditions of the surface usually limit this, especially in hilly country. No satisfactory rule can be given, but in general, the curvature should be as easy, that is, the degree of curve should be as small, as the conditions will permit.

The smaller the degree of curve, the greater will be the length of curve necessary to effect a given amount of curvature, and, consequently, the degree of curve is usually determined by the length of the line or form and extent of the space within the limits of which the required amount of curvature must be effected, this being governed by the topographical features of the surface and other controlling conditions. In level country, the degree of curvature will be determined by the angle of intersection and the tangent distances. (See Art. **34.**) When the angle of intersection and the length of the radius of the curve have been determined, the tangent distances can be calculated by means of formula **13,** or when the angle of intersection and the tangent distances have been determined, the radius of the curve can be calculated. For the same angle of intersection, the length of the curve will vary inversely as the degree of curvature. It is customary to fix a limiting or maximum degree of curvature, according to the requirements of the anticipated traffic, which must not be exceeded.

34. Tangent Distances.—When the degree of curve has been decided, the next step in order is the location of the points on the tangents where the curve begins and ends. The point where the curve begins is called the **point of curve,** and is designated by the letters P. C.; the point where the curve terminates is called the **point of tangency,** and is designated by the letters P. T. According to the principle stated in Art. **26,** 2, these two points are equally distant from the point of intersection of the tangents. The distance of the P. C. and P. T. from the P. I. is called the **tangent distance.** The chord connecting the P. C. and P. T. of a curve is commonly called its **long chord.**

In Fig. 23, let AB and CD be tangents intersecting at the point E. From the principle stated in Art. **26,** 6, we know that $BOC = FEC$; hence, the angle $BOE = \frac{1}{2} FEC$. From the right triangle EBO, we have

$$\frac{BE}{BO} = \tan BOE = \tan \tfrac{1}{2} FEC$$

If we now let I denote the angle of intersection FEC, and T the tangent distance BE, and, remembering that BO is the radius R, substitute these values in the foregoing expression, we shall have, by clearing of fractions,

$$T = R \tan \tfrac{1}{2} I \qquad \textbf{(13)}$$

From the point of intersection, the tangent distance, as determined by formula **13**, is measured back on both tangents, thus determining the tangent points, as the points B and C, Fig. 23. Plugs are driven at both points and centered as to line and measurement, and guard stakes are driven to indicate their positions. If the numbering of the station runs from B toward C, the stake at B will be marked P. C., and the stake at C marked P. T., each stake being marked also with the number of the station and the plus.

EXAMPLE.—Suppose that the intersection angle EFC, Fig. 23, is equal to 40° 00′ and it is decided to unite the tangents AB and CD by a 10° curve. What is the tangent distance?

SOLUTION.—By applying formula **10**, we find that for a 10° curve the radius R is equal to 573.7 feet. One-half the intersection angle is 20° 00′, and the natural tangent of 20° 00′ = .36397. Hence, by applying formula **13** we find the tangent distance to be

$$T = 573.7 \times .36397 = 208.81 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. The point of intersection of two tangents is at Station 20 + 87 and the angle of intersection is 16° 13′. If the two tangents are united by a 3° curve, what is (*a*) the tangent distance, and (*b*) the station of the P. C.?

Ans. (*a*) 272.13 ft.; (*b*) Sta. 17 + 64.87

2. The point of intersection of two tangents is at Station 5 + 84.32 and the angle of intersection is 59° 20′. If the two tangents are united by an 8° 30′ curve, what is (*a*) the tangent distance, and (*b*) the station of the P. C.?

Ans. (*a*) 384.32 ft.; (*b*) Sta. 2

3. Suppose that the point of intersection of two tangents is at Station 40 + 25, and the angle of intersection is 21° 35′. Assuming the two tangents to be united by a 4° 15′ curve, determine (*a*) the tangent distance, and (*b*) the station of the P. C.

Ans. (*a*) 257.03 ft.; (*b*) Sta. 37 + 67.97

FIELD WORK

35. To Lay Out a Curve With a Transit.—The tangent points for a curve having been set as described in the preceding article, the curve can easily be run in between them. Continuing the example of the preceding article, suppose the point B, Fig. 23, which is the P. C. of a 10° curve to the right, happens to be at Station 58 of the tangent $A\ B$, and that it is desired to run in the curve between B and C, setting a stake at each full station, that is, at the end of each chord of 100 feet. The transit is set up at B, the P. C., and, with the vernier set at zero, the telescope is sighted to the point of intersection E, or it may be backsighted to A, if a more convenient or better sight can be obtained. Since the central angle $B\ O\ G$, measured by a chord of 100 feet, is 10°, the deflection angle $E\ B\ G$ subtended by the same chord will be one-half $B\ O\ G$, or 5°. An angle of 5° 00′ is turned to the right on the vernier, giving the line $B\ G$, a distance of 100 feet is measured from B along the line so given, and at the extremity of this measurement the flag is lined in by the instrument, giving the point G, or Station 59, at which a stake marked 59 is driven. An additional 5° 00′ is then turned off, making 10° 00′ from the tangent, and at the end of another measurement of 100 feet the flag is lined in and a stake marked 60 is set for the point H. The operation is continued by turning in succession angles of 5° 00′ each, and measuring for each angle a chord of 100 feet, until a total angle of 20° 00′, or one-half of the intersection angle, is reached. If the work has been performed correctly, the last deflection will bring the head chainman to the point of tangent C. Then, moving the transit to C, and backsighting to B, the angle $B\ C\ E$, turned from the chord $B\ C$ to the tangent $C\ E$, should also be 20° 00′, $= E\ B\ C$.

The P. C. rarely occurs at a full station, however. When it comes at a substation, the chord between it and the next full station will be a subchord. Had the P. C. of the curve come at the substation, say, $57 + 32$, the length of the subchord, or distance to the next full station, would be $100 - 32 = 68$ feet, and the deflection angle for the subchord of 68 feet would be found as follows: The deflection angle for a full station, that is, for 100 feet, is $5° 00' = 300'$; for 1 foot it is $\frac{300}{100}' = 3'$, and for 68 feet it is $68 \times 3 = 204' = 3° 24'$. This angle is turned off from zero and a stake is set on the line given by the transit at a distance of 68 feet from the P. C., which is at Station 58. The remainder of the curve is then run in as already explained, except that the last chord preceding the P. T. will be a subchord also.

36. Length of Curve.—The length of a curve uniting two tangents, as usually understood, is not the actual length of the arc, but it is the length between the tangent points *measured in chords of 100 feet*, which length is always somewhat less than the length of the arc. Since the degree of curve is equal to the central angle subtended by a chord of 100 feet, the number of such chords in the curve will be equal to the quotient obtained by dividing the total angle at the center by the degree of curve. Since also the total angle at the center is equal to the angle of intersection between the tangents, it follows that when the angle of intersection has been measured and the degree of curve decided on, the length of the curve, expressed in stations of 100 feet, can be found by dividing the angle of intersection by the degree of curve. The following rule is a statement of this principle:

Rule.—*Express the angle of intersection in degrees and decimals by reducing the minutes, if any, to decimals of a a degree, and divide by the degree of curve; the quotient will be the length of the curve, in chords of 100 feet and decimals thereof. This quotient, when multiplied by 100, will be the length of the curve in feet, as measured in chords of 100 feet.*

The P. C. and P. T. having been set and the station of the P. C. determined by subtracting the tangent distance from the station of the P. I., the station of the P. T. is found by adding the calculated length of the curve to the station of the P. C.

Example.—Suppose that the angle of intersection is 32° 42′, that the tangents are to be united by a 6° curve, and that the station of the P. C. is 57 + 32. What is (*a*) the length of the curve and (*b*) the station of the P. T.?

Solution.—(*a*) The angle of intersection 32° 42′ reduced to the decimal form is equal to 32.7°. As each central angle of 6° will subtend a chord of 100 feet, there will be as many such chords in the curve as 6 is contained times in 32.7, which is 5.45; that is, there will be in the curve five chords of 100 feet each, plus one chord of 45 feet, or in all a length of 500 + 45 = 545 feet, which is the required length of the curve. Ans.

(*b*) The length of the curve 5 + 45, added to the station of the P. C., 57 + 32, gives 62 + 77 as the station of the P. T. Ans.

Having set all the full stations on the curve, the last chord measurement is in this case 77 feet, while the total deflection angle from the tangent is 16° 21′, or half of the intersection angle 32° 42′.

Another method of calculating the length of the curve is as follows. The sum of all the deflection angles is equal to one-half the intersection angle. The intersection angle being 32° 42′, one-half of which, or the total deflection for the P. T., is equal to 16° 21′, which, reduced to minutes, equals 981′. The deflection for 100 feet is $\frac{6}{2}^\circ = 3^\circ = 180'$, and the deflection for 1 foot is $\frac{180}{100}' = 1.8'$; then, 981′, the total deflection, divided by 1.8′, gives 545 feet as the required length of the curve. Ans.

EXAMPLES FOR PRACTICE

1. Suppose that 16° 13′ is the angle of intersection between two tangents that are united by a 3° curve, and that the P. C. is at Station 17 + 64.87. What is (*a*) the length of the curve and (*b*) the station of the P. T.?

Ans. { (*a*) 540.56 ft.
(*b*) Sta. 23 + 5.43

2. Suppose that 59° 20′ is the angle of intersection between two tangents that are united by an 8° 30′ curve, and that the P. C. is at Station 2. What is (*a*) the length of the curve and (*b*) the station of the P. T.?

Ans. { (*a*) 698.04 ft.
(*b*) Sta. 8 + 98.04

3. Suppose that 21° 35′ is the angle of intersection between two tangents that are united by a 4° 15′ curve, and that the P. C. is at Station 37 + 67.97. What is (*a*) the length of the curve and (*b*) the station of the P. T.?

Ans. { (*a*) 507.84 ft.
(*b*) Sta. 42 + 75.81

4. Suppose that 20° is the angle of intersection between two tangents that are united by a 4° curve, and that the P. C. is at Station 7 + 47.37 What is (*a*) the length of the curve and (*b*) the station of the P. T.?

Ans. { (*a*) 500 ft.
(*b*) Sta. 12 + 47.37

INTERMEDIATE INSTRUMENT POINTS ON CURVE

37. Method by Zero Tangent.—It happens frequently that there are obstructions in the line of curve so that

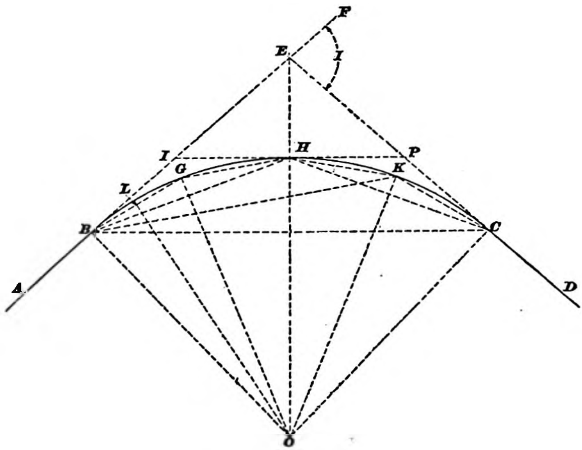

Fig. 25

the entire curve cannot be run in from the P. C. on account of obstructions to the view. This is usually the case in either hilly or timbered country. In such cases the curve is run in as far as the stations are visible from the P. C., and at the last station that can be seen a hub is set and centered,

and the transit moved forward and set up at this intermediate transit point. For example, in Fig. 25, suppose that the station at H, 200 feet from the P. C., which is at B, is the last point on the curve that can be set from the P. C. A plug is driven at H and centered carefully by a tack driven at the point. The transit is now moved forward and set up at H. Since the deflection angle EBH is 10° to the right, an angle of 10° is turned to the left from zero and the vernier clamped. The instrument is then sighted to a flag at B, the lower clamp set, and by means of the lower tangent screw the cross-hair is made to exactly bisect the flag. The vernier clamp is then loosened, the vernier set at zero, and the telescope plunged. The line of sight will then be on the tangent IP, and the deflection angles to K and C can be turned off from this tangent and the stations at K and C located in the same manner that the stations at G and H were located from B. For according to the principle stated in Art. **26**, 2, the angle at IHB between the tangent IH and the chord BH is equal to the angle EBH between the tangent EB and the same chord.

This method of setting the vernier for the backsight when the instrument is moved forward to a new instrument point on the curve is favored by many engineers. It is sometimes called the **method by zero tangent.** The essential principle of the method is that *the vernier always reads zero when the instrument is sighted on the tangent to the curve at the point where the instrument is set, and the deflection angles are made to read from the tangent to the curve at this point in the same manner as though this point were the P. C. of the curve.*

38. Method by Continuous Vernier.—A method of turning off the deflection angles that is commonly employed in railroad practice is what is known as the **method by continuous vernier.** In this method, when the curve has been run in as far as expedient from the P. C. and the instrument is moved forward and set up over another station on the curve, the vernier is set at zero before

taking the backsight on the P. C., where the instrument was previously set up. The backsight is then taken, the centers clamped, the telescope reversed, the plates unclamped, and the regular deflection angle for the next station is turned off the same as though the instrument were at the P. C. The regular deflection angles for the stations following the instrument point are turned off in order, in the same manner, that is, from the zero at the P. C., the same as though the instrument were at the P. C. This will be understood more clearly from an example.

Suppose that the Station *C*, Fig. 25, instead of being the P. T. of the curve is merely a point on the curve 400 feet from the P. C., which is at *B*, and that it is the last point on the curve that can be set with the instrument at *B*. Having located the point *C*, and the transit having been moved forward and set up over this point, the vernier is set at zero and a backsight taken on the P. C. at *B*. According to the principle stated in Art. **26**, 2, the angle *B C E* is equal to the deflection angle *E B C* for the Station *C*; hence, the vernier being set at zero when the backsight is taken to *B*, if the deflection angle for Station *C*, which in this case is 20°, is turned off to the right, the telescope will be in line with the tangent *E D* at the point *C*. The following stations along the curve can therefore be located from this tangent by turning off the deflection angles in consecutive order the same as though the instrument were at the P. C. at *B*, since the deflection angle for each station is greater than that for the preceding station by the deflection angle for a 100-foot chord. For a 10° curve, the deflection angle for a chord of 100 feet is 5°, and in this case, therefore, each station following *C* can be located by deflecting angles of 5°, 10°, 15°, etc. in consecutive order from the tangent *E D*. But since, after backsighting to the P. C., a deflection of 20°, corresponding to the regular deflection angle for Station *C*, was turned off in order to bring the telescope in line with the tangent *E D*, it is evident that adding 5°, 10°, 15°, etc., consecutively, to the deflection angle for Station *C* will give 25°, 30°, 35°,

etc. as the deflection angles for the stations following C in their respective order. Hence, these deflection angles may be used in locating the respective stations when the instrument is at C, the same as though it were at B, if the vernier is set at zero when the backsight is taken to the P. C. at B.

If it is necessary to move the instrument again and set it up at some following station on the curve, from which it is impossible to observe the P. C., the vernier may be set at the deflection angle for the last instrument point C, in this case 20°, and the backsight taken on this point instead of setting the vernier at zero, as when sighting to the P. C. The stations following the new instrument point can then be located by their regular deflection angles, the same as if they were located from the P. C. In like manner, with the instrument set at any station of the curve, and the backsight taken on any station previously located, if previous to taking the backsight the vernier is set at the deflection angle for the station sighted to, any station visible along the curve can then be located by its regular deflection angle, the same as though the instrument were set up at the P. C.

This method possesses the advantage of permitting the deflection angle for each 100-foot station along the curve to be calculated in regular order by adding one-half the degree of curvature to the deflection angle of the preceding station. Any portion of the curve can be run in from the notes without further calculation. The length of curve between any two stations can also be calculated from the difference between their deflection angles, by dividing twice this difference by the degree of curvature.

DEFLECTION OFFSETS

39. Tangent and Chord Deflections.—Let AB, Fig. 26, be a tangent joining the curve $BCEH$ at B. If the tangent AB is prolonged to D, the perpendicular distance DC from the tangent to the curve is called a **tangential**

distance, a tangent deflection offset, or simply a **tangent deflection.** If the chord BC is prolonged to the point G, the distance GE is called a **deflection distance, a chord deflection offset,** or simply a **chord deflection.**

If the values of the radius OB and the chord BC are known, the value of the tangent deflection $DC = FE$ can be found in the following manner: Let OM be drawn to the

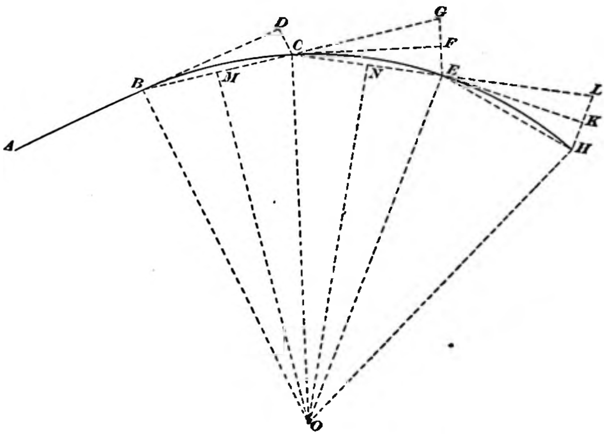

FIG. 26

middle point M of the chord BC; according to the principle stated in Art. **26**, 7, OM is perpendicular to BC, and since also the tangent BD is perpendicular to the radius OB, the right triangles OMB and BDC are similar, and we can write, $OB : BM = BC : DC$. But OB is the radius, BC is a chord, and DC is the tangent deflection. If we denote these values by R, c, and f, respectively, then, since $BM = \frac{1}{2}c$, this proportion can be written in the form $R : \frac{1}{2}c = c : f$, from which

$$f = \frac{c^2}{2R} \qquad \textbf{(14)}$$

This formula will give the tangent deflection for any length of chord irrespective of any other condition.

If the chord BC is prolonged to G, and if the distance $CG = BC = CE$, the chord deflection GE can be found in the following manner: Since BG and CE are perpendicular respectively to OM and ON, the angle $GCE = MON$, and since $BC = CE$, angle $BOC = COE$, and angle MOC, being $\frac{1}{2}$ BOC, is equal to angle CON, which is $\frac{1}{2}$ COE. Hence, angle $COE = MON = GCE$. The triangles OCE and CGE are therefore similar, since both are isosceles, and the angle GCE = angle COE. Hence, we have the proportion $OC : CE = CE : GE$. Denoting the chord deflection GE by d, and substituting R, c, and d for OC, CE, and GE, the above proportion may be written in the form $R : c = c : d$, from which

$$d = \frac{c^2}{R} \qquad (15)$$

From the principles stated in Art. **26,** 3 and 5, we know that, since the chords BC and CE are equal, they form equal angles with a tangent to the curve at C, and since CF is such a tangent and CG is the prolongation of the chord BC, the triangles CFG and CFE are equal; consequently, $GF = FE$ and the chord deflection GE is double the tangent deflection $FE = DC$, as is also shown by the two preceding formulas. It should be well understood, however, that this is the case only when the chords BC and CE are equal.

40. Tangent and Chord Deflections for Subchords. As a basis of calculation, it is convenient to remember that, for a chord of 100 feet preceded by a chord of the same length, the chord deflection for a 1° curve is 1.745 feet. For in Fig. 26, since $BC = CE$ and triangle CGE is similar to triangle OCE, if the angle COE is 1° and the chord CE is 100 feet, then from Art. **27,** we know that the chord deflection GE is 1.745 feet. Assuming the radius of a 2° curve to be equal to one-half that of a 1° curve, etc.,

formula **15** shows that the chord deflection for a 2° curve is double the deflection for a 1° curve, or 3.49 feet, and so on. The tangent deflection, being one-half the chord deflection, will be .873 foot for a 1° curve, 1.745 feet for a 2° curve, etc.

In calculating tangent and chord deflections, distances measured either on chords or tangents are expressed as decimal parts of a station length of 100 feet, which is taken as the unit. Thus, the tangent deflection for 75 feet is expressed as the tangent deflection for .75 of a station. This method of expression, however, is confined entirely to the calculation; the deflection is spoken of as the deflection for 75 feet.

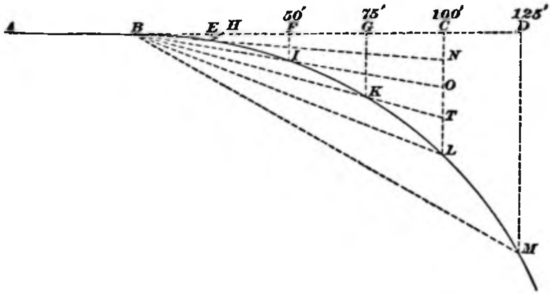

FIG. 27

The tangent to a curve is fixed in direction, being perpendicular to the radius at the point of tangency. Consequently, in a curve of given radius, the value of the tangent deflection depends wholly upon the length of the chord. Formula **14** shows that the tangent deflection is proportional to the square of the chord. Hence, knowing the tangent deflection for a chord of 100 feet, the following rule will give the tangent deflection for a chord of any other length:

Rule.—*Multiply the tangent deflection for a chord of 100 feet by the square of the given chord expressed as the decimal part of a chord of 100 feet.*

EXAMPLE.—Let BKM, Fig. 27, represent a 2° curve. What are the tangent deflections for the chords BH, BI, BK, BL, and BM, having lengths of 25, 50, 75, 100, and 125 feet, respectively?

SOLUTION.—For the chord BL of 100 feet, the tangent deflection CL is 1.745 feet. Hence, for the chords $BH = 25$ feet, $BI = 50$ feet, $BK = 75$ feet, and $BM = 125$ feet, the tangent deflections are, respectively, $EH = .25^2 \times 1.745 = .109$ foot, $FI = .50^2 \times 1.745 = .436$ foot, $GK = .75^2 \times 1.745 = .982$ foot, and $DM = 1.25^2 \times 1.745 = 2.727$ feet.

It is here assumed that the chords and corresponding tangents are of equal lengths. This is not strictly true, but is near enough when the degree of curve is small.

The above principle does not apply to chord deflections, however. The point G, Fig. 26, is in the prolongation of the chord BC, and the value of the chord deflection GE is affected by the direction, and, consequently, by the length of BC. Since OM and ON are, respectively, perpendicular to the chords BC and CE at their middle points, and since the radius OC is perpendicular to the tangent CF, we know that in the triangle GCE, the angle $GCF = \frac{1}{2}\,BOC$ and $FCE = \frac{1}{2}\,COE$. Consequently, the triangle GCE can be isosceles and similar to COE only when the angle $COE = BOC$, that is, when $BC = CE$. Hence, formula **15** applies only *when the two chords preceding the station considered are of equal length.* When these chords are of different lengths, the chord deflection will be given closely by formula **15** if $\frac{1}{2}c\,(c + c')$ is substituted for c^2, where c' is the length of the second chord preceding the station. Or, if the *tangent* deflection f has been computed, the chord deflection d_o will be given closely by the formula

$$d_o = f\left(1 + \frac{c'}{c}\right) \qquad \textbf{(16)}$$

41. Laying Out Curves Without a Transit.—During construction, the engineer is often called upon to restore center stakes on a curve when the transit is not at hand. This can be accomplished reasonably well with a tape, as described in the following example.

Let it be assumed that, in Fig. 28, $A B$ is a tangent and B is the P. C. of a 4° curve, and let it be required to locate each full station on the curve. The points A and B determine the direction of the tangent, the point B being the P. C., which is assumed to be at Station 8 + 25. For a 4° curve the regular chord deflection for 100 feet is $4 \times 1.745 = 6.98$ feet, and the tangent deflection is 3.49 feet.

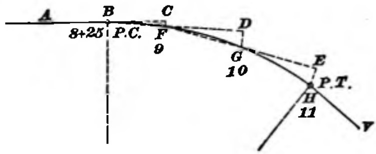

FIG. 28

The distance from the P. C. to the next station C is 75 feet; hence, the tangent deflection $C F = .75^2 \times 3.49 = 1.96$ feet (Art. **40**). The point F is found by first measuring 75 feet from B, thus locating the point C in the line $A B$ prolonged, then from C measuring $C F$, = 1.96 feet, at right angles to $B C$; the point F thus determined will be Station 9. Next the chord $B F$ is prolonged 100 feet to D; as $B F$ is only 75 feet, formula **16** gives for $D G$ the value $3.49 \times (1 + \frac{75}{100}) = 6.11$ feet. This distance is measured at right angles to $B D$; the point G thus determined will be Station 10. The point H, which is Station 11, and the P. T. of the curve, is determined in the same manner, except that, as the chords $F G$ and $G H$ are each 100 feet long, the regular chord deflection of 6.98 feet is used for $E H$. A stake is driven at each station thus located. Although a chord deflection is not at right angles to the chord theoretically, yet the deflection is so small, as compared with the length of the chord, that for curves of ordinary degree it is usually measured at right angles.

EXAMPLES FOR PRACTICE

NOTE.—In order that the student can compare his results, the answers to the following exercises are given to three decimal places, although two decimal places are sufficient in practice.

1. In a 5° curve, what are the tangent and chord deflections for a chord of 67 feet following one of 100 feet?

Ans. $\begin{cases} f = 1.958 \text{ ft.} \\ d = 4.880 \text{ ft.} \end{cases}$

2. In a 7° 30′ curve, what are the tangent and chord deflections for a chord of 23.5 feet following one of 100 feet?

Ans. $\begin{cases} f = .361 \text{ ft.} \\ d = 1.897 \text{ ft.} \end{cases}$

3. In a 6° 15′ curve, what are the tangent and chord deflections for a chord of 100 feet following one of 84 feet?

Ans. $\begin{cases} f = 5.451 \text{ ft.} \\ d = 10.030 \text{ ft.} \end{cases}$

4. In an 8° 45′ curve, what are the tangent and chord deflections for a chord of 100 feet following one of 72 feet?

Ans. $\begin{cases} f = 7.628 \text{ ft.} \\ d = 13.120 \text{ ft.} \end{cases}$

MIDDLE ORDINATE

42. Relation Between Radius, Chord, and Middle Ordinate.—It is often convenient to know the ordinate to a curve at the middle point of a chord, commonly called the **middle ordinate** of the chord. Knowing the positions of two points on a curve, the point on the curve midway between them can be located easily by means of the middle ordinate of the chord connecting the two known points. Or, a point on the curve at a distance from either point equal to the distance between the points can be located, as will be explained farther on.

The radius of curvature, length of chord, and middle ordinate have a fixed relation, and when any two are given the other can easily be determined. The relation between these values can be established and expressed as follows:

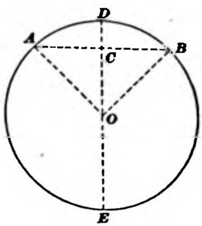

FIG. 29

Let AB, Fig. 29, be any chord of a circle whose radius is $OA = OB$, and let the chord DE be a diameter of the circle perpendicular to the chord AB at its middle point C. From geometry we know that when two chords of a circle intersect, the product of the two segments of one chord is equal to the product of the two segments of the other. Hence, we can at once write the equation

$$EC \times CD = AC \times CB$$

If we now denote the radius of the circle by R, the chord $A B$ by c and the middle ordinate $C D$ of this chord by m, then, since $C D = m$, $E C = 2 R - m$, $A C = C B = \frac{c}{2}$, by substituting these values in the foregoing equation, we get

$$(2 R - m) m = \frac{c^2}{4},$$

which easily becomes

$$2 R m = \frac{c^2}{4} + m^2 \qquad \textbf{(17)}$$

From this equation we can easily write the value of any one of the quantities R, c, or m in terms of the other two, thus:

$$R = \frac{c^2}{8 m} + \frac{m}{2} \qquad \textbf{(18)}$$

$$c = 2 \sqrt{2 R m - m^2} \qquad \textbf{(19)}$$

$$m = R - \sqrt{R^2 - \frac{c^2}{4}} \qquad \textbf{(20)}$$

These are fundamental formulas that apply exactly to all cases. Formula **20**, as derived algebraically from formula **17**, has both the + and − signs before the radical, but since m is always less than R, it is evident that the − sign must be used.

EXAMPLE 1.—What is the radius of a curve in which the middle ordinate to a chord of 60 feet is .71 foot?

SOLUTION.—Substituting known values in formula **18**, we have

$$R = \frac{60^2}{8 \times .71} + \frac{.71}{2} = 634.16 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 2.—In an 8° curve, what is the length of a chord whose middle ordinate is .69 foot?

SOLUTION.—The radius of an 8° curve as given in the Table of Radii and Deflections is 716.78. Substituting known values in formula **19**, we have

$$c = 2 \sqrt{2 \times 716.78 \times .69 - .69^2} = 62.89 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. What is the radius of a curve in which the middle ordinate of a chord of 50 feet is .5 foot? Ans. 625.25 ft.

2. What is the length of the middle ordinate of a chord of 100 feet in a 6° curve? Ans. 1.31 ft.

3. The radius of a curve is 1,146.28 feet. What is the length of a chord whose middle ordinate is .55 foot? Ans. 71.01 ft.

4. What is the length of the middle ordinate to a chord of 100 feet in a curve whose radius is 800 feet? Ans. 1.56 ft.

TO DETERMINE DEGREE OF CURVE FROM MIDDLE ORDINATE

43. It is sometimes necessary to determine the radius or the degree of a curve in an existing track when no transit is available for measuring it. By measuring the middle ordinate of any convenient chord, the degree of the curve can be calculated from the relative values of the ordinate and chord. Since the track is likely not to be in perfect alinement, it is well to measure the middle ordinate of different chords in different parts of the curve. As also the middle ordinate of a chord measured to the inner rail will somewhat exceed the middle ordinate of the same chord measured to the outer rail, the ordinate of each chord should be measured to both rails and the average of the two taken as the value of the ordinate. Having measured the middle ordinate of one or more chords, the radius of curvature can be determined by applying formula **18.** For calculating the degree of curve, either of the two following methods may be applied.

44. First Method.—Equating the value of R expressed by formula **11** with that expressed by formula **18,** we have

$$\frac{5{,}730}{D_o} = \frac{c^2}{8\,m} + \frac{m}{2}$$

Since any method of determining the degree of curve from the middle ordinate as measured on the rail can be only approximate by reason of the imperfect alinement of the track, and since also the last term of this equation has a

very small value as compared with the other terms, it can be dropped from the equation without material error, so that by solving for D_c we shall have

$$D_c = \frac{45,840\, m}{c^2}, \qquad (21)$$

in which c and m are both measured in the same unit. If we denote the length of the chord, expressed as the decimal part of a station length of 100 feet, by c_{00}, this may also be expressed by the closely approximate formula

$$D_c = \frac{55\, m}{12\, c^2_{00}} \qquad (22)$$

EXAMPLE.—Suppose that $A\,B$, Fig. 30, is a chord of 50 feet, and that its middle ordinate $a\,b$, as measured, is .44 of a foot. What is the degree of the curve?

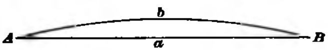

FIG. 30

SOLUTION.—By substituting the given values in formula **21**, we have

$$D_c = \frac{45,840 \times .44}{50 \times 50} = 8.07 \text{ (nearly)}$$

Or, by substituting the given values in formula **22**, we have

$$D_c = \frac{55 \times .44}{12 \times .50 \times .50} = 8.07 \text{ (nearly)}$$

For this result it would be assumed that the original curve was an 8° curve.

45. Convenient Rules for Determining the Degree of Curve.—By reference to formula **21**, it will be seen that for any given length of chord c the degree of curve D_c varies directly as the middle ordinate m. Although this is not exactly the case, as was shown in the derivation of this formula, it is so nearly the case that for practical purposes the condition may be assumed. This assumption is of considerable practical value, as it affords a simple method of determining the degree of curve when the middle ordinate is measured to a chord of given length. For convenience, let formula **21** be written in the form

$$c^2\, D_c = 45,840\, m$$

For a chord of 100 feet in a 1° curve, c has a value of 100, while D_c is unity. By substituting these values in this equation and solving for the value of m, we get

$$m = \frac{100 \times 100}{45{,}840} = .218, \text{ very closely,}$$

which is the value of the middle ordinate to a chord of 100 feet in a 1° curve. As just stated, formula **21** shows that for any given length of chord the degree of curve varies directly as the middle ordinate. Hence, we have the following rule for determining the degree of a curve :

Rule I.—*Measure the middle ordinate to a chord of 100 feet, express it in feet and decimals of a foot, and divide by .218 ; the quotient will be the degree of the curve.*

This is a convenient rule except that the value .218 is not a very convenient divisor. We can choose any convenient value for the ordinate, however, and by substituting it in formula **21**, determine the length of chord in a 1° curve for which the value chosen is the middle ordinate. Remembering that D_c is equal to unity, by substituting a value of .2 for m in the above equation and solving for the value of c, we get

$$c = \sqrt{45{,}840 \times .2} = 95.75, \text{ very closely,}$$

which, in a 1° curve, is the length of chord, expressed in feet and decimals of a foot, whose middle ordinate has a value of .2 of a foot. Hence, in a curve of any degree, the degree of curve is equal to the middle ordinate of a chord of this length divided by .2, and since dividing .2 is the same as multiplying by 5, the following rule may be used for determining the degree of a curve :

Rule II.—*Measure the middle ordinate to a chord of 95.75 feet, express it in feet and decimals of a foot, and multiply by 5 ; the result will be the degree of the curve.*

In like manner, if we substitute a value of .1 for m in the above equation and solve for the value of c, we shall get

$$c = \sqrt{45{,}840 \times .1} = 67.71, \text{ very closely,}$$

which, in a 1° curve, is the length of chord, expressed in feet and decimals of a foot, whose middle ordinate has a value of .1 of a foot. Hence, in a curve of any degree, the degree of curve is equal to a chord of this length divided by .1, and since dividing by .1 is the same as multiplying by 10, we have the following convenient rule for determining the degree of a curve:

Rule III.—*Measure the middle ordinate to a chord of 67.71 feet, express it in feet and decimals of a foot, and multiply by 10; the result will be the degree of the curve.*

Again, if we substitute a value of 1 inch, $= \frac{1}{12}$ foot, for m in the preceding equation and solve for the value of c, we shall get

$$c = \sqrt{45{,}840 \times \tfrac{1}{12}} = 61.81, \text{ very closely,}$$

which, in a 1° curve, is the length of chord, expressed in feet and decimals of a foot, whose middle ordinate has a value of 1 inch. Hence, the following rule will give the degree of a curve directly:

Rule IV.—*Measure the middle ordinate to a chord of 61.81 feet, express it in inches and decimals of an inch; the result will be the degree of the curve.*

46. Second Method.—Let CD, Fig. 31, be the middle ordinate of any given chord AB of the curve ADB, and let AD be a chord drawn from the extremity A of the given chord to the point D, where its middle ordinate intersects the curve. If we draw the radial line OE to the middle point E of the chord AD, it will evidently divide the angle AOD into two equal parts, so that $AOE = EOD = \frac{1}{2}\,AOD = \frac{1}{4}\,AOB$. (See Art. **26**, 5.) From the principle stated in Art. **26**, 7, it is also evident that this radial line OE is perpendicular to the chord AD, and since the triangles ADC and ODE each have one right angle, while

FIG. 31

the angle $A\,D\,O$ is common to both, the triangles are similar and the angle $D\,A\,C$ is equal to the angle $D\,O\,E$, which is equal to the angle $A\,O\,E$, and to one-fourth the angle $A\,O\,B$ subtended by the given chord. We can therefore write

$$\frac{D\,C}{A\,C} = \tan D\,A\,C = \tan \tfrac{1}{4}\,A\,O\,B$$

Since $A\,C$ is one-half the given chord, $D\,C$ is the middle ordinate and $A\,O\,B$ is the angle subtended by the given chord; if we denote the given chord by c, its middle ordinate by m, and the central angle subtended by the chord by a, this expression may be written

$$\frac{m}{\frac{1}{2}\,c} = \tan \tfrac{1}{4}\,a \qquad \textbf{(23)}$$

This is a general formula applying to any chord of a circle and may be expressed by the following general principle:

The quotient obtained by dividing the middle ordinate to any chord by one-half the chord is equal to the natural tangent of one-fourth the central angle subtended by the chord.

Since the degree of a curve is the central angle subtended by a chord of 100 feet, and since dividing the middle ordinate by one-half the chord is the same as dividing twice the middle ordinate by the whole chord, we may write the following rule for determining the degree of curve by means of the middle ordinate to a chord of 100 feet:

Rule I.—*Twice the middle ordinate to a chord of 100 feet, divided by 100, is equal to the natural tangent of one-fourth the degree of curve.*

For chords of lengths other than 100 feet, the quotient obtained by dividing twice the middle ordinate by the chord, formula **23**, will be the tangent of one-fourth the central angle subtended by the chord. In order to obtain the degree of curvature in such cases, the following rule may be applied, which, though not exact, will give it very closely.

Rule II.—*Multiply the central angle by 100 and divide the product by the length of the chord employed.*

EXAMPLE.—Suppose that $A\ B$, Fig. 30, is a chord of 50 feet, and that the middle ordinate $a\ b$ is .44 of a foot. What is the degree of curve?

SOLUTION.—By substituting the given values in formula **23**, we have

$$\frac{2 \times .44}{50} = .0176 = \tan \tfrac{1}{4} a = \tan 60\tfrac{1}{2}' \text{ (closely)},$$

from which we get for the central angle the value

$$a = 4 \times 60\tfrac{1}{2}' = 242',$$

and by applying the preceding rule, we have for the degree of curve

$$242' \times \tfrac{100}{50} = 484' = 8° \ 4'. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

NOTE.—The answers to the following exercises can be obtained by either of the methods just explained. In order to obtain the exact answers by the second method, however, the angle must be determined to the nearest second, although results sufficiently accurate for the purpose can be obtained by taking the angle to the nearest minute.

1. The length of chord is 50 feet, the middle ordinate is .35 foot; what is the degree of curve? Ans. 6° 25′

(The original curve probably 6° 30′.)

2. The length of chord is 40 feet, the middle ordinate is .21 foot; what is the degree of curve? Ans. 6.02°

(The original curve probably 6°.)

3. The length of chord is 25 feet, the middle ordinate is .22 foot; what is the degree of curve? Ans. 16.13°

(The original curve probably 16°.)

4. The length of chord is 35 feet, the middle ordinate is .27 foot; what is the degree of curve? Ans. 10.10°

(The original curve probably 10°.)

OTHER USES OF MIDDLE ORDINATE

47. Middle Ordinate to Chord of Two Stations.—Let $A\ D\ B$, Fig. 32, be a portion of any circular curve, in which $A\ D, = D\ B$, is the regular distance between stations, usually 100 feet, and $A\ B$ is a chord connecting two alternate stations. If

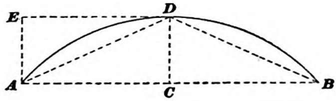

FIG. 32

we let c_2 denote the chord $A\ B$, m_2 denote its middle

ordinate CD, and R denote the radius of the curve, then by formula **17** we have

$$2 R m_2 = \frac{c_2^2}{4} + m_2^2$$

But since $\frac{c_2}{2}$, $= AC$, is the base, and m_2, $= CD$, is the altitude, of the right triangle ACD, if by c we denote the regular station chord AD, which is the chord of half the arc ADB and the hypotenuse of the triangle, we can write the equation

$$\frac{c_2^2}{4} + m_2^2 = c^2,$$

and by substituting c^2 for the last member of formula **17** as written above, this equation becomes

$$2 R m_2 = c^2,$$

from which

$$m_2 = \frac{c^2}{2R} \qquad (24)$$

This equation gives the same value for m_2 as formula **14** gives for f, which should evidently be the case. For, if we draw a line DE tangent to the curve at the middle point D of the arc, it will be parallel to the chord of the whole arc AB, so that the perpendicular distance AE between the extremity of the chord and the extremity of the tangent will be equal to the middle ordinate CD.

48. To Lay Out a Curve by Middle Ordinate.—One of the most expeditious methods of laying out a curve without the aid of a transit is by means of the middle ordinate to a chord that subtends an arc of twice the length subtended by the chord for a regular station.

Let AB, Fig. 33, be a tangent, and B the P. C. of the curve $BCDE$, which it is desired to lay out on the ground when no transit is available. The distance BC', equal to 100 feet or one station length, is measured in line with the tangent AB, the head chainman keeping in range by means of a stake at B and a flag held at A, and a temporary stake is driven at the extremity C' of the measured distance. The

tangent deflection $C' C$ is then calculated by formula **14**; this distance is measured at right angles to $B C'$, the prolongation of the tangent, and a stake is driven at the point C on the curve. The middle ordinate $C M$ of the long chord $B D$ is then measured as nearly at right angles to the chord as can be judged and a temporary stake is set at M. The middle ordinate $C M$ can be calculated by formula **24**; it is equal to the tangent deflection $C' C$, as has been shown. The rear flag is then held at B, the rear end of the chain is

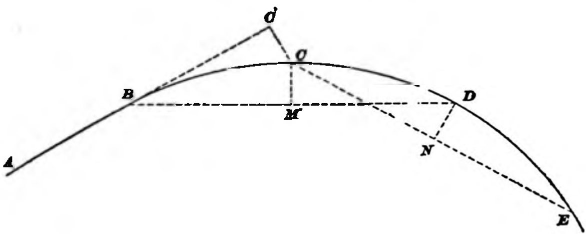

FIG. 33

held at C, and the head chainman stretches out the chain and swings the forward end around until it is in range with the stake at M and the flag at B, thus determining the point D on the curve, at which a stake is then driven. Continuing, a temporary stake is set at the extremity N of the middle ordinate to the long chord $C E$ as measured from D, and, with the rear end of the chain held at D, the forward end is brought in range with this temporary stake and the flag held at C, thus determining the point E. The remaining points on the curve are located in the same manner.

49. Ranging in the Head Flag on Curves.—The facility with which a curve can be laid out in the field will depend largely on the ability of the head flagman, who is also the head chainman, to keep in line and find quickly the approximate positions of the various points on the curve at which stakes are to be driven, so that when lined in at each

point by means of signals from the transitman, it will only be necessary to move his flag a few inches in order to be at the true point on the curve. In order to facilitate the field work of laying out curves, the following method is suggested to aid the head chainman in keeping the line.

Let us suppose that the curve is 6° and is to be laid out, as shown in Fig. 34, with stakes set at intervals of 50 feet.

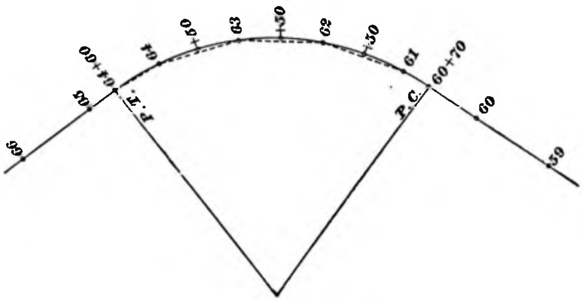

FIG. 34

With the instrument set up over the P. C., which is at Station 60 + 70, the first stake to be set is at Station 61, which is 30 feet from the P. C. The rear chainman holds the 30-foot mark on the chain at the P. C., and the head chainman holds his flag at the forward end of the chain and moves it to the right or left, according as the transitman may signal, until at the point for Station 61, at which point the stake for this station is driven. The rear chainman now moves up to this station and holds the 50-foot mark on the chain at the stake just set, while the head chainman with his flag held at the forward end of the chain gets the point for Station 61 + 50. The rear chainman remaining at Station 61 allows the chain to be drawn forward to its full length of 100 feet and then holds the rear end at the stake, while the head chainman measures the distance and is ready to get the point for Station 62. He ranges in his flag by holding

it in such a position that by sighting from the flag to Station 61 the line of sight passes in front of the stake at Station 61 + 50 by an amount about equal to the middle ordinate of the given curve for a chord of 100 feet; the correct line is then given by the transitman. The rear chainman then moves up to Station 62 and holds the 50-foot mark on the chain at this station while the head chainman gets the point for Station 62 + 50, as before, and then holds the rear end of the chain at the same station while the head chainman gets the point for Station 63, and so on around the curve to Station 64, which is the last full station on the curve. He then holds the 60-foot mark on the chain at Station 64 while the head chainman gets the point for the P. T., which is at Station 64 + 60. In this case, the P. T. is only 10 feet beyond 64 + 50, and it will therefore be unnecessary to set a stake at the latter point, since the additional 10 feet in the length of the chord is unimportant.

Before starting to run out the curve, the head chainman should ascertain the value of the middle ordinate for a chord of 100 feet in a curve of the given degree. He can obtain this from the transitman, or, when the stakes are to be set at intervals of 50 feet, can himself calculate it by multiplying the degree of curve by the decimal .218, or, near enough for this purpose, by the decimal .22; the result will be the middle ordinate in feet. (See Art. **45.**) By estimating with the eye a distance from the preceding station equal to the ordinate, and ranging in the flag in the manner just described, the head chainman, after a little practice, can place his flag within a few inches of the required point, unless the curve happens to be on unusually rough ground.

If stakes are set on the curve only at the regular stations, 100 feet apart, substantially the same method can be employed for ranging in the head flag after the first two consecutive stakes 100 feet apart have been set. In this case the middle ordinate of the long chord, or the distance from the preceding stake to the line of sight from the flag to the second preceding stake, can be calculated by formula **24,** by merely substituting the value of c and R. In a 1° curve,

the middle ordinate for a chord of two stations, as given by this formula, is equal to $\frac{100 \times 100}{2 \times 5,730} = .873$ foot. Hence, the ordinate for ranging in the head flag on a curve, when the stakes are set at the regular stations only, can be found near enough for the purpose by multiplying the degree of curve by the decimal .87; the result will be the ordinate in feet.

PASSING OBSTACLES ON CURVES

50. Different methods may be employed for passing obstacles on curves, depending on the conditions encountered and the ingenuity of the engineer in adapting the method to them. Three of the simplest and most common of the methods employed will be explained.

Suppose that it is required to run out the curve *A E H*, Fig. 35, with several obstacles in the direct line of the curve, as shown, Station 3 being the P. C., and the regular stations on the curve being in the positions indicated by the numbers 4, 7, 8, etc. The positions of Stations 5 and 6 are indicated by the letters *C* and *D*, the figures 5 and 6 being omitted for want of space. The stations are to be located in their proper positions on the curve, between the obstructions, wherever it is possible to do so. In addition to this, it is customary to mark with a tack or otherwise the point where the line of the curve intersects each obstruction.

Beginning at the point of curve *A*, which is at Station 3, we can run in the curve as far as the first obstruction, which is the building *P*, setting the stakes on the curve at Stations 4 and 5, and a tack in the side of the building *P* at the point where the line of curve intersects it, according to the deflection angle as determined by its distance from Station 5. We cannot proceed further in the regular manner, however, because Station 6 cannot be seen from the P. C. The most expeditious method of passing the buildings and locating the subsequent stations on the curve will depend on the conditions encountered. The three methods here explained are all susceptible of more or less modification.

51. First Method: by Equilateral Triangle.—The easiest way of passing a single obstacle in the line of the

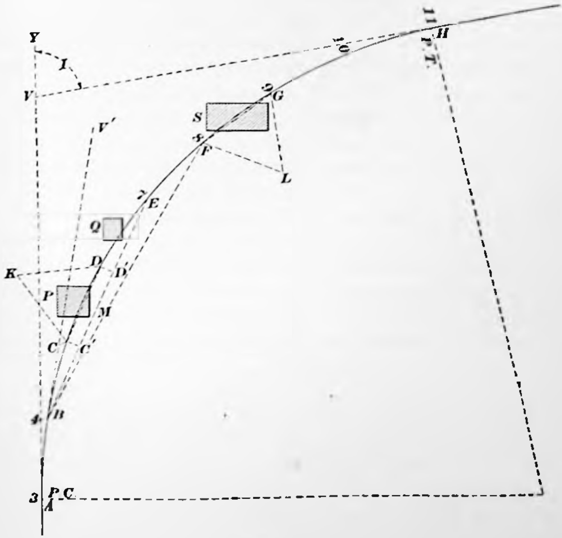

FIG. 35

curve, such as the building P, Fig. 35, is by means of an equilateral triangle, substantially as explained in Art. **30,** *Compass Surveying, Part 1.*

The instrument is moved forward and set up at the point C, which is at Station 5, is sighted back to the P. C., which is at Station 3, the telescope is reversed, and the deflection angle for Station 6 is turned off the same as if no obstruction existed; the telescope will now be sighted on the line CD, although the point D, or Station 6, will not be visible, because the building P obstructs the view. The angle DCK, equal to 60°, is then turned in either direction, as may be most advantageous for passing the building; in

the figure it is shown turned to the left. The distance CK is then measured on the line thus determined, equal to the distance CD, in this case 100 feet, and a transit point K is set in the line at the extremity of this distance. The transit is then moved forward and set up at K, sighted back on C, and the angle CKD, equal to 60°, turned to the left, giving the line KD. A distance equal to CK, in this case 100 feet, is measured on this line, and a transit point D is set at the extremity of the measurement. This point will be Station 6 on the line of the curve and in its proper position at a distance of 100 feet from Station 5. The transit is now moved forward and set up at D, the vernier set at the reading for Station 5, and an angle of 60° turned to the right from this reading. The telescope is then backsighted on K, and the vernier is turned back to the regular deflection for Station 6. The instrument is now sighted on a tangent to the curve at Station 6, and any point on the curve visible from this station can easily be established by turning off a deflection angle corresponding to its distance from the instrument. The point back on the curve where the line of curve intersects the building P is marked by a tack or otherwise, as is also the point forward on the curve where the line of curve intersects the building Q. No other stations can be seen from Station 6, however, though the point for Station 7 can be set from Station 6 by the method just explained, and from this point Station 8 can be located directly. Station 9 cannot be located from Station 7 by reason of the intervening building S, but by setting up the transit at Station 8, the building can be passed and this station located, and the remainder of the curve can then be completed regularly.

52. Second Method: by Long Chord.—With the conditions as shown in Fig. 35, however, Stations 5, 6, 7, and 8 can be located from the point B, which is Station 4. Station 7 can be located by means of the long chord BE and its deflection angle $V'BE$, turned from the tangent BV' to the curve at B. Stations 5 and 6 can then be located by means

of the ordinates $C' C$ and $D' D$ measured to the curve at right angles to the chord $B E$ from the points C' and D' located on the chord. Station 8 can be located by means of the deflection angle $V' B F$ and the chord $E F$, both of which are measured in the regular manner. The operations in detail and the calculations necessary to locate Stations 5 and 6 are as follows:

With the transit set at B and the vernier set at zero, the telescope is backsighted to A and reversed, and the deflection corresponding to Station 7 is turned. If it is found that the line of sight will miss both obstructing buildings P and Q, the distance $B E$ is measured, and the stake for Station 7 is set in the line given by the instrument. The length of the chord $B E$ can be calculated from the deflection angle $V' B E$ that is turned from the tangent $V' B$, in order to give the direction of the chord. From formula **12**, we have for the length of the chord $B E$ the value

$$c = 2 R \sin D$$

In this case the distance from B, measured on the curve, is three full stations, and therefore the distance $B E$ can be taken from a table of long chords, such as may be found in most engineers' field books.

If desired, Stations 5 and 6 can be located from the chord $B E$ by calculating the ordinates $C' C$ and $D' D$ to the curve at Stations 5 and 6, respectively, and the distances $M C'$ and $M D'$ from the center M of the chord to each respective ordinate, as measured along the chord. In this case these ordinates are of equal length, since they are at equal distances from the center M of the long chord $B E$, but the same general method will apply to any case.

From Fig. 36, which represents in a somewhat exaggerated manner the portion of the curve between Station 4 and 7, it is evident that $C' C = M P - N P$. Since the angle $B O P$ is equal to the angle $V' B E$, which is the deflection angle D for the long chord $B E$, and the distance $O M = O B \times \cos B O P = R \cos D$ we can write

$$M P = R (1 - \cos D)$$

Also, since in this case the arc CP is one-half of the arc CD, which latter arc is equal to the arc BC, the

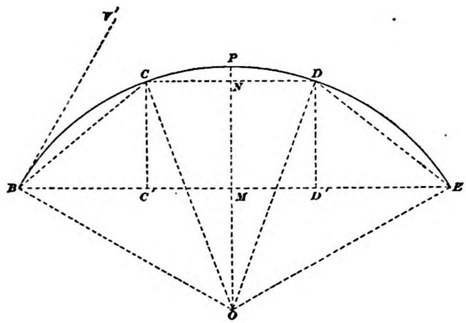

FIG. 36

angle COP is equal to one-third the angle BOP, or $\frac{1}{3}D$, we can also write

$$NP = R\,(1 - \cos\tfrac{1}{3}D)$$

Hence, we have for the ordinate $C'C$ the value

$$C'C = R\,(1 - \cos D) - R\,(1 - \cos\tfrac{1}{3}D) = R\,(\cos\tfrac{1}{3}D - \cos D)$$

The value of the ordinate $D'D$ is found in the same manner, and in this case is the same.

In order to locate the point C' on the chord BE, we have

$$MC' = NC = R\sin\tfrac{1}{3}D$$

The distance MD', locating the point D', is found in the same manner, and in this case is the same.

Having determined these values, in order to locate Stations 5 and 6, we have merely to bisect the chord BE and lay off the distances MC' and MD' on it from the middle point, then measure the ordinates $C'C$ and $D'D$ from the points so determined, thus locating the points C and D, which are the stations required. Or, what amounts to the same thing, we measure along the chord BE the distance BC', equal to one-half the length of the chord minus

the distance MC', thus locating the point C', and from this point measure the distance $C'D'$, equal to twice the distance MC', thus locating the point D'; the ordinates $C'C$ and $D'D$ are then measured from these points.

53. Third Method: by Concentric Parallel Curve. Let it be required to run a curve $ABCD$, Fig. 37, connecting

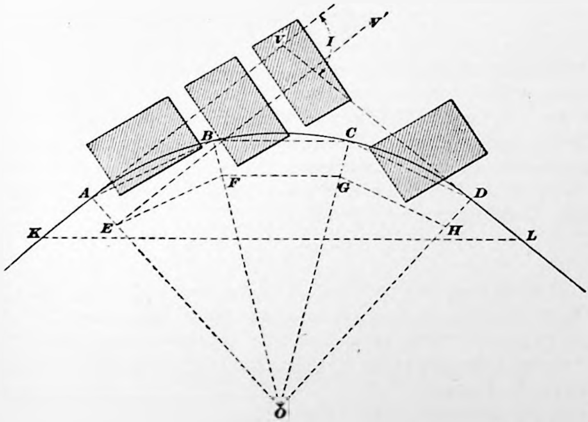

FIG. 37

the tangents KA and DL, and suppose that obstacles lie directly in the path of the curve, and that the point of intersection V is also inaccessible, as shown. A concentric parallel curve $EFGH$ may be run in such position as to avoid the obstacles, and from the stations on this parallel curve the corresponding stations on the required curve can be located. The intersection angle I is equal to the sum of the angles VKL and VLK, and by measuring these angles and the distance KL, the distances VK and VL can be calculated by the general principles of sines. The tangent distances VA and VD can then be

calculated by formula **13.** The distance KA being the difference between the distances VK and VA, and the distance LD the difference between the distances VL and VD, the points A and D, which are, respectively, the P. C. and P. T. of the curve, can at once be located.

Having located the P. C. and P. T., the equal distances AE and DH are measured perpendicular, respectively, to the tangents KA and DL and of such length that the parallel curve $EFGH$ connecting their extremities E and H will avoid the obstacles, and transit points are set at E and H in the positions thus determined. It is evident that while the angle AOD remains constant, the chords EF, FG, and GH in the curve $EFGH$ are shorter than the corresponding chords AB, BC, and CD of the required curve, since they are nearer the common center O of the two curves. The triangles AOB and EOF are similar and therefore their sides are proportional. Hence,

$$OA : OE = AB : EF$$

If we denote the radius OA of the required curve by R_1, the radius OE of the parallel curve by R_2, the chord AB of the required curve by c_1, and the corresponding chord EF of the parallel curve by c_2, then from this proportion we can write the formula

$$c_2 = c_1 \frac{R_2}{R_1} \qquad \textbf{(25)}$$

The deflection angle $V'EF$ is equal to VAB, because the corresponding sides are parallel, and for similar reasons all the deflection angles of the parallel curve are the same as the corresponding ones for the required curve. The transit is set up at E, backsighted to A, and the angle AEV', equal to 90°, is turned, giving the direction of the tangent EV', from which the deflection angles for the curve $EFGH$ are turned off in the regular manner and a transit point is set at each station by measuring each chord EF, FG, and GH of the proper length, as calculated by formula **25.** The transit is then set at a point on the parallel curve, as at F, and

by backsighting to another point on the curve, as E, and turning off the deflection angle, the line of sight is directed on a tangent to the curve at the instrument point. Then, by turning a right angle from this tangent and measuring from F the distance FB equal to AE, a stake may be set locating the point B on the required curve. In a similar manner each of the stations on the required curve can be located.

EXAMPLE.—Suppose the required curve $ABCD$, Fig. 37, is a 7° curve and that the offset distance $AE, = DH$, necessary to avoid the obstacles, is 90 feet, what is the length of each of the chords EF, FG, and GH of the parallel curve in order that the corresponding chords of the required curve will each be 100 feet in length?

SOLUTION.—The radius of a 7° curve is 819.02 feet, which is the radius $OA = R_1$; the radius OE is therefore $819.02 - 90 = 729.02 = R_2$. The length of each of the chords AB, BC, and CD is 100 feet, $= c_1$. By substituting these values in formula **25**, the length of each of the chords EF, FG, and GH is found to be equal to

$$c_2 = 100 \times \frac{729.02}{819.02} = 89.01 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Determine the middle ordinate to a chord connecting two alternate stations in a 4° curve, the consecutive stations being 100 feet apart. Ans. 3.49 ft.

2. The degree of curve is 5°; what is the middle ordinate to a chord of 300 feet? Ans. 9.81 ft.

3. Assuming that AEH, Fig. 35, is a 6° curve, and that the stations are 100 feet apart, determine the length of the long chord BE, the ordinate $C'C$, and the distance MC'.

$$\text{Ans.} \begin{cases} BE = 298.90 \text{ ft.} \\ C'C = 10.45 \text{ ft.} \\ MC' = 50.00 \text{ ft.} \end{cases}$$

4. Referring to Fig. 37, let the required curve $ABCD$ be a 4° curve, let the chords AB, BC, and CD each be 50 feet in length, and let the offset distance $AE, = DH$, necessary to avoid the obstacles, be 75 feet. Determine the length of the chords EF, FG, and GH of the parallel curve. Ans. 47.38 ft.

FIELD NOTES FOR CURVES

54. As stated in another section, various styles of field notebooks are published, in which the pages are ruled to suit the different kinds and methods of field work. The following, which are the field notes of a portion of a line containing a curve, represents a good form for recording the field notes of a curve that is run in by the method of zero tangent explained in Art. **37.**

Station.	*Deflection.*	*Tot. Angle.*	*Mag. Bearing.*	*Ded. Bearing.*	*Remarks.*	*June 20 1894*
9						
8						
7						
6+95	*4°54' P.T.*	*15°00'*	*N. 35°20' E.*	*N. 35°15' E.*		
6+50	*4°00'*					
6	*3°00'*					*70°00'*
5+50	*2°00'*				*5+80*	*Centerline of Highway*
5	*1°00'*				*5+60*	
4+50	*2°36'*	*5°12'*				
4	*1°36'*				*Int. Angle = 15°00'*	*4° Curve R*
3+50	*0°36'*				*T. = 188.01 ft.*	*Def. Angle for 50 ft. = 1°00'*
3+20	*P.C. 4° R*				*P.C. = 3+20*	*Def. Angle for 1 ft. = 1.2'*
3					*Length of Curve = 375 ft.*	
2					*P.T. = 6+95*	
1						
0			*N. 20°15' E.*	*N. 20°15' E.*		

In the first column the station numbers are recorded. In the second column are recorded the deflections with the abbreviations P. C. and P. T., together with the degree of curve and the abreviation *R* or *L*, according as the line curves to the right or left. At each transit point on the curve, the total or central angle from the P. C. to that point is calculated and recorded in the third column. This total angle is double the deflection angle between the P. C. and the transit point. In the above notes, there is but one intermediate transit point between the P. C. and the P. T. The deflection from the P. C. at Station 3 + 20 to the intermediate transit point at Station 4 + 50 is 2° 36′. The total angle is double this deflection, or 5° 12′, which is recorded on the same line in the third column. The record of total

angles at once indicates the stations at which transit points are placed. The total angle at the P. T. will be the same as the angle of intersection, if the work is correct. When the curve is finished, the transit is set up at the P. T., and the bearing of the forward tangent taken, which affords an additional check upon the previous calculations. The magnetic bearing is recorded in the fourth column, and the deduced, or calculated, bearing is recorded in the fifth column.

When the method by continuous vernier is employed for running in the curve, the notes will be substantially the same as given, with the exception that the deflection angles will increase continuously from the P. C. throughout the entire curve by adding the proper amount for each station to the deflection angle for the preceding station. The deflection angles will then have the values 3° 36′, 4° 36′, 5° 36′, 6° 36′, and 7° 30′ for Stations 5, 5 + 50, 6, 6 + 50, and 6 + 95 of the foregoing notes, respectively, instead of starting anew for Station 5.

TABLE OF RADII AND DEFLECTIONS

Degree	Radii	Chord Deflection	Tangent Deflection
° ′			
0 5	68754.94	.145	.073
10	34377.48	.291	.145
15	22918.33	.436	.218
20	17188.76	.582	.291
25	13751.02	.727	.364
30	11459.19	.873	.436
35	9822.18	1.018	.509
40	8594.41	1.164	.582
45	7639.49	1.309	.654
50	6875.55	1.454	.727
55	6250.51	1.600	.800
1 0	5729.65	1.745	.873
5	5288.92	1.891	.945
10	4911.15	2.036	1.018
15	4583.75	2.182	1.091
20	4297.28	2.327	1.164
25	4044.51	2.472	1.236
30	3819.83	2.618	1.309
35	3618.80	2.763	1.382
40	3437.87	2.909	1.454
45	3274.17	3.054	1.527
50	3125.36	3.200	1.600
55	2989.48	3.345	1.673
2 0	2864.93	3.490	1.745
5	2750.35	3.636	1.818
10	2644.58	3.781	1.891
15	2546.64	3.927	1.963
20	2455.70	4.072	2.036
25	2371.04	4.218	2.109
30	2292.01	4.363	2.181
35	2218.09	4.508	2.254
40	2148.79	4.654	2.327
45	2083.68	4.799	2.400
50	2022.41	4.945	2.472
55	1964.64	5.090	2.545
3 0	1910.08	5.235	2.618
5	1858.47	5.381	2.690
10	1809.57	5.526	2.763
15	1763.18	5.672	2.836
20	1719.12	5.817	2.908
25	1677.20	5.962	2.981
30	1637.28	6.108	3.054
35	1599.21	6.253	3.127
40	1562.83	6.398	3.199
45	1528.16	6.544	3.272
50	1494.95	6.689	3.345
55	1463.16	6.835	3.417
4 0	1432.69	6.980	3.490
5	1403.46	7.125	3.563
10	1375.40	7.271	3.635
15	1348.45	7.416	3.708
20	1322.53	7.561	3.781
25	1297.58	7.707	3.853
30	1273.57	7.852	3.926
35	1250.42	7.997	3.999
40	1228.11	8.143	4.071
45	1206.57	8.288	4.144
50	1185.78	8.433	4.217
55	1165.70	8.579	4.289
5 0	1146.28	8.724	4.362
5	1127.50	8.869	4.435
10	1109.33	9.014	4.507
5 15	1091.73	9.160	4.580
20	1074.68	9.305	4.653
25	1058.16	9.450	4.725
30	1042.14	9.596	4.798
35	1026.60	9.741	4.870
40	1011.51	9.886	4.943
45	996.87	10.031	5.016
50	982.64	10.177	5.088
55	968.81	10.322	5.161
6 0	955.37	10.467	5.234
5	942.29	10.612	5.306
10	929.57	10.758	5.379
15	917.19	10.903	5.451
20	905.13	11.048	5.524
25	893.39	11.193	5.597
30	881.95	11.339	5.669
35	870.79	11.484	5.742
40	859.92	11.629	5.814
45	849.32	11.774	5.887
50	838.97	11.919	5.960
55	828.88	12.065	6.032
7 0	819.02	12.210	6.105
5	809.40	12.355	6.177
10	800.00	12.500	6.250
15	790.81	12.645	6.323
20	781.84	12.790	6.395
25	773.07	12.936	6.468
30	764.49	13.081	6.540
35	756.10	13.226	6.613
40	747.89	13.371	6.685
45	739.86	13.516	6.758
50	732.01	13.661	6.831
55	724.31	13.806	6.903
8 0	716.78	13.951	6.976
5	709.40	14.096	7.048
10	702.18	14.241	7.121
15	695.09	14.387	7.193
20	688.16	14.532	7.266
25	681.35	14.677	7.338
30	674.69	14.822	7.411
35	668.15	14.967	7.483
40	661.74	15.112	7.556
45	655.45	15.257	7.628
50	649.27	15.402	7.701
55	643.22	15.547	7.773
9 0	637.27	15.692	7.846
5	631.44	15.837	7.918
10	625.71	15.982	7.991
15	620.09	16.127	8.063
20	614.56	16.272	8.136
25	609.14	16.417	8.208
30	603.80	16.562	8.281
35	598.57	16.707	8.353
40	593.42	16.852	8.426
45	588.36	16.996	8.498
50	583.38	17.141	8.571
55	578.49	17.286	8.643
10 0	573.69	17.431	8.716
10	564.31	17.721	8.860
20	555.23	18.011	9.005
30	546.44	18.300	9.150
40	537.92	18.590	9.295
10 50	529.67	18.880	9.440
11 0	521.67	19.169	9.585
10	513.91	19.459	9.729
20	506.38	19.748	9.874
30	499.06	20.038	10.019
40	491.96	20.327	10.164
50	485.05	20.616	10.308
12 0	478.34	20.906	10.453
10	471.81	21.195	10.597
20	465.46	21.484	10.742
30	459.28	21.773	10.887
40	453.26	22.063	11.031
50	447.40	22.352	11.176
13 0	441.68	22.641	11.320
10	436.12	22.930	11.465
20	430.69	23.219	11.609
30	425.40	23.507	11.754
40	420.23	23.796	11.898
50	415.19	24.085	12.043
14 0	410.28	24.374	12.187
10	405.47	24.663	12.331
20	400.78	24.951	12.476
30	396.20	25.240	12.620
40	391.72	25.528	12.764
50	387.34	25.817	12.908
15 0	383.06	26.105	13.053
10	378.88	26.394	13.197
20	374.79	26.682	13.341
30	370.78	26.970	13.485
40	366.86	27.258	13.629
50	363.02	27.547	13.773
16 0	359.26	27.835	13.917
10	355.59	28.123	14.061
20	351.98	28.411	14.205
30	348.45	28.699	14.349
40	344.99	28.986	14.493
50	341.60	29.274	14.637
17 0	338.27	29.562	14.781
10	335.01	29.850	14.925
20	331.82	30.137	15.069
30	328.68	30.425	15.212
40	325.60	30.712	15.356
50	322.59	31.000	15.500
18 0	319.62	31.287	15.643
10	316.71	31.574	15.787
20	313.86	31.861	15.931
30	311.06	32.149	16.074
40	308.30	32.436	16.218
50	305.60	32.723	16.361
19 0	302.94	33.010	16.505
10	300.33	33.296	16.648
20	297.77	33.583	16.792
30	295.25	33.870	16.935
40	292.77	34.157	17.078
50	290.33	34.443	17.222
20 0	287.94	34.730	17.365

STADIA AND PLANE-TABLE SURVEYING

STADIA SURVEYING

PRELIMINARY DEFINITIONS AND PRINCIPLES

1. Definitions.—The term **stadia surveying** is commonly applied to a method of surveying in which distances are determined by observing what length of a graduated rod, held on a distant point, is included between two horizontal cross-wires in a telescope (usually that of a transit or a plane table) through which the rod is viewed. The rod is called a **stadia rod**, and the cross-wires are called **stadia wires.** The length of the stadia rod included between the stadia wires bears a certain fixed relation to the distance of the rod from the instrument.

Stadia measurement is of great value in all kinds of field work, especially in the minor operations of topographic and hydrographic surveys. It is the quickest and best way of measuring distances of any considerable length where great accuracy is not required.

2. Optical Principles Involved.—In Fig. 1, $L\,N$ represents a double convex lens, the faces of which are spherical surfaces whose radii are $O_1\,C_1$ and $O_2\,C_2$. The line $O_1\,O_2$ passing through the centers of these surfaces is called the **principal axis** of the lens. It is shown in physics that rays of light r, r, r, parallel to the axis of the lens, converge, after being refracted, at a point F on the axis. This point is called

the **principal focus** of the lens. The distance $F C_2$ from the principal focus to the lens is called the **principal focal distance** of the lens, and is generally represented by f.

It is also shown in physics that the rays from any point O_1 on the principal axis converge, after refraction, at another point O_2 on the axis. These two points are called **conjugate**

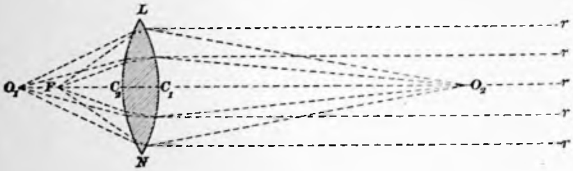

FIG. 1

foci, and their distances $O_1 C_2$ and $O_2 C_1$ from the lens are called **conjugate focal distances**. A law of optics is that the sum of the reciprocals of any two conjugate focal distances of a lens is equal to the reciprocal of the principal focal distance of the lens. Hence, representing $O_1 C_2$ by f_1 and $O_2 C_1$ by f_2,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

3. The object glass of a transit or plane table consists of a combination of lenses forming a compound lens. The same general principles and formulas that are applicable to a single lens apply to a compound lens. As the lens is always very thin compared with the conjugate focal distances, its thickness may be neglected, and the lens represented by a single line, as in the following article.

STADIA FORMULAS

4. Horizontal Sights.—Let $L N$, Fig. 2, represent the objective lens, or object glass, of a telescope; $A B$, a portion of a vertical rod, the center of this portion being on the axis $m M$ of the lens; and $a b$, the image of $A B$ on the plane of the cross-wires. The axis of the lens is supposed to be

horizontal and therefore perpendicular to AB and ab. From the formula of Art. 2, f representing the focal distance,

$$\frac{1}{CM}+\frac{1}{Cm}=\frac{1}{f}, \text{ whence } \frac{1}{Cm}=\frac{1}{f}-\frac{1}{CM}$$

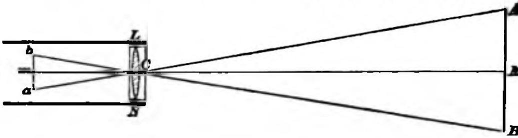

FIG. 2

Also, the triangles ACB and aCb are similar, and, therefore,

$$\frac{ab}{Cm}=\frac{AB}{CM}$$

whence

$$\frac{1}{Cm}=\frac{AB}{ab\times CM}$$

Equating this to the preceding value of $\frac{1}{Cm}$, there results

$$\frac{AB}{ab\times CM}=\frac{1}{f}-\frac{1}{CM}$$

whence, solving for CM,

$$CM=\frac{f}{ab}\times AB+f \qquad (a)$$

As f is constant for any telescope, the distance CM depends only on the two variable quantities AB and ab. The distance ab is made constant by placing two stadia wires no and pq, Fig. 3, on the reticule of the telescope parallel to and equidistant from the ordinary horizontal cross-wire hk, and taking always that part of the image included between a and b.

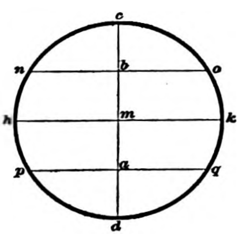

FIG. 3

To CM, which is the distance from the rod to the object glass, must be added the distance c from the object glass to the center of the instrument, in order to obtain the true distance d

from the rod to the center of the instrument. The distance c varies slightly as the object glass is moved in or out for focusing, but this variation is too small to be considered. The distance d is then given by the equation

$$d = CM + c$$

Substituting the value of CM from (a), and representing the distance ab between the stadia wires by r, and the rod reading AB by R, we have

$$d = \frac{f}{r}R + f + c \qquad (1)$$

The ratio $\frac{f}{r}$ is called the **stadia constant**, and $f+c$ is called the **instrument constant**. Representing the stadia constant by s and the instrument constant by i, formula **1** becomes

$$d = sR + i \qquad (2)$$

This formula assumes that the line of sight is perpendicular to the rod.

5. Inclined Sights.—As comparatively few lines of sight are horizontal, the transit used for stadia measurements should have a level tube attached to the telescope and an arc or circle for reading vertical angles.

In Fig. 4, let MN represent an inclined line of sight from a transit set over the point I to a point N on a rod held vertically on a point P, whose distance from I is required. Let MP_1 be the horizontal projection of IP or MN.

If the rod is held in the position $P'A'$ at right angles to the line of sight, and the portion $A'B'$ intercepted by the stadia wires is read, the length of MN can be found by formula **2**, Art. **4**. Representing $A'B'$ by R', that formula gives

$$MN = sR' + i \qquad (a)$$

The right triangle NMP_1, in which V represents the vertical angle NMP_1, as measured by the vertical arc of the transit, and d represents the horizontal distance MP_1, gives

$$d = MN \cos V$$

or, substituting the value of MN from (a),

$$d = (sR' + i) \cos V$$

6. It is inconvenient and inaccurate to hold the rod perpendicular to an inclined line of sight. In practice, the rod is usually held vertical, as shown at PA, Fig. 4, and the horizontal distance MP_1 or d is expressed in terms of the

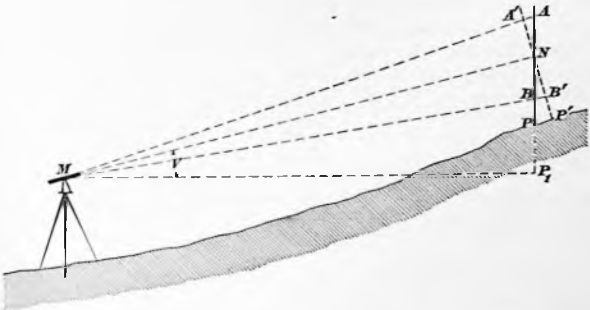

FIG. 4

rod reading $R(= AB)$ determined by the stadia wires on the rod thus held. As the angle AMB is very small, no appreciable error will result if A' and B' are considered to be right angles. Assuming this to be the case, the right triangles ANA' and BNB' give

$$A'N = AN \cos ANA'$$
$$NB' = NB \cos BNB'$$

Adding these equations member to member and noticing that the angle BNB' is equal to ANA',

$$A'N + NB' = (AN + NB) \cos ANA'$$

that is, $A'B' = AB \cos ANA'$, or

$$R' = R \cos ANA' \qquad (a)$$

As the sides of the angle ANA' are, respectively, perpendicular to those of NMP, or V, these two angles are equal, and equation (a) may be written

$$R' = R \cos V$$

Substituting this value of R' in the last equation of the preceding article,

$$d = (sR \cos V + i) \cos V$$

7. Vertical Distances.—In Fig. 4, NP_1 is the difference of elevation between the center M of the transit (the intersection of the horizontal and the vertical axis of the telescope) and the point N. Denoting this difference by v, the right triangle NMP_1 gives

$$v = MP_1 \tan V = d \tan V$$

or, substituting the value of d from the formula of Art. **6**, and performing the multiplications,

$$\begin{aligned} v &= sR \cos^2 V \tan V + i \cos V \tan V \\ &= sR \cos^2 V \frac{\sin V}{\cos V} + i \cos V \frac{\sin V}{\cos V} \\ &= sR \sin V \cos V + i \sin V \end{aligned}$$

or, writing $\frac{1}{2} \sin 2V$ instead of $\sin V \cos V$ (for $\sin 2V = 2 \sin V \cos V$),

$$v = \tfrac{1}{2} sR \sin 2V + i \sin V$$

Fig. 4 shows a line of sight inclined above the horizontal. This formula and the formula of Art. **6** apply equally well when the line of sight is inclined below the horizontal. When the line of sight is inclined above the horizontal, the vertical angle is an angle of elevation, and is recorded in the notes as plus (+); when the line of sight is inclined below the horizontal, the vertical angle is an angle of depression, and is recorded in the notes as minus (−).

8. It is usually desired to determine the difference of elevation between the point over which the instrument is set and that on which the rod is held, whereas the formula of Art. **7** gives the difference of elevation between the center of the instrument and the point observed on the rod. If the middle cross-wire is made to intersect the rod at a point whose height above the ground is equal to that of the instrument, the result obtained by the formula of Art. **7** will be equal to the difference of elevation between the point over which the instrument is set and that on which the rod is held. In ordinary work, the observer estimates a distance on the rod about equal to the height of the instrument. If the difference of elevation is desired as accurately as possible, the observer generally measures the height of the instrument

by holding a rod alongside of it or in some other convenient manner. He calls out the height to the rodman, who marks an equal height on the rod, usually by setting a target or tying a piece of cloth around the rod at the required height.

EXAMPLE.—The length intercepted on the rod is 7.00 feet, and the angle that the line of sight makes with a horizontal line is 18° 23′. If the stadia constant is 100 and the instrument constant 1.00 foot: (*a*) what is the horizontal distance of the rod from the center of the instrument? (*b*) what is the difference of elevation between the center of the transit and the point where the line of sight intersects the rod, as indicated by the center cross-wire?

SOLUTION.—(*a*) Here $s = 100$, $R = 7$, $i = 1$, and $\cos V = \cos 18° 23' = .94897$. Substituting these values in the formula of Art. **6**,

$$d = (100 \times 7 \times .94897 + 1) \times \cos 18° 23' = 631.3 \text{ ft.} \quad \text{Ans.}$$

(*b*) Here $\sin V = \sin 18° 23' = .31537$, and $\sin 2V = \sin 36° 46' = .59856$. Substituting these values and those given above in the formula of Art. **7**,

$$v = \tfrac{1}{2} \times 100 \times 7 \times .59856 + 1 \times .31537 = 209.8 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The length intercepted on the stadia rod when held at right angles to the horizontal line of sight is 4.45 feet, the instrument constant is 1.00 foot, and the stadia constant, 100; what is the distance of the stadia rod from the center of the instrument? Ans. 446.0 ft.

2. The length intercepted on the stadia rod is 8.67 feet, and the angle that the line of sight makes with the horizontal is 25° 21′. If the stadia constant is 100 and the instrument constant 1.00 foot: (*a*) what is the horizontal distance of the rod from the center of the instrument? (*b*) what is the difference of elevation between the center of the instrument and the point where the line of sight intersects the rod, as indicated by the center cross-wire?

Ans. $\begin{cases} (a)\ 709.0 \text{ ft.} \\ (b)\ 335.9 \text{ ft.} \end{cases}$

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9. Values of the Constants.—The values of the instrument and stadia constants are usually determined by the instrument maker. The instrument constant varies from about .75 to 1.33 feet in different transits according to the size and power of their telescopes. Its value is usually marked on a card attached to the inside of the instrument box.

The stadia constant is customarily made equal to 100; so that, in a horizontal line of sight, the stadia wire will intercept

a distance of 1 foot on a rod whose distance from the instrument is 100 feet plus the instrument constant. Thus, if the stadia wires intercept a distance of 8.37 feet on the rod, the distance from the rod to the transit would be 837 feet plus the instrument constant. For ordinary topographical work, especially for long distances, it is sufficiently close to take for the distance 100 times the length intercepted on the rod, the instrument constant being disregarded.

10. Determination of the Constants.—If it is desired to determine or to check the constants, a line, from 400 to 800 feet long, is run on ground as level as practicable, taking care that the atmospheric conditions are favorable. (Refraction is least between about 8 and 10 A. M. and increases with the temperature.) The rod is held on this line at distances of 50, 100, 200 feet, etc. from the center of the instrument, and the space intercepted by the stadia wires at each setting of the rod is carefully read. Then the distances and the corresponding readings are taken in pairs and substituted in formulas given in this article; from each pair a value is found for s and one for i. Finally, the means are taken of all the values of s and of all the values of i.

Let d_1 and d_2 be two of the measured distances, and R_1 and R_2 the corresponding readings of the rod. Then, from formula 2, Art. 4,

$$d_1 = sR_1 + i \qquad (a)$$

$$d_2 = sR_2 + i \qquad (b)$$

Subtracting (a) from (b),

$$d_2 - d_1 = s(R_2 - R_1)$$

whence

$$s = \frac{d_2 - d_1}{R_2 - R_1} \qquad (1)$$

also,

$$i = d_1 - sR_1 = d_1 - \frac{(d_2 - d_1)R_1}{R_2 - R_1}$$

or, reducing,

$$i = \frac{d_1 R_2 - d_2 R_1}{R_2 - R_1} \qquad (2)$$

In determinations of this kind, the rod should be read to thousandths by means of a vernier. Although it is easier to find i by the formula $i = d_1 - sR_1$, it is advisable to find it independently by formula 2 of this article.

EXAMPLE.—To determine the stadia and the instrument constant from the following data:

DISTANCE MEASURED, FEET	ROD READING, FEET
50	.488
100	.988
200	1.988
300	2.991
400	3.986

SOLUTION.—Taking 50 feet for the value of d_1 and 100 feet, 200 feet, etc. successively for the values of d_2, and applying formulas 1 and 2, we find:

First, for $d_1 = 50$ ft. and $d_2 = 100$ ft.,

$$s = \frac{100 - 50}{.988 - .488} = 100.000$$

$$i = \frac{50 \times .988 - .488 \times 100}{.988 - .488} = 1.200 \text{ ft.}$$

Second, for $d_1 = 50$ ft. and $d_2 = 200$ ft.,

$$s = \frac{200 - 50}{1.988 - .488} = 100.000$$

$$i = \frac{50 \times 1.988 - .488 \times 200}{1.988 - .488} = 1.200 \text{ ft.}$$

Third, for $d_1 = 50$ ft. and $d_2 = 300$ ft.,

$$s = \frac{300 - 50}{2.991 - .488} = 99.880$$

$$i = \frac{50 \times 2.991 - .488 \times 300}{2.991 - .488} = 1.258 \text{ ft.}$$

Fourth, for $d_1 = 50$ ft. and $d_2 = 400$,

$$s = \frac{400 - 50}{3.986 - .488} = 100.057$$

$$i = \frac{50 \times 3.986 - .488 \times 400}{3.986 - .488} = 1.172 \text{ ft.}$$

Tabulating these results,

	s	i
	100.000	1.200
	100.000	1.200
	99.880	1.258
	100.057	1.172
	4)399.937	4)4.830
means	99.984	1.208

Ans. $\begin{cases} s = 99.984 \\ i = 1.208 \end{cases}$

If very accurate determinations are desired, observations as above should be taken at different hours of the day and an average of the results should be used.

EXAMPLE FOR PRACTICE

Determine the stadia and the instrument constant from the following data:

Distance Measured, Feet	Rod Reading, Feet
100	1.000
150	1.504
200	2.011
250	2.516
300	3.019

Ans. $\begin{cases} s = 99.030 \\ i = .970 \end{cases}$

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11. Precision.—The precision of stadia measurements depends on the accuracy of the determination of the constants, the care with which the rod readings are taken, and very largely on the state of the atmosphere. Other things being equal, greatest accuracy is obtained between 8 and 10 A. M., when refraction is least. If the atmosphere is hazy or unsteady from the effects of heat, as is often the case in summer, especially along a railroad track, a less degree of precision can be obtained than under the opposite conditions. Long sights should be avoided. With a good transit and under favorable atmospheric conditions, the error should not exceed $\frac{1}{500}$, and with very careful work may be as low as $\frac{1}{1000}$.

STADIA RODS

12. Kinds of Rods Used in Stadia Work.—An ordinary self-reading leveling rod is very often used in making stadia measurements. However, more satisfactory and more rapid work can be done with a rod made especially for the purpose. Various patterns have been designed, the principal idea being so to mark the graduations of the rod that they will be most easily and quickly read by the instrumentman. Fig. 5 shows four patterns of graduations in three of which the tenths of a foot are indicated by the principal angular points. Pattern (*a*) is one of the styles used by the United States Coast and Geodetic Survey, and patterns (*b*)

and (*c*) are used by the United States Lake Survey. Pattern (*d*) is easily graduated, each foot being subdivided into tenths by the black rectangles and the white spaces between them. One foot is here shown as the unit of graduation, although a yard or a meter is sometimes used. In pattern (*a*), each twentieth of a foot, and in pattern (*c*) each fiftieth of a foot, is also designated by an angular point. If readings to hundredths of a foot are desired, as is generally the case, they are estimated along the diagonal lines

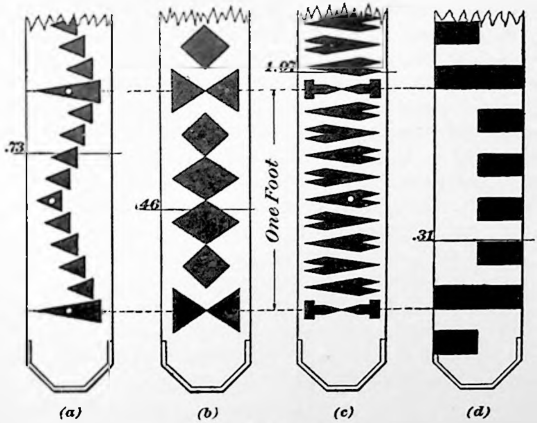


Fig. 5

of the graduations. Each diagonal line represents in pattern (*a*), $\frac{5}{100}$ foot; in pattern (*b*), $\frac{10}{100}$ foot; and in pattern (*c*), the long diagonals represent $\frac{4}{100}$ and the short diagonals $\frac{2}{100}$ foot. Thus, if the full line crossing each stadia rod, as shown in Fig. 5, represents the intersection of the line of sight, the readings on patterns (*a*), (*b*), (*c*), and (*d*) are, respectively, .73, .46, 1.07, and .31. In pattern (*d*), as in an ordinary self-reading leveling rod, the number of hundredths of a foot from the nearest tenth mark to the intersection of the line of sight must be estimated. With a little

practice, close reading of any of the various patterns of stadia rods is readily acquired. Stadia rods are generally about 12 feet long and are made in two equal sections, which are connected by a hinge for convenience in handling when the rod is not in use.

13. Regraduating a Stadia Rod.—In order that the stadia measurements may readily be determined from the rod readings when the stadia constant differs considerably from 100, the stadia rod may be regraduated so that one of its full divisions shall be intercepted between the stadia wires when the distance from the rod to the center of the instrument is 100 feet plus the instrument constant. The length of the new main graduations will be equal to 100 feet divided by the stadia constant. Thus, if the stadia constant is found to be 98, the length of the new graduations on the rod will be $100 \div 98 = 1.02$ feet. The entire length of the stadia rod should thus be laid off in equal divisions 1.02 feet long, and each division should be subdivided so that decimal portions can be read. Regraduating a rod is objectionable in that the rod is thus made unfit for use with any other transit or for leveling purposes. It is always preferable to employ a rod divided into true feet and fractions of a foot, and compute the distances by means of formula **2**, Art. **4**, and the formulas of Arts. **6** and **7**, or, still better, by means of stadia tables, as will be explained further on.

PRACTICAL STADIA WORK

MAKING THE OBSERVATIONS

14. Organization of Party.—The stadia party may consist merely of an **instrumentman**, or **observer**, and a **stadia rodman**. If the survey is large and rapid progress is desired, a **recorder** and one or more additional stadia rodmen are necessary. In this case the observer merely makes the observations and calls them out to the recorder. The observer generally has charge of the work of the party and should indicate to the rodmen the points on which the

stadia rod should be held for observation. The recorder keeps all the notes and should make sketches of objects that it is desired to locate, showing their position with reference to the points located by the transit. He should repeat to the observer the result of each observation as given out by the observer, before recording it in the notebook.

15. Location of Points.—A point over which the instrument is set is called an **instrument point.** A point on which a stadia rod is held for observation is called a **stadia point.** A stadia point is generally located with reference to the meridian or to some other line. The azimuth of the line of sight to the point to be located is obtained as explained in *Transit Surveying*, Part 1. The distance along this line of sight to the point is obtained from the stadia reading and (if the sight is inclined) the vertical angle. From the stadia reading and the vertical angle is also obtained the difference of elevation between the stadia point and the instrument point.

16. Taking the Readings.—Observing the length intercepted on a rod by the stadia wires is called **taking a stadia reading.** To take a stadia reading on a point where a rod is held, the telescope is first directed toward the rod, and then raised or lowered by means of the tangent screw until one of the stadia wires, generally the lower, coincides with a full division mark on the rod, and the number of full divisions and fractions of a division to the point intersected on the rod by the other stadia wire is read. The difference between the readings of the two wires is the stadia reading. Thus, if the lower wire is made to coincide with the 3-foot mark on the rod and the upper wire intersects the rod at a point whose reading is 5.35, the stadia reading is 5.35 − 3.00, or 2.35. The process is the same if a regular stadia rod is used.

17. As the stadia wires are placed equidistant from the regular cross-wire of the telescope, the latter will intersect the rod at a point midway between the points intersected by the stadia wires. The distance on the rod intercepted

between the middle cross-wire and one of the stadia wires is, therefore, one-half of the regular stadia reading. In very long sights, where the distance intercepted between the stadia wires is greater than the length of the rod, or where a portion of the rod is not visible, the middle wire and one of the stadia wires are used in taking the stadia reading, this reading being practically one-half the true reading. Thus, if the rod is held on a distant point and the lower stadia wire is made to read 1 foot and the middle cross-wire reads 8.24, the difference $8.24 - 1$, or 7.24, is one-half the true reading, and the true stadia reading is 7.24×2, or 14.48. Such half readings are frequently taken as checks on regular readings.

18. If the line of sight is inclined, the angle that it makes above or below the horizontal should be read from the vertical limb. It should be recorded in all cases if differences of elevation are to be determined, but it is not usually considered in connection with horizontal distances unless the vertical angle is more than 3°. When the angle is less than 3°, the line of sight may be assumed to be horizontal and the distance found by formula **2**, Art. **4**.

If the notes are kept by a recorder, the observer calls out the stadia reading as soon as obtained. If the observer keeps the notes, as soon as the stadia reading is taken, but before it is recorded, he should move the telescope up or down until the middle cross-wire intersects the rod at a point whose height above the ground is equal to the height of the instrument above the ground. The stadia reading is then recorded, after which the observer reads and records the vertical angle and the azimuth, if the latter has not already been recorded.

Care should be taken not to touch the lower clamp or tangent screw until all the sights from any instrument point have been taken and their azimuths recorded. It is generally best to check the sights from a station after the last one has been taken by sighting again to the first point observed from that station and noting if the vernier reading is the same as when the first sight on that station was taken. If one instrument point is located from another, the stadia reading

and vertical angle should be checked by backsights. The readings given by a backsight should be about the same as those given by the corresponding foresight, and the means between them are assumed to be the correct readings.

STADIA NOTES

19. Form of Notes.—For stadia notes, a regular transit book is commonly used. The numbers and letters designating the stadia stations and points on which stadia readings are taken are recorded in the first column of the left-hand page, the azimuths in the second column, the stadia readings in the third column, and the vertical angles in the fourth column. These four columns are filled out in the field when the observations are made. The horizontal distances are recorded in the fifth column and the elevations in the sixth column. These two columns are filled out when the notes are reduced, which is usually done in the office.

The notes are generally begun at the top of the page and the observations from each station are grouped as shown in the form on next page. The right-hand page of the book is used for sketches and remarks. Each instrument station is usually designated, both in the notes and sketches, by a dot enclosed in a small square, followed by the number of the station; thus, ⊡ 1, ⊡ 2, mean, respectively, *Station 1*, *Station 2*. A plain triangle or a square is often used for the same purpose.

At the top of the first left-hand page are written the location of the tract surveyed and the date the survey is begun. On the opposite page are written the names and positions of the members of the party. At the end of the notes is recorded the date on which the survey is completed. It is also advisable to record the date on which the stadia reductions are made and the notes platted, and the names of the persons who do this part of the work.

20. Reducing the Notes.—Before the notes can be platted, the horizontal and vertical distances must be determined from the stadia readings and the vertical angles. This operation is usually called **reducing the notes.** The

Sta.	Azimuth	Stadia	Vert. Angle	Hor. Dist.	Elev.	⊡ 1 = Old corner stone. Elev. assumed as 100.0.
	Readings from ⊡ 1 Elev. = 100.0					Property line from ⊡ 1 to A
A	230° 0′	6.18	0° 0′			Corner post.
B	100° 24′	4.47	−18° 10′			Artesian well.
⊡ 2	125° 14′	12.10	−11° 45′			
	Readings from ⊡ 2 Elev. =					
⊡ 1	305° 14′	12.16	+11° 53′			

vertical distances are not recorded, but they are used when obtained to determine the elevations of the stadia points with reference to some known or assumed datum. The elevation of any stadia point is obtained by first obtaining the difference in elevation between that point and the instrument point from which the readings on the former point are taken, and then adding that difference to or subtracting it from the elevation of the instrument point, according as the vertical angle is one of elevation or one of depression.

In stadia work, it is customary to calculate and record horizontal distances to the nearest foot only, and elevations to the nearest tenth of a foot only, except on instrument stations, where the differences of elevation are determined and recorded to the nearest hundredth of a foot. Such results are as close as are justified by the degree of accuracy usually attained in stadia measurement.

21. If the line of sight is horizontal, the distance corresponding to a stadia

reading is obtained by formula **2**, Art. **4**. This formula is generally used if the angle of inclination does not exceed 3°. Since the stadia constant is commonly 100, the distance required is 100 times the stadia reading, plus the instrument constant, which, if considered, is generally taken as 1 foot. Thus, if the stadia reading in a horizontal sight is 6.18, the distance is $100 \times 6.18 + 1$, or 619 feet. If the line of sight is inclined, the horizontal distance may be obtained by the formula of Art. **6** and the difference of elevation by the formula of Art. **7**. These calculations are generally and more conveniently made by the aid of a table, the use of which is explained in the following article.

It is sometimes desirable to reduce and plat the notes in the field. The platting is done on a sheet attached to a light drawing board. The azimuths are laid off with a protractor, as explained in *Transit Surveying*, Part 1.

22. Stadia Reduction Table.—The formula of Art. **6** giving the horizontal distance between an instrument and a stadia point, may be written

$$d = s\,R\cos^2 V + i\cos V \qquad (a)$$

Let e be the difference between the stadia constant s and 100, so that $s = 100 \pm e$, the upper sign applying when s is greater than 100, the lower when s is less than 100. This value substituted in equation (a) gives

$$\begin{aligned} d &= (100 \pm e)\,R\cos^2 V + i\cos V \\ &= \left(100\cos^2 V \pm 100\cos^2 V \times \frac{e}{100}\right)R + i\cos V \qquad (b) \end{aligned}$$

In order to facilitate the application of this equation, the Stadia Reduction Table given at the end of this Section has been prepared. That table contains, in the column headed Horizontal Distances, values of $100\cos^2 V$ for values of V varying by intervals of 2′ from 0° to 31°. At the bottom of each column of horizontal distances are given values of $i\cos V$ for three values of i; namely, .75, 1.00, and 1.25 feet, and for an angle equal to that at the top of the column plus 30′, this being the mean between the angle at the top and the angle plus 60′; a mean value that can be used with

sufficient accuracy in place of any value of V included between those limits.

Let the value of 100 $\cos^2 V$ taken from the table, for any given value of V, be denoted by d_1, and the corresponding value of $i \cos V$ (the mean value mentioned above being here used instead of V) be denoted by I_d. Equation (b) may then be written

$$d = \left(d_1 \pm d_1 \times \frac{e}{100}\right)R + I_d \qquad (1)$$

If, as is usually the case, the stadia constant is 100, or very nearly 100, the term containing e is omitted, and the formula becomes

$$d = d_1 R + I_d \qquad (2)$$

23. The formula of Art. 7 for the difference of elevation between the instrument and the stadia point may be written

$$v = \frac{(100 \pm e) R \sin 2V}{2} + i \sin V$$

$$= \left(\frac{100 \sin 2V}{2} \pm \frac{100 \sin 2V}{2} \times \frac{e}{100}\right)R + i \sin V \qquad (a)$$

The stadia table contains, in the column headed Difference of Elevation, values of $\frac{100 \sin 2V}{2}$ for values of V as stated in the preceding article. At the bottom of each of these columns are given values of $i \sin V$, for the three values of i and the mean angles mentioned in the article just referred to. If the values of $\frac{100 \sin 2V}{2}$ and $i \sin V$, as taken from the table, are denoted by v_1 and I_v, respectively, equation (a) takes the form

$$v = \left(v_1 \pm v_1 \times \frac{e}{100}\right)R + I_v \qquad (1)$$

When the stadia constant is 100, or very nearly 100, this formula reduces to the form

$$v = v_1 R + I_v \qquad (2)$$

24. It has been stated that the angles in the table vary by intervals of 2′; in other words, the table contains only even numbers of minutes. For any odd number of minutes, it is sufficiently accurate to substitute either of the two even

numbers between which it lies. Thus, if the angle V is 23° 15′, either 23° 14′ or 23° 16′ may be used in its place. Should greater accuracy be desired, a mean of the values (horizontal distance or difference of elevation) corresponding to 23° 14′ and 23° 16′ may be taken.

EXAMPLE 1.—The stadia reading is 3.96 feet; the vertical angle, when the line of sight intersects the rod at a height equal to the height of the instrument, is 10° 26′; the stadia constant is 100; and the instrument constant is 1 foot. To determine by the table: (*a*) the horizontal distance from the center of the instrument to the stadia point; (*b*) the difference of elevation between the instrument point and the stadia point.

SOLUTION.—(*a*) The horizontal distance given in the table for 10° 26′ is found to be 96.72($= d_1$). At the bottom of the 10° column of horizontal distances and opposite a value of i = 1.00 (the instrument constant) is found .98($= I_d$). As the stadia constant is 100, formula **2**, Art. **22**, should be used. In this case, we have,

$$d_1 = 96.72,\ R = 3.96,\ I_d = .98$$

Substituting these values in the formula,

$$d = 96.72 \times 3.96 + .98 = 383.99 \text{ ft. Ans.}$$

(*b*) Likewise, the difference of elevation corresponding to 10° 26′ is found to be 17.81($= v_1$). The value of I_v for a value of i = 1.00 is found at the bottom of the column of differences of elevation, under 10°, to be .18. To apply formula **2**, Art. **23**, we have,

$$v_1 = 17.81,\ R = 3.96,\ I_v = .18$$

Substituting these values in that formula,

$$v = 17.81 \times 3.96 + .18 = 70.71. \text{ Ans.}$$

EXAMPLE 2.—If in the preceding example the elevation of the instrument point is 102.46 feet, and the vertical angle is one of depression, what is the elevation of the stadia point?

SOLUTION.—The difference of elevation between these points was found to be 70.71 ft., and, since the vertical angle is one of depression, this distance is subtracted from the elevation of the instrument point. The elevation of the stadia point is, therefore, 102.46 − 70.71, or 31.75 ft. Ans.

EXAMPLE 3.—The stadia reading being 8.24; the vertical angle, + 20° 40′; the elevation of the instrument point, 240.72 feet; the stadia constant, 97.8; and the instrument constant, 1.25, to find: (*a*) the horizontal distance d from the instrument point to the stadia point; and (*b*) the elevation of the stadia point.

SOLUTION.—(*a*) The distance d is determined by formula **1**, Art. **22**. In this case, e = 100 − 97.8 = 2.2, and $\frac{e}{100}$ = .022. The

value of d_1 corresponding to an angle of 20° 40′ is 87.54. At the bottom of the column of horizontal distances, under 20°, and horizontally opposite the expression i = 1.25, is found 1.17(= I_d). As R = 8.24, formula **1**, Art. **22**, gives

$$d = (87.54 - 87.54 \times .022)8.24 + 1.17 = 706.60 \text{ ft. Ans.}$$

(*b*) The values of v_1 and I_v, to be substituted in formula **1**, Art. **23**, are found from the table to be 33.02 and .44, respectively; therefore, applying that formula,

$$v = \left(33.02 - 33.02 \times \frac{2.2}{100}\right)8.24 + .44 = 266.51 \text{ ft.}$$

Adding this to the elevation 240.72 of the instrument point (since the vertical angle is +), 507.23 ft. is obtained as the elevation of the stadia point.

EXAMPLES FOR PRACTICE

NOTE.—In calculating the horizontal distances and differences in elevation in the following examples, use the Stadia Reduction Table.

1. The intercepted distance on the vertical stadia rod is 4.28 feet, and the vertical angle is −20° 25′. Assuming the instrument constant to be .75 foot and the stadia constant to be 100, what are: (*a*) the horizontal distance from the center of the instrument to the stadia rod; (*b*) the difference in elevation between the center of the instrument and the point observed on the rod?

Ans. { (*a*) 376.6 ft. (*b*) 140.2 ft.

2. The intercepted distance on the vertical stadia rod is 3.27 feet, and the vertical angle is −28° 16′. Assuming the instrument constant to be 1 foot and the stadia constant to be 100: (*a*) what is the difference in elevation between the center of the instrument and the point observed on the rod? (*b*) If the height of the center of the instrument above datum is 256.28 feet, and the reading of the center cross-wire on the stadia rod is 8.39 feet, what is the elevation of the stadia point above datum?

Ans. { (*a*) 136.9 ft. (*b*) 111.0 ft.

3. The intercepted distance on the stadia rod is 5.47 feet, and the vertical angle is +18° 14′. Assuming the instrument constant to be 1.25 feet, the stadia constant 100, and the elevation of the instrument point 126.00 feet, find: (*a*) the elevation of the stadia point, if the line of sight intersects the rod at a height equal to the height of instrument; (*b*) the horizontal distance from the center of the instrument to the stadia rod.

Ans. { (*a*) 289.0 ft. (*b*) 494.6 ft.

4. All the other conditions being the same as in the preceding example, what is the horizontal distance, if the stadia constant is 99?

Ans. 489.7 ft.

PLANE-TABLE SURVEYING

THE PLANE TABLE

25. Description.—The **plane table** is an instrument used in the preparation of topographical maps. It consists of a drawing board mounted on a tripod, and a device for sighting in any direction and transferring the line of sight to a sheet or roll of drawing paper on the board. Fig. 6 shows the Johnson plane table, which is the one most generally used in private work.

26. The **drawing board** is made of well-seasoned pine with a piece dovetailed on each end at right angles to the grain of the board, so as to overcome the tendency to warp.

27. The instrument for sighting and transferring the line of sight to the paper on the board is called the **alidade.** It consists of a metal ruler r from which rises an upright carrying the telescope T. The telescope has a vertical movement in a plane that contains the edge of the ruler or is parallel to it. It is equipped with a level tube and a vertical circle for the measurement of vertical angles, and generally has two stadia wires in addition to the regular horizontal and vertical wires. The alidade merely sets on the drawing board when in use; when not in use, it is kept in the instrument box.

28. The **tripod** is of the same general design as that of a transit, except that the legs are heavier and shorter, so that the table may be set firmly and at such a height as is convenient for the purpose of drawing or for sighting through the telescope. The device by which the drawing board is connected to the tripod head, permitting the separate motions necessary for leveling the board and for turning it horizontally, is called the **movement.** Fig. 6 shows the board with

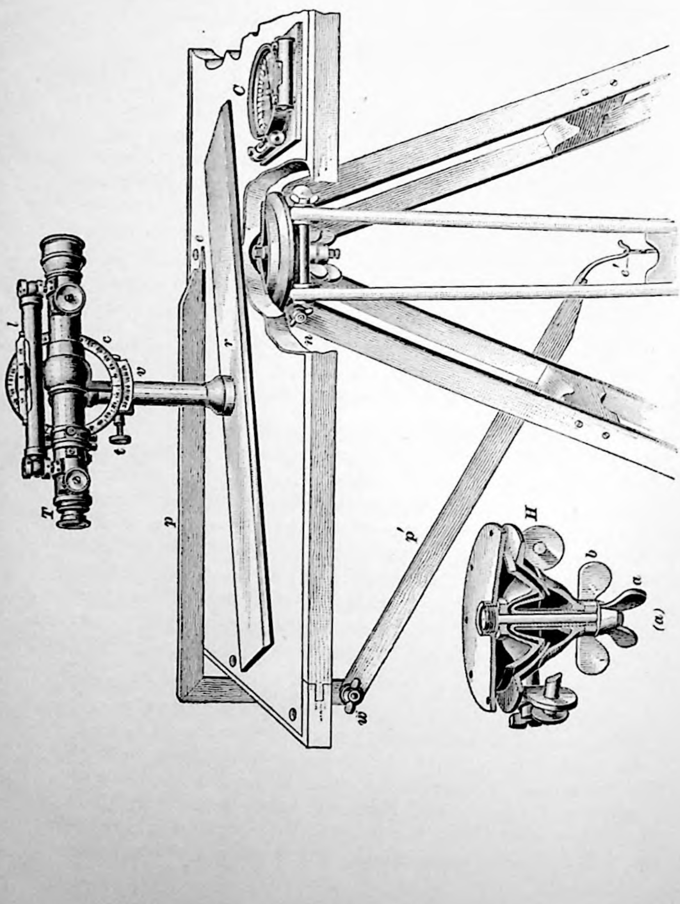


FIG. 6

a portion cut away to show the tripod head and movement. The latter is also shown separately with a portion cut away so that its construction may be seen more plainly. In some plane tables, the leveling is accomplished by means of regular leveling screws.

29. The **declinator** *C*, Fig. 6, is a compass needle mounted in a flat box, whose sides are straight edges parallel to the line determined by the zero marks of the compass graduations. In this type of plane table, the level tubes are attached to the declinator, as shown in the figure. The level tubes are sometimes attached to the alidade ruler.

30. The **plumbing arm** $epp'e'$, Fig. 6, is a device for suspending a plumb-bob so that it will be directly under a point e on the paper representing the point determined on the ground by the plumb-bob.

31. **Plane-Table Paper.**—The drawing paper for plane-table work is sometimes used in rolls, the portions not in use being rolled under the ends or sides of the board. More often the paper is prepared in the form of sheets about the size of the board. It is held on the board by heavy brass clamps or by strong tacks or screws.

The paper should be of good quality, as the expansion and contraction due to atmospheric changes is very large in the poorer grades of paper. It may be seasoned by being exposed for several days alternately to a very damp and to a very dry atmosphere. This considerably lessens the effect of atmospheric changes.

If a great degree of accuracy is desired, doubly mounted sheets of the best paper are used. They are made by pasting a sheet of paper on each side of a piece of tightly stretched muslin with the grain of one sheet at right angles to the grain of the other. Celluloid sheets are sometimes used where there is likely to be an accumulation of moisture on the sheet, such as rain, dew, or water dropping from the trees.

TESTS AND ADJUSTMENTS

32. The Level Tubes.—The level tubes attached to the declinator are tested by placing the declinator in the center of the table and bringing each bubble to the center of the tube by the leveling apparatus. The declinator is then reversed in azimuth; if it is in adjustment, the bubbles will be in the center of the tubes. If not, one-half the error is corrected by means of the adjusting screws, and the other half by again leveling the table. This adjustment is similar to that of the plate levels of a compass or transit. The same method is used if the level tubes are attached to the alidade.

33. The Telescope.—The adjustments of the telescope are the same as the second, third, fourth, and fifth adjustments of a transit, described in *Transit Surveying*, Part 1.

34. The Drawing Board.—If the drawing board is warped or uneven, it should be planed off until a true plane surface is obtained. The board should revolve in a truly horizontal plane. To determine this, after the level tubes have been tested, the table should be leveled carefully and then turned around about 180°. If the bubbles remain in the center of the tubes, the test is satisfactory. If not, one-half the error should be corrected by inserting thin washers between the board and the arms or plate of the movement to which it is attached on that side of the center which it is necessary to raise, in order to bring the bubble toward the center of the tube. This adjustment is repeated until the bubbles remain in the center of the tubes during a complete revolution of the table. Except for very important work, it is not necessary that this adjustment should be perfect.

PLANE-TABLE FIELD WORK

35. Organization of Party.—A plane-table party consists of an instrumentman, generally called a **topographer**, and one or more rodmen. Additional helpers are needed if the survey is of a large area or the country is very rough. The topographer makes the observations and does the platting

and sketching. The rodmen should be able to determine what points are necessary to be located, in order to properly represent on the map the ground covered by the survey.

36. Plane-Table and Triangulation Stations.—The point on the ground over which the plane table is set is called a **plane-table station**, and is generally designated by a small circle with a dot in the center (⊙). In plane-table work, it is customary to denote points or stations on the ground by capital letters, and their platted positions by small letters. Thus, if a point on the ground is called *A*, its platted position, or its position on the map, is called *a*.

Plane-table work is generally based on an imaginary line connecting two known points that are visible from each other. The distance between these points is measured or calculated, and the line connecting them is platted to the scale decided on for the map on such a portion of the sheet as is indicated by the relation of the known points to the rest of the area to be covered by the survey. In the survey of a large area, reference lines forming a net of triangles are generally run with a transit, and from them other points of the survey are located by means of the plane table. The vertexes of these triangles are called **triangulation stations**, and are often designated by a small triangle with a dot in the center (◬). In order that a triangulation station can be seen from the plane-table stations in various directions, a **signal** is often erected after the point has been occupied by the transit. This generally consists of a straight pole, braced so as to stand in a vertical position, and with a piece of cloth attached to the top to make it more easily discernible.

37. Setting Up the Plane Table.—In setting up the plane table, the legs of the tripod are so placed that the tripod head is about level. The board is then made level by means of the level tubes attached to the declinator or to the alidade. If the movement is an adaptation of the ball and socket joint, the board is moved up or down by pressure on the sides until the bubble is in the center of each tube.

By means of the plumbing arm (see Art. 30), the table can be so set that a special point on the map shall be directly over a given station on the ground. In order to accomplish this, it may be necessary to move the tripod legs, or the tripod as a whole, several times. Usually, however, if the platted point is nearly over the station on the ground that it represents, the table is considered to be correctly set, as the map is generally drawn to so small a scale that an error, even of 1 foot, would not show on the map.

The level movement should be clamped as soon as the board is leveled, and the horizontal movement as soon as the desired position of the board is secured.

38. Orienting the Plane Table.—If the survey is based on lines connecting two or more points whose positions are platted on the board, the latter, after being leveled, must be clamped in such a position that each line platted corresponds in direction with, or is parallel to, the line on the ground that it represents. When this condition obtains, the plane table is said to be **oriented, or in orientation, or in position.**

Unless the platted point is in the center of the board, any turning of the table to orient it will change the position of the platted point with reference to the station on the ground. Hence, the plane table is oriented as closely as possible by the eye before attempting to set a platted point accurately over a point on the ground. Thus, in Fig. 7, the Stations A and B on the ground are platted on the plane-table sheet at a and b. The plane table is placed at Station A and is oriented approximately by sighting along the line $a\,b$ and turning the board until the line $a\,b$ is about in the same direction as the line $A\,B$ on the ground. The point a is brought over Station A by moving the tripod and using the plumbing arm, care being taken to keep the board approximately oriented. The table is then leveled and the edge of the alidade ruler is made to coincide with the line $a\,b$, the objective end of the telescope being directed toward B. The table is next turned in azimuth until the telescope is

accurately directed to B. This turning of the board changes the position of a with reference to A; but the error is generally so small that it is not considered. When the edge of the ruler is along $a\ b$, and the telescope directed to B,

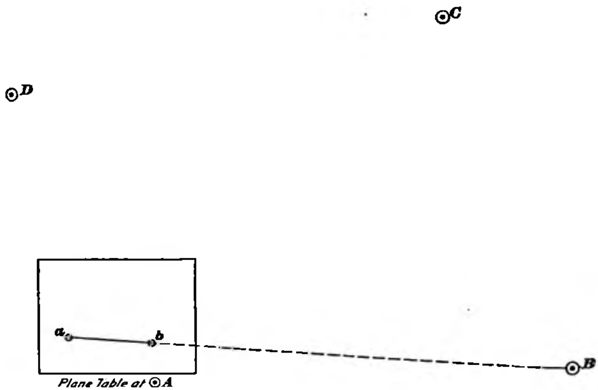


FIG. 7

as just explained, the plane table is oriented. The board is then clamped; and lines of sight to any points, such as C and D, that are to be located and are visible from the station occupied, are platted in the manner explained in the following article.

39. Platting the Line of Sight.—After the plane table is oriented, the telescope is directed to the point to be located, the edge of the alidade ruler being kept in contact with the platted position of the station occupied. A line is then drawn along the edge of the ruler, beginning at the platted point and extending in the direction of the point to be located. Thus, in Fig. 7, the plane table is oriented at Station A, the telescope is directed to the point C, with the edge of the ruler in contact with the point a, which is over Station A on the ground. A line representing this line of sight is then drawn along the edge of the ruler, as shown at $a\,x$

in the left-hand portion of Fig. 8. In platting a line of sight, a hard pencil having a fine point should be used, and the line should be drawn lightly and close to the edge of the ruler.

LOCATION OF POINTS

40. Location by Distance.—If the plane table is oriented at any station, any point visible from that station may be located on the plat by laying off to scale, on the platted line of sight to the point, the distance from the point to the station occupied. That distance may be determined

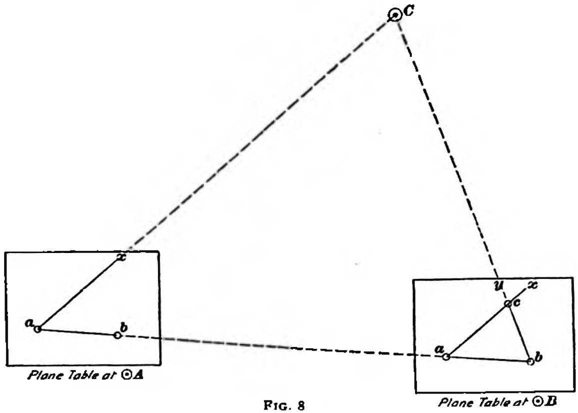


Fig. 8

by either stadia or direct measurement. Thus, in Fig. 8, the plane table having been oriented at Station *A* and the line of sight to the point *C* having been platted at *a x*, the point *c* may be located by obtaining the distance from *A* to *C* and laying off this distance to the scale of the map on the line *a x*.

41. Location by Intersection.—A point may be located on the plat by sighting to it with the plane table oriented from two stations whose locations are platted on the board. The intersection of the platted lines of sight from those two stations to the point will determine the

position of the point on the plat. Thus, in Fig. 8, the plane table is shown in two locations at Stations A and B, these stations being platted at a and b. To locate the point C, the plane table is set up at A and oriented by sighting to B along ab, as explained in Art. 38. The board being clamped, the telescope is directed to the point C with the edge of the ruler in contact with the platted position a of the station occupied, and a line ax is drawn representing this line of sight. The plane table is then set up at B and oriented by sighting to A along ba, and a line of sight by is platted to C. The intersection c of this line with the line ax drawn from a will be the platted location of C.

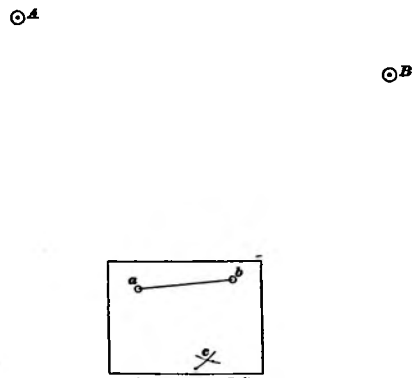
This method of location is based on the principle of geometry that two triangles are similar when two angles of one are equal to two angles of the other. Thus, in Fig. 8, the triangle abc is similar to the triangle ABC on the ground, since the angles abc and bac are platted equal to the angles ABC and BAC. The sides of these triangles are therefore proportional, and as ab represents AB to the scale of the map, ac and bc will represent, respectively, AC and BC to the same scale.

To secure a good determination of the point of intersection, the lines should, if possible, meet at an angle not less than 30° nor greater than 150°. If a point located by intersection is an important one, sights should be taken to it, if practicable, from more than two known points, in order to check its position on the map.

42. Location by Resection.—This method is of great advantage in an extensive survey, as it permits the point occupied by the plane table to be arbitrarily selected. It consists in setting the table over a convenient point from which two or more of the points already platted can be seen, determining the position on the plat of the point thus occupied, and orienting the table at that point, so that other points may be observed from it and platted.

If two points that have already been platted are near at hand and accessible, the station occupied may be located on

the plat by determining the distances from it to those two points. Using these distances, to the scale of the map, as radii, the intersection of arcs swung from the platted points will locate on the map the station occupied. Thus, in Fig. 9, the point occupied by the plane table is Station *C*. It is desired to locate on the map the position of Station *C*, with



Plane Table at ⊙C

FIG. 9

reference to the platted positions *a* and *b* of the points *A* and *B*. The distances of the points *A* and *B* from Station *C* are determined either by stadia or by direct measurement; then, with *a* and *b* as centers and with radii equal, to the scale of the map, to the distances *A C* and *B C*, respectively, arcs are swung. The point of intersection *c* of these arcs is the platted position of the point *C*.

If it is desired to locate additional points from this station, the plane table is oriented by placing the edge of the ruler in contact with *c a* or *c b* and directing the telescope to *A* or *B*, as the case may be.

43. If the station occupied and to be located on the map is on a line that has been platted from another station, its position on the map is determined as follows: Let *C*,

Fig. 10, be a point on the ground to which a line of sight was directed when the table was set and oriented at *A*, and let *a x* be the platted position of that line of sight. The plane table is oriented by placing the edge of the ruler in contact with the line *a x* and directing the telescope to Station *A*. Having clamped the board in this position, the edge of the ruler is placed in contact with another platted point *b*, and the ruler turned about this point until the telescope is directed to the corresponding point (*B* in this case) on the ground, and the line of sight is platted. The intersection *c* of this line with the line *a x* is the platted position of the point *C*.

When the point occupied has not been sighted to before, and its distances from other known points cannot be conveniently obtained, its

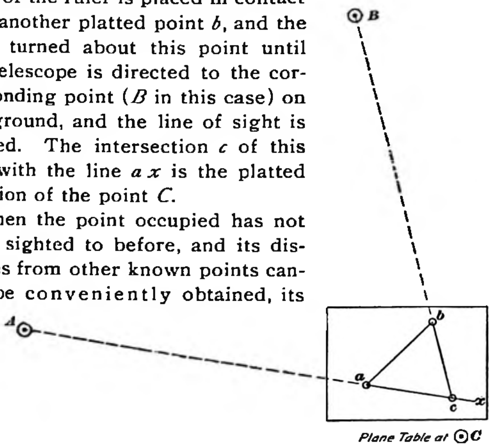


FIG. 10

position on the map is determined by the solution of one or two problems, known, respectively, as the *three-point problem* and the *two-point problem*—the former occurring when three known stations are visible from the station occupied, and the latter when only two stations are visible. The former condition is preferable to the latter, and, whenever possible, the station occupied should be so chosen that three known stations already located on the map can be seen from it.

44. The Three-Point Problem.—Let the plane table be set over a point *P*, Fig. 11, from which three points *A*, *B*, *C*, platted at *a*, *b*, and *c*, respectively, are visible. The

three-point problem consists in the determination of the point p on the map corresponding to the point P on the ground; that is, of a point on the map whose position with reference to a, b, and c shall be the same as that of P with reference to A, B, and C. For convenience, the board is turned so that the points a, b, and c occupy about the same relative positions on the board as are occupied by the points A, B, and C on the ground; in other words, the table is approximately oriented by the eye. A piece of tracing cloth or paper $s\,s\,s\,s$,

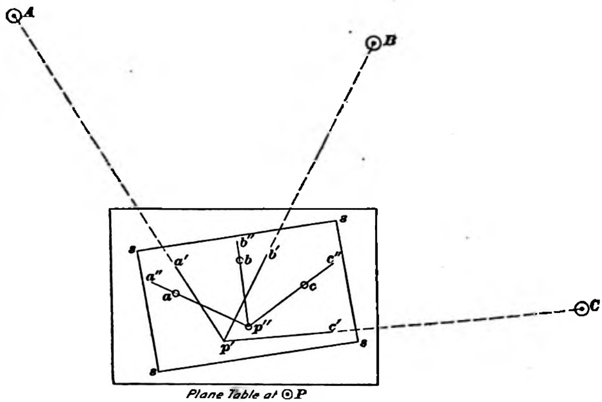


FIG. 11

large enough to cover the three platted points and the estimated location of the station occupied, is fastened to the board over the plane-table paper. A point p' is chosen on the tracing cloth approximately in the same position with reference to the points a, b, and c that Station P has with reference to the points A, B, and C. With the edge of the ruler in contact with the point p', the telescope is directed successively to A, B, and C, and the lines of sight are platted, as shown at $p'a'$, $p'b'$, and $p'c'$. The alidade is then removed and the tracing cloth is unfastened and shifted on the drawing paper to a position in which the lines $p'a'$, $p'b'$,

and $p'c'$ pass through the platted points a, b, and c, as shown at $p''a''$, $p''b''$, $p''c''$, which are then the positions of $p'a'$, $p'b'$, and $p'c'$, respectively, p'' being the position of p'. The point p'' is over the exact position of p and can be pricked through with a fine needle point. The tracing paper is then removed and the alidade is replaced on the table. With the edge of the ruler in contact with the pricked point p and one of the platted points, such as a, the table is turned in azimuth until the telescope is directed to the point on the ground that the platted point (a in this case) represents. By this operation, the plane table is oriented and the board is then clamped. If desired, the position of p may be checked by sighting to the points B and C with the edge of the ruler in contact, respectively, with the points b and c, and platting the lines of sight. These lines will intersect at p if the work has been accurately done.

Care should be exercised to keep the tracing cloth free from wrinkles and not to stretch it in the process of fastening over the drawing paper.

45. The Two-Point Problem.—Let C, Fig. 12 (a), be a station from which two points A and B, platted at a and b, are visible. It is required to determine the platted position of C. To do this, a fourth point D is chosen in such a position that A, B, and C are visible from it and that the lines of sight from C and D to A and B will make with each other angles sufficiently large for good intersections. The plane table is set up at D, approximately oriented by the eye, and clamped. A point d is chosen on the board about in the same position with reference to a and b that the point occupied has with reference to A and B. The point D is located directly under d by means of the plumbing arm. With the edge of the ruler in contact with d, the telescope is directed successively to A, B, and C and the lines of sight are platted, as shown at da', db', and dc'. The plane table is then set up at Station C (the station to be located on the map) and by placing the edge of the ruler on the line $c'd$ and directing the telescope to Station D, the board is oriented with

reference to the line CD. A point c'' on the line $c'd$ is chosen to represent the Station C. (This is generally done by estimating the distance from C to D and laying it off from d to the scale of the map, but the distance dc'' does not affect the result, and, therefore, c'' may be taken anywhere on dc'.) With the edge of the ruler in contact with c'', the telescope is directed successively to the points A and B, and the lines of sight are platted. These lines will intersect the lines da'

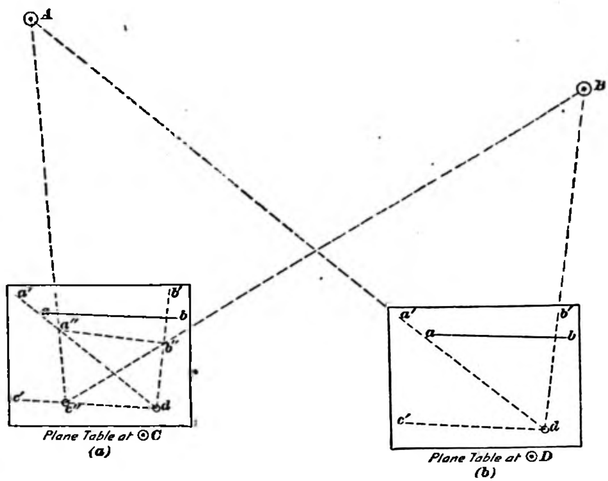


FIG. 12

and db' at a'' and b'', respectively. The points A and B are thus located by intersection, with reference to DC, at a'' and b'', and the line $a''b''$ is parallel to AB. To orient the plane table, the platted line ab must be made parallel to AB. This is now readily accomplished by placing the edge of the ruler on the line $a''b''$ and setting a rod or other mark in line at any convenient point at least 500 feet from the plane table. The board is then unclamped, and, with the edge of the ruler on the line ab, turned until the line of sight bisects the rod.

The distance between the points a and a'' is so small compared with the length of the sight to the rod, that the line $a\,b$ may be considered to coincide with the former position of the line $a''\,b''$, which was parallel to $A\,B$. Therefore, the board may be considered as oriented. By sighting to A with the alidade in contact with a, and to B with the alidade in contact with b, the platted lines representing these sights will intersect at a point c, which is the platted position of the station occupied. The board is now in position to locate any additional points from Station C.

The auxiliary lines necessary for the solution of this problem are sometimes drawn on tracing cloth fastened to the board, and the position of the point c when obtained is pricked through to the drawing paper. The tracing cloth is then removed.

46. Compass Orientation.—When the plane table is in orientation, a magnetic meridian may be established by means of the declinator, which is placed on the board and moved until the needle points to the north point of the compass. A line drawn along the side of the declinator box that is parallel to the needle will indicate a magnetic meridian. The plane table may be oriented roughly, that is, within the degree of accuracy of the compass, at any succeeding station by placing the side of the declinator box in contact with the magnetic meridian as drawn on the board and turning the board until the needle points to the north point of the compass box. **Compass orientation** is often used in the preliminary operation of the solution of the two-point and the three-point problem, and also in running rough traverse lines to locate unimportant roads, streams, fence lines, etc.

STADIA REDUCTION TABLE

Minutes	0°		1°		2°		3°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	.00	99.97	1.74	99.88	3.49	99.73	5.23
2	100.00	.06	99.97	1.80	99.87	3.55	99.72	5.28
4	100.00	.12	99.97	1.86	99.87	3.60	99.71	5.34
6	100.00	.17	99.96	1.92	99.87	3.66	99.71	5.40
8	100.00	.23	99.96	1.98	99.86	3.72	99.70	5.46
10	100.00	.29	99.96	2.04	99.86	3.78	99.69	5.52
12	100.00	.35	99.96	2.09	99.85	3.84	99.69	5.57
14	100.00	.41	99.95	2.15	99.85	3.89	99.68	5.63
16	100.00	.47	99.95	2.21	99.84	3.95	99.68	5.69
18	100.00	.52	99.95	2.27	99.84	4.01	99.67	5.75
20	100.00	.58	99.95	2.33	99.83	4.07	99.66	5.80
22	100.00	.64	99.94	2.38	99.83	4.13	99.66	5.86
24	100.00	.70	99.94	2.44	99.82	4.18	99.65	5.92
26	99.99	.76	99.94	2.50	99.82	4.24	99.64	5.98
28	99.99	.81	99.93	2.56	99.81	4.30	99.63	6.04
30	99.99	.87	99.93	2.62	99.81	4.36	99.63	6.09
32	99.99	.93	99.93	2.67	99.80	4.42	99.62	6.15
34	99.99	.99	99.93	2.73	99.80	4.47	99.61	6.21
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.32
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38
42	99.99	1.22	99.91	2.97	99.78	4.71	99.58	6.44
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61
50	99.98	1.45	99.90	3.20	99.76	4.94	99.55	6.67
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.79
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
i = .75	.75	.01	.75	.02	.75	.03	.75	.05
i = 1.00	1.00	.01	1.00	.03	1.00	.04	1.00	.06
i = 1.25	1.25	.02	1.25	.03	1.25	.05	1.25	.08

STADIA REDUCTION TABLE—*Continued*

Minutes	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.49	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.27
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.30	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
i = .75	.75	.06	.75	.07	.75	.08	.74	.10
i = 1.00	1.00	.08	1.00	.10	.99	.11	.99	.13
i = 1.25	1.25	.10	1.24	.12	1.24	.14	1.24	.16

STADIA REDUCTION TABLE—*Continued*

Minutes	8°		9°		10°		11°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
$i = .75$	.74	.11	.74	.12	.74	.14	.73	.15
$i = 1.00$	.99	.15	.99	.17	.98	.18	.98	.20
$i = 1.25$	1.24	.18	1.23	.21	1.23	.23	1.22	.25

STADIA REDUCTION TABLE—*Continued*

Minutes	12°		13°		14°		15°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.82	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
$i = .75$	.73	.16	.73	.18	.73	.19	.72	.20
$i = 1.00$	.98	.22	.97	.23	.97	.25	.96	.27
$i = 1.25$	1.22	.27	1.22	.29	1.21	.31	1.20	.33

STADIA REDUCTION TABLE—*Continued*

Minutes	16°		17°		18°		19°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.97	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.18	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.49	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
i = .75	.72	.21	.72	.23	.71	.24	.71	.25
i = 1.00	.96	.28	.95	.30	.95	.32	.94	.33
i = 1.25	1.20	.36	1.19	.38	1.19	.40	1.18	.42

STADIA REDUCTION TABLE—*Continued*

Minutes	20°		21°		22°		23°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
$i =$ .75	.70	.26	.70	.27	.69	.29	.69	.30
$i =$ 1.00	.94	.35	.93	.37	.92	.38	.92	.40
$i =$ 1.25	1.17	.44	1.16	.46	1.15	.48	1.15	.50

STADIA REDUCTION TABLE—*Continued*

Minutes	24°		25°		26°		27°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
i = .75	.68	.31	.68	.32	.67	.33	.67	.35
i = 1.00	.91	.41	.90	.43	.89	.45	.89	.46
i = 1.25	1.14	.52	1.13	.54	1.12	.56	1.11	.58

STADIA REDUCTION TABLE—*Continued*

Minutes	28°		29°		30°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	77.96	41.45	76.50	42.40	75.00	43.30
2	77.91	41.48	76.45	42.43	74.95	43.33
4	77.86	41.52	76.40	42.46	74.90	43.36
6	77.81	41.55	76.35	42.49	74.85	43.39
8	77.77	41.58	76.30	42.53	74.80	43.42
10	77.72	41.61	76.25	42.56	74.75	43.45
12	77.67	41.65	76.20	42.59	74.70	43.47
14	77.62	41.68	76.15	42.62	74.65	43.50
16	77.57	41.71	76.10	42.65	74.60	43.53
18	77.52	41.74	76.05	42.68	74.55	43.56
20	77.48	41.77	76.00	42.71	74.49	43.59
22	77.42	41.81	75.95	42.74	74.44	43.62
24	77.38	41.84	75.90	42.77	74.39	43.65
26	77.33	41.87	75.85	42.80	74.34	43.67
28	77.28	41.90	75.80	42.83	74.29	43.70
30	77.23	41.93	75.75	42.86	74.24	43.73
32	77.18	41.97	75.70	42.89	74.19	43.76
34	77.13	42.00	75.65	42.92	74.14	43.79
36	77.09	42.03	75.60	42.95	74.09	43.82
38	77.04	42.06	75.55	42.98	74.04	43.84
40	76.99	42.09	75.50	43.01	73.99	43.87
42	76.94	42.12	75.45	43.04	73.93	43.90
44	76.89	42.15	75.40	43.07	73.88	43.93
46	76.84	42.19	75.35	43.10	73.83	43.95
48	76.79	42.22	75.30	43.13	73.78	43.98
50	76.74	42.25	75.25	43.16	73.73	44.01
52	76.69	42.28	75.20	43.18	73.68	44.04
54	76.64	42.31	75.15	43.21	73.63	44.07
56	76.59	42.34	75.10	43.24	73.58	44.09
58	76.55	42.37	75.05	43.27	73.52	44.12
60	76.50	42.40	75.00	43.30	73.47	44.15
i = .75	.66	.36	.65	.37	.65	.38
i = 1.00	.88	.48	.87	.49	.86	.51
i = 1.25	1.10	.60	1.09	.62	1.08	.63

TOPOGRAPHIC SURVEYING

TOPOGRAPHY AND SLOPE MEASUREMENT

DEFINITIONS AND METHODS

1. Topography is the detailed representation of the physical features of a region, including not only the geographical location of its boundaries and all important divisions of, and objects on, its surface, but also the form of its surface with respect to its elevations and depressions. A point is located on a plane surface when the length and direction of a line from that point to a point of reference are known, or when its coordinates with respect to two axes are known. The topographical location of a point includes also its elevation above a given level surface. In determining the topography of a region, points are located topographically in sufficient number and in such positions that the change in the surface between any two adjacent points will be reasonably uniform, so that the form of the intervening surface can be inferred from the points located. From this it is evident that the points located should be at the most abrupt changes, both in the outline, such as angles, corners of conspicuous objects, etc., and in the configuration of the surface, such as the crests of ridges, bottoms of ravines, etc.

2. Topographic surveying is the operation of determining the topographical features of any portion of the earth's surface, including the location of points within the limits of the district surveyed and the relative elevations or depressions of the surface at the different points. It thus

determines the positions and forms of prominent objects and the inequalities of the surface. Three general methods, differing with regard to the instruments used, are employed in making topographic surveys; namely, the transit and level method, the stadia method, and the plane-table method.

3. Transit and Level Method.—In this method, objects are located by means of a compass or transit for the azimuths and a chain or tape for the linear measurements, while the relative elevations are determined by means of a leveling instrument, sometimes supplemented by a clinometer or slope level.

This method is well adapted to surveys for the location of railroads and to similar surveys that relate to lines rather than to areas, and in which the topography is required to cover only comparatively narrow strips of country contiguous to the lines. In such surveys, the entire process is based on the line of the survey, which is usually alined with a transit and measured with a chain or tape, and the elevations are taken over it with a leveling instrument.

4. Stadia Method.—Points are located by means of a transit for the azimuths. The transit is equipped with a level on the telescope, a vertical arc or circle, and stadia wires. The distances and the differences of elevation are determined by stadia measurement. This method is adapted to all kinds of surveys in which a great degree of accuracy is not required. It is without question the best method of making a general topographical survey of considerable extent, and is especially convenient for preliminary railroad location surveys. The stadia method was officially adopted by the United States Lake Survey in 1864.

5. Plane-Table Method.—Points located by the plane table are at once platted on the map, which is thus prepared in the field without the intermediate process of reading and recording angles and distances. This method is well adapted to mapping, especially for filling in the details after the principal lines of a survey have been determined by other means. It has been used extensively for this purpose by

the United States Coast and Geodetic Survey and the United States Geological Survey. It is also adapted for smaller surveys, such as that of a park, in which it is desired to locate very numerous objects within a small area, and in surveys for rough maps, the time for making which is limited and in which only some of the principal points are located accurately, the other features being sketched in by eye.

If the area to be covered is long and narrow, as in a railroad survey, the line of survey is taken as a line of reference for the location of all points. The elevations are determined from a line of levels, which is generally run along with the survey line. At suitable intervals along the line, generally at each station (unless the ground is very regular), cross-sections are taken at each side of the line and at right angles to it; in these the rates of slope of the surface are determined.

6. The Rate of Slope.—The rate of slope is determined either by measuring the horizontal and vertical distances between two points in the slope or by measuring the angle of the slope. In the former method, the vertical distance that some point of the slope is above or below the beginning of the slope is measured with a level and rod, and the horizontal distance with a tape. A hand level, described in Art. **7**, is generally used in this work. The angle of the slope is determined by means of a clinometer, described in Art. **9**. Slopes of the natural surface are generally designated by the vertical rise or fall in any given number of feet measured horizontally. Thus, a slope of 1 in 20 indicates a rise or fall of 1 foot in 20 feet measured horizontally. In earthwork, such as a railroad or other embankment, a slope is generally designated by the ratio of the horizontal measurement to the vertical measurement, the latter being reduced to 1. Thus, a slope of 20 feet vertical in 30 feet horizontal is designated as a $1\frac{1}{2}$ to 1 slope, and is usually expressed thus: $1\frac{1}{2} : 1$. Slopes are sometimes designated by the slope angle. Thus, a 45° slope indicates an angle of 45° with the horizontal.

7. The Hand Level.—The usual form of this level is shown in Fig. 1; it is called the **Locke level**, from the name of the inventor. It consists of a brass tube *A B* about 6 inches long, usually finished in bronze or nickel plated, and having on the top near the object end a small spirit level *C*. Beneath the level is an opening in the tube through which, by means of a reflecting prism placed below it, the bubble can be seen when the eye is placed at the small opening *D* in the eye end. The reflecting prism occupies one-half the cross-section of the tube and is set at an angle of 45° to the line of sight; any object toward which the level

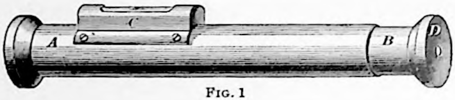


FIG. 1

is directed can be seen through the part of the tube unobstructed by the mirror. A cross-wire is placed directly below the center of the level in such position that, as reflected, it will bisect the bubble when the line of sight is horizontal. If the level is held to the eye and directed toward a distant object, and its object end is raised or lowered until the bubble is bisected by the cross-wire, the point on the distant object with which the cross-wire coincides, as seen through the unobstructed portion of the tube, is at the same elevation as the eye.

8. Cross-Sectioning With a Hand Level.—This may be done by a topographer and one rodman. Usually, however, the topographer has three assistants—one rodman and two tapemen. The latter merely make the horizontal measurements as directed by the topographer or rodman. A metallic tape, 50 or 100 feet long, and a regular self-reading level rod are generally used; though for extensive cross-sectioning, a special rod, similarly graduated, but lighter and longer, is sometimes preferred. Before beginning the work, the topographer, by standing alongside the rod, measures the height of his eye above the ground, which height is a constant quantity to be subtracted from all rod readings except

when the observations are extended by means of a backsight and foresight. The topographer, standing at the station on the line of survey where a cross-section is to be taken, determines by the eye the right-angle line on which the elevations are to be taken, and generally selects a point some distance off in line with which the rodman is kept when the rod readings are taken. The readings are taken at the points where the slope seems to change or at points where the top or bottom of the rod is level with the eye of the topographer. The points where elevations are taken are located by horizontal measurements from the survey line or from points already located. On a steep ascending slope the topographer, after determining the right-angle line, walks along it up the slope until his eye is about level with the top of the rod, when the latter is held at the station on the survey line. The rod reading on the station, less the height of the topographer's eye above the ground, determines the height above the station of the point where he is standing. On an ascending slope that is not steep, and on a descending slope, the topographer stands at the station while the rodman holds the rod on the first point on which a reading is to be taken. The topographer then stands at the point formerly occupied by the rod, and the rodman proceeds up or down the slope to give another sight. By continuing the sights, the cross-section is made to cover as much ground on each side of the survey line as may be desired. The topographer, by taking a rod reading on a point whose elevation has already been determined, determines the height of his eye above the known point. It is often desirable to continue this sight to a point still further from the survey line. The eye of the topographer remains at the same height as he turns around and takes a rod reading on the latter point, the elevation of which above or below the preceding point is equal to the difference between the two rod readings. These are similar to a backsight and foresight in direct leveling.

It is sometimes desired to take a sight on the rod when its top is below the level of the topographer's eye. This is accomplished by "shinning the rod"; that is, by the

rodman raising the rod off the ground so that the top is sufficiently high. The bottom is generally held against his legs or body to steady it. After the reading has been taken by the topographer, the rodman measures with the rod the distance from the ground to the point to which the bottom of the rod was raised. This distance is called out to the topographer, who adds it to the rod reading. If the ground is flat, the length of a sight is limited by the distance away that the figures on the rod can be plainly read. The topographer must decide, from the nature of the ground and from the purpose for which the topography is desired, how far on each side of the survey line the cross-section should extend. Ascending slopes are recorded as + and descending slopes as −.

9. The Clinometer.—By means of a **clinometer,** or **slope level,** the angle that a slope makes with the horizontal

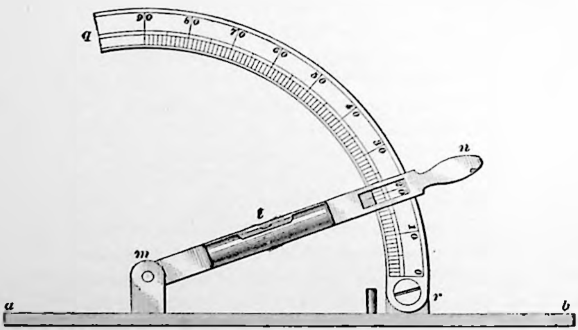


FIG. 2

is determined. A simple form of this instrument is shown in Fig. 2; it consists of a straight bar *a b* about 6 inches long, to which is attached the movable arm *m n*, which carries a spirit level *l* and turns on a hinge at *m*. The direction of the arm with reference to the bar *a b* is shown by the quadrant scale *q r*, which is graduated to degrees. If the bar *a b* is placed on any sloping surface and the arm *m n* raised until

the bubble is at the center of the level tube, the arm will be horizontal and its reading on the graduated quadrant will be the angle that the slope makes with the horizontal. Since the bar *a b* is short and the surface of the ground uneven, in order that the slope of the surface of the ground, as measured, shall be its average slope, a board about 10 feet long, called a **slope board**, or **slope rod**, is used with the clinometer. This board has one straight edge with a portion of the opposite edge parallel. The straight edge is placed on the sloping surface of the ground and the bar *a b* of the clinometer is placed on the opposite parallel edge of the board. It is sometimes convenient to attach permanently the clinometer to the slope board.

10. Abney Level and Clinometer.—As now commonly made, the clinometer is combined with the hand level, and the

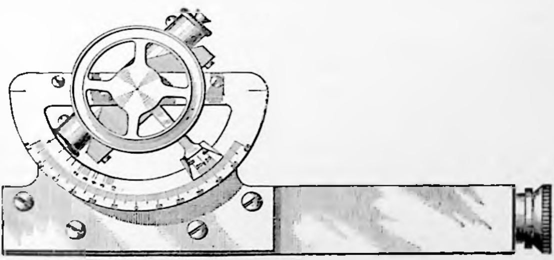


FIG. 3

combined instrument is known as the **Abney level and clinometer.** Such an instrument is shown in Fig. 3. It is similar to an ordinary hand level in every way, except that the small spirit level on top of the main tube is movable in a vertical plane, so that when the tube is given any inclination, the level can be turned to a horizontal position, as indicated by the bubble, when the angle of the inclination of the tube with the horizontal is shown by means of a graduated arc attached to the instrument. The main tube is square in cross-section, and its lower side is straight, so that it can be applied to any surface. When the spirit level is set in a

position parallel to the main tube so as to read zero on the graduated arc, the bubble can be seen through the eyepiece, as in any hand level, and the instrument can be used as a hand level.

The graduations on the inner arc to the left of the center indicate the slope ratio, the left-hand edge of the vernier arm being used as an index. The extreme left-hand graduation indicates a slope of 1 to 1, the next $1\frac{1}{4}$ to 1, etc. up to 10 to 1.

11. Cross-Sectioning With the Clinometer.—If the simple form of clinometer is used, the angle of each slope of the cross-section is measured with it, and the horizontal distance with a tape. Thus, if the angle of a descending slope is found to be 5° and the horizontal distance is 90 feet, it is recorded $-\frac{5°}{90}$. Each of the different slopes of a cross-section is thus measured and recorded. If the Abney level and clinometer is used to determine the angle of the slope, it is directed to a point as far above the ground as the instrument is when held to the eye of the observer. This is readily done by sending an assistant up or down the slope and sighting about to the top of his head. While sighting thus on a line parallel to the surface of the ground, the level tube and the vernier arm, which is attached to it, are moved until the bubble is in the center of the tube. The angle is then read on the graduated arc, the vernier reading to 5 minutes. The advantage of this instrument is that it may be used as a hand level when differences of elevation are desired, as in slopes that are rough and steep, and as a clinometer when the slopes are long and flat. In the latter case, it is sometimes not even necessary to do any horizontal measuring, the angle of the slope determining the elevation for any desired distance.

12. Computing Slope Distances From Slope Angles.—Having determined the slope angle, the horizontal distance of the slope equals the difference of elevation between the top and the bottom of the slope multiplied by the natural cotangent of the slope angle. The difference of elevation between the top and the bottom of the slope equals

the horizontal distance multiplied by the natural tangent of the slope angle. Thus, if the slope angle is 5°, the horizontal distance for a difference of elevation of 10 feet is 10 cot 5°, or $10 \times 11.4 = 114$ feet, and the difference of elevation in a horizontal distance of 100 feet is 100 tan 5°, or $100 \times .087 = 8.7$ feet. If a clinometer is used in determining slopes, it is sometimes convenient to prepare a table for field use giving the natural tangent and cotangent, correct to three figures, of angles up to 45°, each quarter degree up to 2°, each half degree up to 15°, and each degree up to 45°. Such a table can be inserted in the topographer's field book.

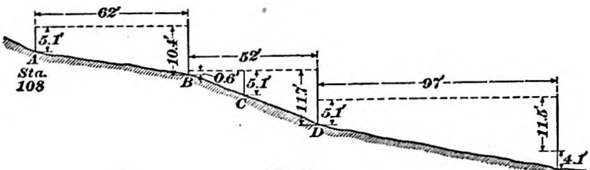


FIG. 4

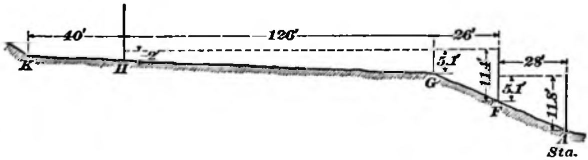


FIG. 5

13. Cross-Sectioning Illustrated.—Fig. 4 represents the **right slope** at Station 108 of a railroad survey; that is, the slope on the right-hand side of the survey line. Fig. 5 represents the **left slope**. The topographer, having determined that his eye is 5.1 feet above the ground, stands at the station and, by looking along the line of survey, generally both forwards and backwards, determines a line about at right angles to the survey line. The right slope, Fig. 4, is taken first. The rodman walks down the slope until he reaches B, where the slope changes. The rod is held at this point, and the topographer, by means of the hand level, finds

that 10.4 feet on the rod is level with his eye. From this is deducted 5.1 feet, the height of his eye, and the remainder, 5.3 feet, is recorded as the difference in elevation between the points A and B. If there are tapemen in the party, they measure the horizontal distance from A to B while the reading is being taken. If not, the rodman takes one end of the tape while the topographer keeps the other and the measurement is made after the sight is taken on the rod. The distance from A to B is found to be 62 feet, and the slope, which is a descending one, is recorded $-\frac{5.3}{62}$.

The topographer then proceeds down the slope to C, where his eye is about level with the bottom of the rod at B. The rod reading on B is 0.6 foot. The rodman proceeds to D, where the slope again changes. The topographer turns around at C and obtains the rod reading on D, which is 11.7. The difference of these rod readings, $11.7 - 0.6 = 11.1$, is the difference in elevation between B and D. Since the elevation of point C is not desired, its location is not recorded. The distance from B to D is found to be 52 feet, and the second slope is recorded $-\frac{11.1}{52}$.

The topographer moves forwards to the point D, and the rodman holds the rod at E, the foot of the slope. The top of the rod is below the level line of sight from the topographer's eye, so the rodman "shins the rod," holding it against his body sufficiently high to be intersected by the level line of sight. The rod reading is found to be 11.5 feet. The rodman then measures with the rod the distance from the ground to the point on his body to which the bottom of the rod was raised. This distance, 4.1 feet, is called out to the topographer, who adds it to the rod reading and then deducts the height of the eye. The distance from D to E is found to be 97 feet. This slope is recorded $-\frac{10.5}{97}$.

Having previously determined to continue the cross-section for about 200 feet on each side of the line, no further sights are necessary, since the topographer sees from the notes that a distance of $62 + 52 + 97 = 211$ feet has already been covered by the sights taken. The party then returns to the station to determine the left slope. Since the slope is quite

steep, the topographer decides, after having determined a line at right angles to the survey line, to proceed up the slope to F until his eye is about level with the top of the rod when held on the station. The rod reading is 11.8 feet and the distance from A to F is 28 feet. Deducting the height of the eye from the rod reading, the difference in elevation between A and F is found to be 6.7 feet. The rodman then holds the rod on F and the topographer goes to G, the top of the slope. The rod reading on F is 11.4 feet and the distance from F to G is 26 feet. Deducting the height of the eye, the elevation of G above F is found to be 6.3 feet. These two sights are $+\frac{6.7}{28}$ and $+\frac{6.3}{26}$. Since they are part of the same slope, they are sometimes added together and recorded thus, $+\frac{13.0}{54}$. The topographer, wishing to continue his line of sight at the same level, turns around and takes a reading on the rod at H, which is as far away as he can read the figures on the rod. The rod reading is 2.0 feet, and the distance from G to H is 126 feet. Since the height of the eye was 5.1 feet above the ground, this height minus the rod reading on H gives 3.1 feet as the difference of elevation between G and H. This is recorded $+\frac{3.1}{126}$.

The same slope continues to K, so the distance from H to K is measured and found to be 40 feet. This is added to the distance GH and is recorded with the slope as determined, $+\frac{3.1}{126}$ for 166 feet, which means that the ground slopes at the rate of 3.1 feet in 126 feet for a distance of 166 feet. The horizontal measurements on the left slope ($28 + 26 + 126 + 40 = 220$ feet) exceed the distance from the survey line that it is desired to cover. Therefore, no further sights are necessary, and the cross-section at Station 108 is completed.

At a station where the line of survey changes direction, the cross-section is generally taken on a line that bisects the angle formed by the two courses of the survey line.

14. Cross-Section Notes.—The field notes of cross-section work are usually kept in a regular transit book. A convenient form is shown on the following page, the left-hand

page of the note book being shown. This page is usually ruled off in six columns. The numbers of the stations are recorded in the first column. In the second column is put the elevation of the station. The third and fourth columns are used for the cross-section notes to the left of the line; and the fifth and sixth columns, for the cross-section notes to the right of the line. For clearness, a heavy pencil line may be drawn over the third and the fifth vertical line of the book so as to mark the slope columns. It is sometimes preferred to record the elevations in the fourth column, leaving the second, third, fifth, and sixth columns for the cross-section notes. The

Sta.	Elev.	Left	Slope	Right	Slope
109	112.1	$+\frac{4.7}{58}$	$+\frac{7.3}{91}+\frac{5.8}{70}$	$-\frac{2.4}{51}-\frac{8.9}{40}$	$-\frac{11.2}{85}$ for 140′
108	106.4	$+\frac{3.1}{126}$ for 166′	$+\frac{6.3}{26}+\frac{6.7}{28}$	$-\frac{5.3}{62}-\frac{11.1}{52}$	$-\frac{10.5}{97}$

right-hand page is used for sketches and general notes, such as the location and description of bench marks, the width and depth of streams and the direction of their currents, general character of the soil, timber, vegetation, buildings, and other important features. It is usually ruled and cross-ruled and has a red line down the center. The latter is used to represent the line of the survey in any sketches made on this page. The notes should begin at the bottom of the page so that when facing in the direction in which the survey line is being run, the right slope will be on the right-hand

side of the topographer and objects on either side of the line can be sketched in their natural positions with reference to the line.

15. Sketches and Eye Measurements.—In topographical surveying, the work can often be much facilitated by estimating certain unimportant distances by the eye. The distances thus estimated may be recorded by written notes or represented by means of sketches drawn freehand. Such notes, or sketches, as the case may be, are

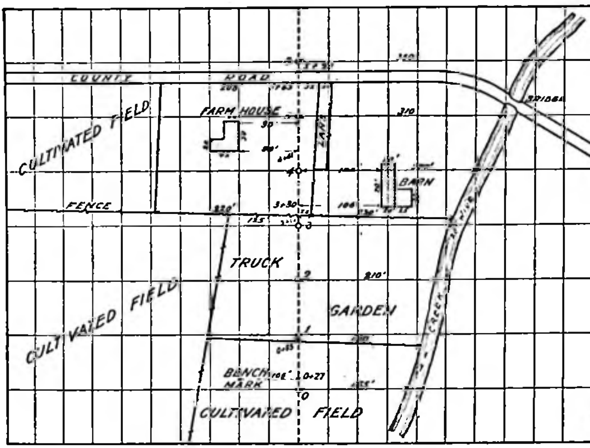


Fig. 6

placed on the right-hand page of the notebook. After some practice, the topographer is able to estimate distances by the eye with some degree of approximation, though this method is not to be relied on when accurate results are required. In taking slopes, the length of the last one may usually be estimated by the eye. More distant objects, especially those that lie beyond the probable limits within which the location will be made, may be sketched in by the aid of the eye alone.

Freehand sketches, when made intelligently, are of much value in topographical work, especially when supplemented by a few judicious measurements. The measurements are written in connection with the sketches and represent certain distances or dimensions accurately, and by the aid of these, other distances or dimensions can be sketched in with a reasonable degree of approximation. Such sketches frequently enable the topographical draftsman to plat correctly features concerning which he would be in doubt if they were recorded only by the written notes. Many topographical features can be represented by means of rough freehand sketches, with dimensions and distances marked, much more quickly and clearly than they can be recorded by means of written descriptions.

Fig. 6 represents a portion of the right-hand page of a topographer's notebook, on which is drawn a rough freehand sketch showing the topographical features of a farm through which the line of survey passes. This sketch, in connection with the regular notes written on the left-hand page of the notebook, gives very complete information regarding the locality. The center line of the page is assumed to represent the line of survey.

TOPOGRAPHY OF LIMITED AREAS

16. General Description of Method.—The method here described may be used for making the topographical survey of a limited area that is to be laid out, as a new town site, an addition to a city, a park, a cemetery, or is to be devoted to any other purpose requiring a knowledge of the topography of the surface.

If the boundaries are not clearly defined, the entire tract is first surveyed and its boundary lines determined and marked. Then, in order to determine the topography, the tract is usually divided, as far as possible, into squares or parallelograms of uniform size, whose sides may have any dimensions from about 25 to 100 feet or more, as the conditions may require. The form chosen for these will depend somewhat on the form of the tract, and their size will

depend on the physical features of the ground and on the degree of accuracy required. The corners of the squares or parallelograms are either defined by stakes or are located by ranging out and measuring from stakes already set.

Fig. 7 is assumed to represent a tract of land that is to be surveyed for the purpose of determining the topography of its surface. It is assumed that a traverse survey locating the boundaries has already been made. The tract is to be divided by means of lines in two perpendicular directions, and 100 feet apart. In dividing tracts in this manner, it is

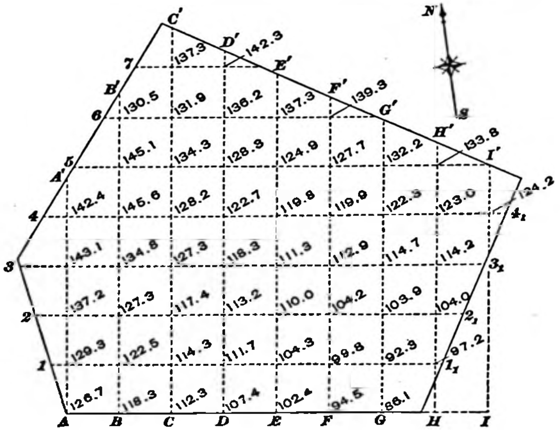


FIG. 7

customary to designate by letters the lines that extend in one direction, and by figures the lines at right angles to that direction. The point at the intersection of any two lines is then designated by the letter and figure of the respective intersecting lines. Thus, in Fig. 7, the lines perpendicular to the side decided on for a base are designated as lines *A*, *B*, *C*, etc., while the lines parallel to the base are designated as lines *1*, *2*, *3*, etc. The intersection of the line *D* and the line *1* is designated as *D 1*, the intersection of the line *E* and

the line *5* is designated as *E 5*, etc. The intersections of the boundary lines with the dividing lines are designated by the letters or numbers by which the latter lines are designated, affected with an accent or subscript, as A', B', 2_1, 3_1, etc. (See Fig. 7.)

17. Staking Out the Tract.—Different methods may be followed in laying out a tract. Any method is satisfactory that accurately defines the positions of the points of intersection, so that they can be readily located when the levels are being taken. For this purpose it is not usually necessary to mark all the points of intersection, but a sufficient number of points should be marked by stakes so that the remaining points can be located easily and quickly by merely ranging them in from the points that are marked. The rougher and more irregular the surface of a tract, the more stakes must be set, and a tract of irregular form usually requires a comparatively greater number of stakes than a tract of rectangular form. If the tract is rectangular in form and its boundary lines are complete and unbroken, so that the stakes can be set on each boundary line along the entire length of the side, and if the tract is comparatively level, so that the stakes on one boundary line can be seen from the corresponding stakes on the opposite boundary line, it will not usually be necessary to set stakes at the points on the interior of the tract. For, in taking the levels over such a tract, the leveling rod can be ranged in between the stakes on the boundary lines.

The surface of the tract represented in Fig. 7 is somewhat irregular, as shown by the elevations written at the intersections of the lines. For these conditions, the following method may be adopted in staking out the tract: The south side is taken as a base line, and stakes are set along this line at intervals of 100 feet, these stakes being marked *A*, *B*, *C*, etc. At each of these points lines are run across the tract at right angles to the base line, stakes being set at intervals of 100 feet. The base line is extended beyond the tract so that the lines *H* and *I* may be drawn. Each interior

stake is marked by the number of intervals from, and the letter of, the stake from which the line is run. Thus, the stakes in the line run from the point *A* are marked *A 1*, *A 2*, *A 3*, etc. No cross-lines are run, the squares being completed in the sketches and on the map by drawing lines through *A 1*, *B 1*, *C 1*, etc. and *B 1*, *B 2*, *B 3*, etc. After the stakes at *A 1* and *B 1* have been set, the stake at *1* is put in line with them. Likewise, the stake at *2* is put in line with *A 2* and *B 2*, and so on, locating the stakes that are not at the corners of the squares. Care should be taken to have the stakes, as shown by the sketch in the notebook, numbered in the same manner as on the ground. If the surface of the tract were comparatively level, or of uniform slope, so that the stake on any line could be seen from the corresponding stake on the opposite line, these would be the only stakes necessary to be set, since all the remaining points of intersection could be located by ranging them in from these stakes with sufficient accuracy for the purpose of taking the levels.

18. Taking the Levels.—After the required number of stakes have been set, the levels are taken over the tract, determining the elevations at all points of intersection and at any intermediate points where the slope changes abruptly. Such an intermediate point is generally located in a direct line between two intersections by its distance from the intersection having the lower letter or number. This distance is measured with a tape, approximated by pacing, or merely estimated by the eye, according to the conditions and to the degree of accuracy required, and is recorded as a plus. The tops of knolls and the low points of depressions are generally located when they are not crossed by the line between two intersections.

The levels should be taken in such order as is advantageous, which will depend on the nature of the ground, the object being to take rod readings at each of the intersections and other points with as few settings of the level as possible. To be sure that rod readings are taken at all the

intersections, those taken from each setting are checked off on the sketch or sketches in which they are all shown.

19. Form of Notes.—The notes are substantially the same as ordinary level notes, with the addition on the right-hand page of sketches, showing the form and dimensions of the tract surveyed, the manner in which it is divided, and the method of numbering the stations. Each station is designated by its letter and number, since the levels are not usually taken successively along one line.

TOPOGRAPHY BY TRANSIT AND STADIA

20. Survey of a Large Area.—In making the survey of a large area by stadia measurement, it is customary, before beginning the stadia work proper, to establish a number of controlling points whose positions with reference to each other are determined and from which the various stadia lines can be begun. The azimuths and lengths of the lines joining these established points are determined and a line of levels is run to ascertain the elevations of these points with reference to a common datum. The tract is generally divided into a series of triangles, the angles and sides of which are carefully measured, or computed, thus forming an accurate framework on which to build the map by filling in the details determined by stadia measurement.

This framework sometimes consists of two or more random lines carefully run through the tract, with tie-lines connecting them at convenient points.

21. Survey of a Small Area.—In the survey of a small tract, it is not usually necessary to establish a framework for the map. The magnetic bearing of any line of the survey can be observed, the azimuth of the line calculated from its bearing, and this azimuth taken as a basis from which to determine the azimuths of the other lines of the survey.

Fig. 8 represents a small tract to be surveyed. It is assumed that the magnetic bearing of that part of Mead

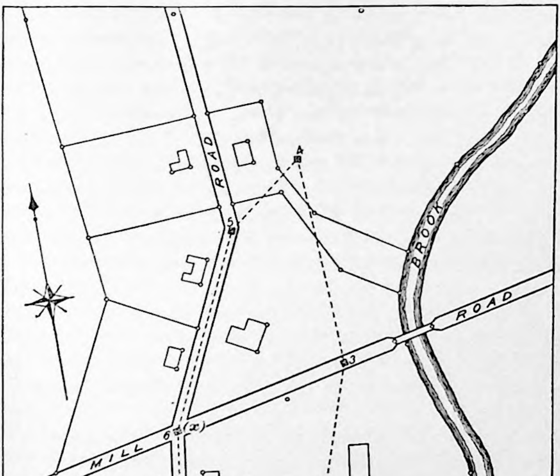


FIG. 8

Road, south of Mill Road, as observed from the angle in Mead Road, is found to be N 7° 30′ E. It is also assumed that from the records of another survey, the elevation of the center of Mill and Mead Roads at their intersection is known to be 173.2 feet. It is required to locate the brook forming the eastern boundary of the tract, the roads, fences, and buildings, and also to determine the elevations of a sufficient number of points to show the topography of the tract.

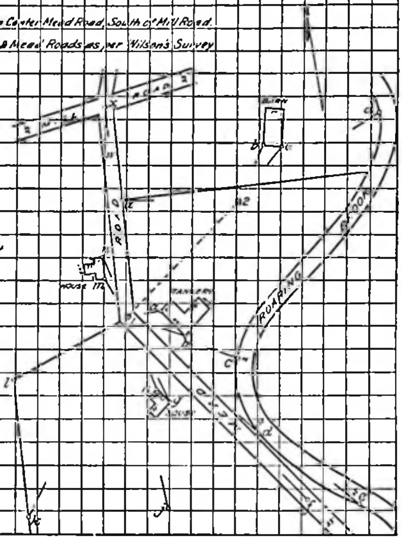
The party consists of an observer, a recorder, and two rodmen. The survey is conducted as follows: The transit is first set up over a point in the center of Mead Road at the angle south of Mill Road, which point is Station 1 of the survey, and is so marked in the notes and figures. It is decided to close the survey at the intersection of the center lines of Mead and Mill Roads, and, since the number of intervening stations is not yet known, this point is temporarily marked x, as shown in the notes and sketches following this description.

Considering the azimuth to be measured from the north point clockwise, the azimuth of the center line of Mead Road is 7° 30′, since the bearing of this line is N 7° 30′ E. The vernier is set at 7° 30′, the telescope is directed to the rod held on the point x, and the lower clamp is set. By this operation, the transit is oriented on the center line of Mead Road. The stadia reading is taken and called out to the observer. The vertical angle is read and called out. The plate clamp is then loosened and the telescope is directed to the stadia rod held successively on the points to be located. The stadia rod, vertical angle, and azimuth are read and recorded for each sight. These sights, called **side shots**, are preferably taken in succession by turning to the right, but this plan cannot usually be adhered to, as it is desirable that the rodmen shall cover the ground with as little unnecessary walking as possible. The points to which side shots are taken are commonly indicated in the notes and sketches by the small letters of the alphabet, beginning with *a* at each instrument point. Thus, side shots *a* and *b* are taken to two adjacent corners of the tannery (see notes and sketches on pages

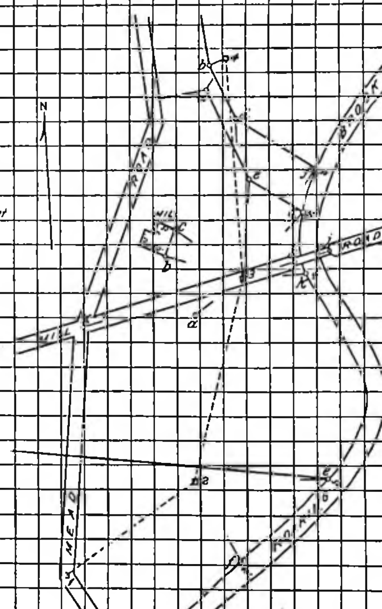
22 to 25), *c*, *d*, and *e*, to points on the edge of Roaring Brook; *f*, to the center of Mead Road; *g* and *h*, to corners of the house south of the station occupied; and *j*, to a point where the slope of the ground changes. Such a point, located merely to show the elevation of the ground, is commonly called a **contour point** (marked *c. p.*), as it is used only to determine the contours of the tract, as will be explained later. Contour points are located when, in the judgment of the observer, the slope of the ground is not sufficiently well shown by the elevation of the other points that have been located. Side shots *k* and *l* are taken to locate the fence line, and *m* and *n* to locate the house north of Station 1.

All the desired points adjacent to Station 1 having been located, one of the rodmen selects a position suitable for Station 2. Before sighting to this point, the observer sets the vernier again at 7° 30′ and checks the position of the instrument by sighting again on the rod held at point *x*. If it is found that the instrument has moved slightly out of position, the telescope is again directed to the rod by means of the lower tangent screw. The upper plate is then unclamped and the telescope is directed to the rod held on the point selected for Station 2. The stadia reading is taken and the vertical angle and the azimuth are read. The latter is found to be 60°. The transit is then set up at Station 2 and oriented on Station 1, as explained in *Transit Surveying*, Part 1. The vertical angle and stadia reading from Station 2 to Station 1 are taken as a check. Thus, the stadia reading from Station 1 to Station 2 is 4.20 and the vertical angle is 1° 33′, whereas, on the backsight, the stadia reading is 4.22 and the vertical angle is 1° 37′. The means between the two stadia readings and the two vertical angles are generally taken as the correct readings. Side shots to points adjacent to Station 2 are taken, and Station 3 is then located. In a similar manner, Stations 4 and 5 and points adjacent to them are located. When the instrument is at Station 3, it is noticed that the point *x* is plainly visible, so a stadia reading is taken on it. The distance thus obtained is used as a check on the

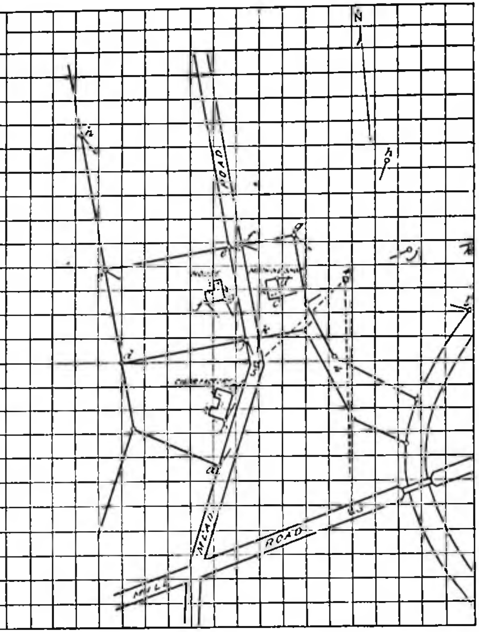
Stadia Survey of Meadville Suburb May 27th 1901.						Observer–J. R. Gavin Recorder–Gonaver Rodmen – Griffith, Steele
Sta	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation	
	Readings	from □1,	Elev. 177.42.			□1 Angle in Center Mead Road, South of Mill Road.
x	7° 30′	6.30	− 0° 29′	631	173.2	= Center Mill & Mead Roads as per Wilson's Survey
a	90° 43′	.91	− 2° 04′	92	174.1	
b	120° 18′	1.66	− 2° 12′	167	171.0	
c	126° 31′	3.15	− 2° 06′	316	165.8	
d	143° 43′	4.60	− 1° 25′	461	166.0	
e	141° 32′	7.85	− 0° 38′	786	168.7	
f	146° 04′	6.74	− 0° 34′	675	170.7	
g	167° 19′	2.47	− 0° 50′	248	173.8	
h	172° 22′	1.97	− 1° 20′	198	172.8	
i	181° 17′	4.99	+ 0° 12′	500	179.2	Contour point
k	221° 45′	5.79	+ 1° 02′	580	187.8	
l	256° 01′	3.47	+ 1° 50′	348	188.5	
m	342° 03′	1.17	+ 1° 16′	118	180.0	
n	350° 16′	1.71	+ 0° 52′	172	180.0	
□2	60° 00′	4.20	1° 33′	M = 422	165.77	
	Readings	from □2	Elev. 165.77			
□1		4.22	+ 1° 37′			
a	286° 01′	3.21	+ 2° 01′	322	177.1	
b	32° 02′	2.36	− 0° 48′	237	162.5	
c	41° 40′	2.60	− 1° 03′	261	161.0	
d	68° 32′	4.61	− 1° 22′	462	154.8	



Sta	Azimuth	Stadia	Vertical Angle	Hor. Distance	Elevation	
	Readings	from □2	(cont.) Elev. =	165.77.		
e	90° 45'	3.59	− 1° 18'	360	157.6	
f	154° 53'	2.28	− 0° 54'	229	162.2	
□3	17° 30'	5.47	+ 0° 17'	M = 545	168.61	
	Readings	from □3,	Elev. = 168.61.			
□2		5.42	− 0° 19'			
a	245° 30'	1.73	− 1° 53'	174	162.9	Contour Point
x	256° 45'	4.58	+ 0° 40'	459		Tie Line
b	287° 43'	2.19	− 1° 16'	220	163.7	
c	307° 24'	2.21	− 1° 41'	222	162.1	
d	359° 04'	4.06	− 1° 26'	407	158.4	
e	7° 33'	2.49	− 2° 50'	249	156.3	
f	38° 21'	3.39	− 3° 07'	339	150.1	
g	47° 20'	2.31	− 4° 18'	231	151.2	
h	76° 45'	2.39	− 0° 20'	240	167.2	
j	76° 46'	1.39	− 0° 34'	140	167.2	
k	82° 52'	1.66	− 5° 41'	165	152.9	
□4	357° 35'	5.61	− 0° 46'	M = 559	161.29	
	Readings	from □4,	Elev. = 161.29			
□3		5.55	+ 0° 44'			
a	215° 41'	0.96		97	161.3	
b	255° 10'	0.53	+ 1° 13'	54	162.5	

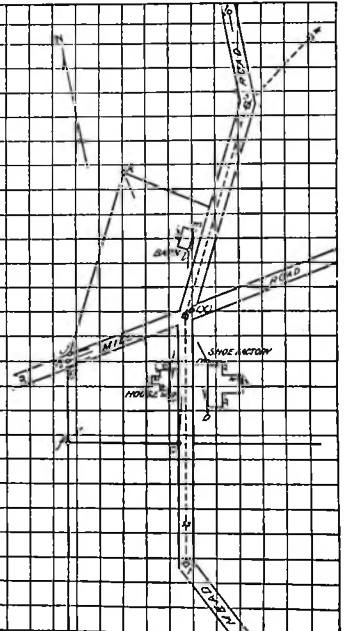


Sta	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation
	Readings	from □4.	(cont) Elev. =	161.29	
c	275° 17′	1.09	+ 1° 19′	110	163.8
d	301° 11′	1.34	+ 0° 54′	135	163.4
e	303° 46′	2.84	+ 0° 47′	285	165.2
f	308° 36′	2.57	+ 0° 43′	258	164.5
g	339° 25′	1.79	+ 0° 08′	180	161.7
h	32° 19′	4.29	− 1° 24′	430	150.8
i	70° 03′	2.67	− 1° 38′	268	153.7
k	77° 32′	6.61	− 1° 21′	662	145.7
l	102° 12′	4.03	− 1° 54′	404	147.9
□5	230° 54′	2.53	+ 0° 44′	M = 252	164.51
	Readings	from □5.	Elev. = 164.51		
□4		2.49	− 0° 45′		
a	208° 02′	2.73	+ 0° 28′	274	166.7
b	218° 53′	1.63	+ 0° 38′	164	166.3
c	228° 02′	0.96	+ 0° 53′	97	166.0
d	277° 15′	3.68	+ 1° 59′	369	177.3
e	307° 47′	4.87	+ 1° 04′	488	173.6
f	326° 20′	2.18	+ 1° 02′	219	168.5
g	338° 43′	1.99	+ 0° 48′	200	167.3
h	327° 47′	7.71	− 0° 15′	772	161.1
j	343° 48′	0.71	+ 0° 19′	72	164.9
k	13° 31′	0.72	− 0° 34′	73	163.8



110—15

Sta.	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation	
	Readings from □5, (cont.) Elev. = 164.51					
l	355° 45′	6.02	+ 0° 27′	603	169.2	
X=□6					173.35	Should be 173.20
						as per Wilson's Survey
	Readings from □6. Elev. = 173.20					= Original B.M.
□5		5.55	− 0° 59′			
a	159° 12′	1.27	− 0° 14′	128	172.7	
b	172° 20′	2.34.	+ 0° 22′	235	174.7	
c	190° 14′	3.22	+ 0° 53′	323	178.2	
d	201° 46′	2.02	+ 1° 12′	203	177.5	
e	207° 24′	1.51	+ 1° 13′	152	176.4	
f	232° 18′	4.52	+ 1° 25′	453	184.4	
g	254° 03′	3.44	+ 1° 18′	345	181.0	
h	256° 45′	3.39	+ 0° 50′	340	178.1	
j	260° 02′	3.29	+ 1° 14′	330	180.3	
k	342° 47′	3.96	+ 0° 04′	397	173.7	
l	10° 14′	1.65	− 1° 07′	166	170.0	
m	13° 11′	2.18	− 1° 07′	219	168.9	
□1	187° 30′	6.27	+ 0° 25′			
	Survey finished May 28th 1901					
	Stadia Readings reduced by Gonaver—June 1st 1901					
	Notes platted by Griffith—June 1st 1901					



platting of the survey. The last point sighted from Station 5 is the point x at the intersection of Mead and Mill Roads, which was the first point sighted from Station 1 and now becomes Station 6. When the transit is set up and oriented at this station, the azimuth readings on all the instrument points can be checked by sighting to Station 1. If the field work has been performed accurately, the azimuth reading for this sight should be $7° 30' + 180° = 187° 30'$. As the survey progresses, the recorder makes sketches showing the lines run, the objects located, etc., being careful to designate each point in the sketches by the same letter that is used for the same point in the first column of the notes. In the location of a building, two adjacent corners are generally located by stadia measurements, and the sides are measured by the rodmen or by the recorder and one rodman. The complete notes and sketches of this survey are shown in the preceding pages. The platting of these notes is explained in *Mapping*, Part 2.

TOPOGRAPHY BY PLANE TABLE

22. Survey of a Large Area.—The tract is generally divided into a system of triangles called the **control.** The vertexes of these triangles are called **controlling stations.** The triangles are carefully located by the transit and the chain or tape, while a line of levels is run to determine the elevations of the controlling stations. After they have been established and platted, generally on a number of plane-table sheets, additional points within the triangles are determined by some of the plane-table methods already explained.

From these points, which are so selected as to obtain the best outlook, the topographical features adjacent to each are located by stadia measurement and at once platted. The difference of elevation between the station occupied and each point observed is determined by means of the vertical angle and the Stadia Reduction Table. The elevation of the observed point is then obtained by adding or subtracting the difference of elevation to the elevation of the station occupied, as explained in *Stadia and Plane-Table Surveying*.

The elevation of each point located is marked on the map as soon as determined, and the topographer, with the ground before him, is able to determine if a sufficient number of points have been located to represent the slope of the ground with the degree of accuracy desired for the map. He is also able to see from the map if his locations from the station occupied cover the ground sufficiently close to the area previously mapped. Random lines are sometimes run through heavily wooded sections of country where the surface is not visible from any of the principal points.

23. Survey of a Small Area.—A survey of a small area is made with the plane table in a similar manner to that used for a small stadia survey, no preliminary triangulation being necessary unless a great degree of accuracy is desired. Traversing with the plane table is sometimes done to locate roads, streams, and other features, or to locate and determine the elevation of points in heavily wooded country where the surface is not visible from the principal stations.

CONTOURS

LOCATION OF CONTOURS

24. Definition.—A **contour** is a line that connects all points having the same elevation. It may be also defined as the intersection of the surface of the ground with a horizontal plane.

25. Contour Intervals.—The vertical distance between two adjacent contours is called a **contour interval.** This distance is generally the same for each map, but varies with the degree of accuracy with which it is desired to show the surface. If it is desired to represent the surface accurately, a small interval, such as 2 feet, 5 feet, or 10 feet, is used. The interval customarily used in mapping by the United States Coast and Geodetic Survey is 20 feet. In mountains or very rough country, where sketching is largely relied on,

an interval of 100 feet is usually sufficiently close to show all necessary features.

Contours are generally taken at elevations that are multiples of the interval. Thus, 10-foot contours are taken at elevations of 10, 20, 30, 40 feet, etc., and 5-foot contours at elevations of 5, 10, 15, 20 feet, etc.

26. Determining the Contours.—The contours between two adjacent points whose elevations have been determined are generally platted as a part of the office work, as will be explained in *Mapping*, Part 2. Since the slope of the surface is assumed to be about uniform from each point to the one adjacent in each direction, it is necessary to locate a sufficient number of points to show the surface with a degree of accuracy sufficient for the purpose and in harmony with the scale of the map.

27. Sketching the Contours.—This method is used as an auxiliary to the location of points when the contours are platted in the field and is especially advantageous when the map is a rough one and is hastily made. Sketching is an important part of the topographer's work and a difficult one in which to acquire proficiency. It affords a wide scope for the exercise of judgment and skill, and its accuracy depends on the ability to see the principal features of the country and to represent them on paper so that they give the same impression as is given by the natural features.

28. Direct Location of Contours.—It is sometimes advantageous to determine on the ground the location of each contour by locating a sufficient number of points of equal elevation to properly represent the contour when platted. The number of points located depends on the roughness of the country and the accuracy with which it is desired to represent the topographical features. The plane table is most commonly used in the direct location of contours, and it is generally supplemented by a Y level. The transit with stadia wires is sometimes used. The process with plane table and level is as follows: Fig. 9 represents a portion of a tract of ground on which the contours at

intervals of 20 feet are to be located directly. The points *A*, *B*, and *C* have been previously located and their elevations have been determined, as shown in the figure. The plane table is set up and oriented at a point *P*, from which is visible as much as possible of the ground to be covered, and the station occupied is platted on the map. It is decided to locate contour *100*; so the level is set up above this elevation at a point of good outlook. A backsight reading of 4.2 is taken on station *A* and the height of instrument is thus found

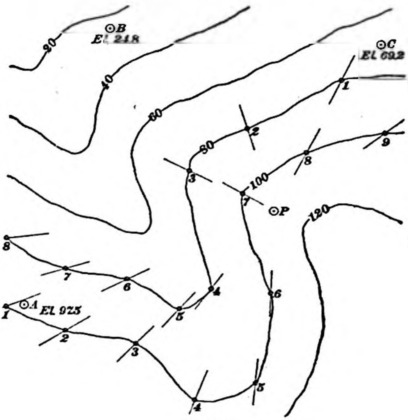


FIG. 9

to be 97.5 + 4.2 = 101.7. All points in contour *100* are 101.7 − 100 = 1.7 feet lower than the level line of sight. The target is set at 1.7 feet on the rod and the contour is then traced on the ground as follows: The first point *1* of the contour is determined by moving the rod up or down the slope until the target is intersected by the line of sight. This point is then sighted on from the plane table and a stadia reading taken. The line of sight is platted lightly and the distance is laid off on it to the scale of the map. The rodman then determines about where it is necessary to locate another point to properly show the contour. Here he again

moves up or down hill until he reaches a point where the target is cut by the line of sight. This point *2* is located by the plane table, in a manner similar to that used for locating the first point. Points *3*, *4*, *5*, *6*, and *7* are located in the same manner. After point *7* is located, it is noted by the levelman that his position will have to be changed to trace the contour further, so he moves to another position beyond the last point located, takes a backsight on point *7*, and proceeds to locate the other points as before. Each contour point when platted is marked lightly with its elevation and not with numbers *1*, *2*, *3*, etc. When a number of points of equal elevation have been platted, the topographer delineates the contour by drawing a continuous line through the contour points, curving it where it appears to change its direction. No more points on contour *100* being visible from the plane table, the leveler sets a temporary turning point by a foresight of 11.8, the elevation of which is $102.3 - 11.8 = 90.5$, and again by a backsight of 1.4 and a foresight of 10.9, a temporary turning point is set whose elevation is $90.5 + 1.4 - 10.9 = 81.0$. Selecting a point with a good outlook, the level is set up and a backsight of 1.8 gives a height of instrument of $81 + 1.8 = 82.8$. The target on the rod is then set at 2.8 and points *1*, *2*, *3*, *4*, *5*, *6*, and *7* are located on contour *80*. In a similar manner portions of contours *60*, *40*, and *20* are platted, after which another position is chosen for the plane table and the contours are extended still farther over the tract. The farthest point located on each contour in the direction the contours are being extended is generally marked with a temporary stake, and this point is sighted on first from the next succeeding position of the plane table. Thus, point *9* on contour *100*, and point *8* on contour *80* will each be marked with a stake. It is often of advantage to have a stadia rodman who holds a rod on the contour point when determined by the leveler and his rodman. While the topographer is taking the reading, the level rodman is selecting the next contour point. If the contour intervals are 5 feet, two rodmen are sometimes employed, one tracing one contour and the other the next one above; the

leveler generally determines points in two contours from the same setting of the instrument, the topographer alternately locating points in each contour.

29. Contours in Cross-Sectioning.—In cross-sectioning, 5-foot contours are sometimes located directly with the hand level or rod at a point 5 feet above the ground. The process is as follows: Fig. 10 represents a cross-section at Station 102 in a survey. The elevation of the station is 177.2 feet, and it is desired to locate 5-foot contours for 200 feet on each side of the station.

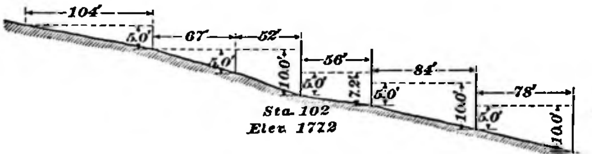


Fig. 10

The topographer, standing at the station, determines by the eye the right-angle line on which the cross-section is to be taken. The elevation of the eye, when the hand-level staff is held at the station, is 177.2 + 5.0 = 182.2. The first contour below the station is 175 and a point at this elevation is determined by the rodman moving the rod down the slope until the level line of sight intersects the rod at 182.2 − 175 = 7.2. This point is then located by measurement from the station and found to be 56 feet. It is recorded thus: $\frac{175}{56}$. The topographer then moves to the point just located, determines a point in contour *170* in substantially the same manner as explained for contour *175*, and so on.

CONTOUR LINES

30. Description.—In representing topography by means of a map, the area surveyed is conceived to be projected on a horizontal plane, represented by the plane of the paper, on which the inequalities of surface, important natural features and other conspicuous objects are shown in their true

relative positions. The simplest method and the one most generally employed for representing the topography of a given surface is by means of lines joining points of equal elevation, called **contour lines,** or simply **contours.** A map containing the outlines of a given area or tract, together with the contour lines showing the relative elevations of different portions of its surface, is called a **contour map** of that surface.

31. Example of Contour Lines.—The manner in which the relative elevations of different portions of a given

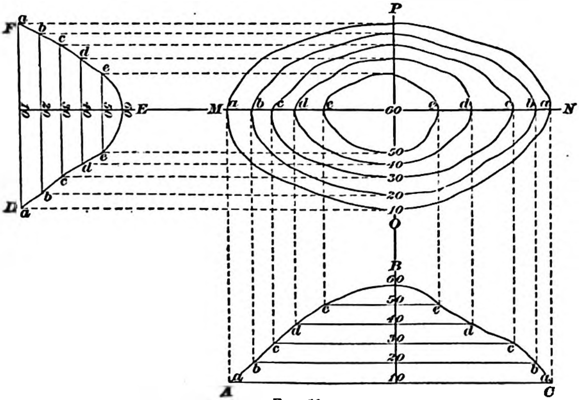


Fig. 11

surface can be represented by contour lines is illustrated in Fig. 11. Let $MPNO$ represent the top view of a hill projected on a horizontal surface, and let ABC represent a vertical section of the hill on the line MN, and DEF represent a vertical section on the line OP. In the horizontal outline $MPNO$, the lines aa, bb, cc, dd, and ee may represent imaginary lines drawn around the hill through points of equal elevation; these lines are assumed to be at intervals 10 feet apart vertically. Suppose that this hill is surrounded by water, the surface of which reaches just to the line aa.

Since this line passes around the hill through points of equal elevation, and since the surface of the water is level, and consequently has the same elevation at all points, if the surface of the water reaches the line *a a* at any point, it will just touch it at all points around the hill, so that this line will represent the flow line or shore line of the water. In the vertical section *A B C*, the elevation of the surface of the water will be represented by the line *a a*; it will also be represented by the corresponding line *a a* in the vertical section *D E F*. Now, suppose the hill to be gradually submerged in water by the water rising in successive heights of 10 feet, corresponding to the vertical intervals between the lines. At each successive rise of the water, its flow line, or shore line around the hill, corresponds to one of these lines. The horizontal lines shown in the vertical sections *A B C* and *D E F* correspond, respectively, to the positions of the surface of the water when at the successive elevations. Thus, when the water has risen to the line *b b* in the horizontal plan *M P N O*, this line will represent the shore line of the water around the hill. The horizontal line *b b* in the vertical section *A B C* and the corresponding horizontal line *b b* in the vertical section *D E F* are lines in these respective sections that represent the elevation of the surface of the water at such a stage.

Each of the lines *a a*, *b b*, *c c*, *d d*, and *e e* that represent the shore line of the water around the hill at the successive stages is evidently a line through points of equal elevation and therefore corresponds to a contour line. These contour lines can be constructed from the vertical sections, or profiles, in the following manner: The points *a*, *b*, *c*, *d*, and *e*, where the respective horizontal lines intersect the surface of the hill in the vertical section *A B C*, are projected on the line *M N*, and the corresponding points in the vertical section *D E F* are also projected on the line *O P*. The corresponding points thus projected on *M N* and *O P* are connected by continuous lines, as shown in the figure; these lines are lines of equal elevation, or contour lines. In order to determine the position of the contour lines and the

approximate form of the hill with a reasonable degree of accuracy and completeness, the elevations of the surface should be taken on a sufficient number of sections across the hill so that the contour lines can be drawn with the required degree of accuracy. The two vertical sections represented in the figure are sufficient for the purpose of illustration, but in actual practice a greater number will usually be required.

HYDROGRAPHIC SURVEYING

SURVEY OF OUTLINE OF A BODY OF WATER

1. Definition and Object of Hydrographic Surveying.—**Hydrographic surveying** comprises all surveys of lakes, streams, reservoirs, or other bodies of water. Its object may be:

1. To obtain sufficient information from which to draw an outline map of the body of water surveyed.

2. To determine, in addition, the elevations of a sufficient number of points on the bottom below the water surface to define the subaqueous contours of the containing valley or basin.

3. To determine the form of a portion of the bottom of the sea, a bay, harbor, or navigable river, for purposes of navigation. In this case, it is necessary to locate the navigable channels, and the obstructions to navigation, such as shoals, rocks, sunken wrecks, etc.

2. Limitations.—Hydrographic surveying consists merely in making the measurements necessary for acquiring such information as is outlined above. The measurement of the velocity and discharge of rivers and streams, and the planning and execution of works of improvement, such as the reclamation of submerged areas or the construction of breakwaters, sea walls, dams, etc., belong to hydraulic engineering, and will not be treated here.

3. Traverse Survey.—The survey of a body of water to determine its outline may be conducted as an ordinary traverse by any of the methods described in *Transit Surveying*,

Part 1. The courses are run at convenient distances from the water's edge and the shore line is determined by measurement from the line of the survey. The position of the ordinary low-water line is usually defined, but in many cases the high-water line is also determined and noted.

A good way to make an outline survey of a body of water is by means of a deflection traverse, using a transit and a

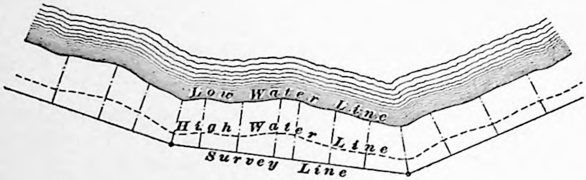


FIG. 1

chain or tape. This method is commonly used and is satisfactory for ordinary surveys of this kind. The outline survey of a body of water can also be made by the transit and stadia; this method of surveying is fully described in *Stadia and Plane-Table Surveying.* The entire survey of a small

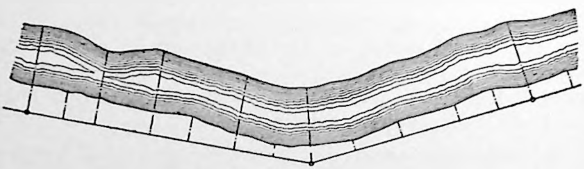


FIG. 2

river or stream, including the location of soundings, can be made with the stadia.

Prominent objects on shore may be located by direct measurement from the line of survey. If at a considerable distance, they may be located by triangulation, as described in *Transit Surveying*, Part 2. The distances from the line of the survey to the high- and low-water lines are usually measured by offsets, as illustrated in Fig. 1. If it is not necessary to obtain a close approximation of the shore outline,

the offset measurements can be omitted and the shore line between the survey stations sketched in by the eye.

In the case of a small stream, a traverse run along one bank is usually sufficient. The offsets should be measured to the edge of the water, and the width of the stream also measured at sufficiently close intervals to give the required information, as shown in Fig. 2. In the case of a small lake, the traverse is run entirely around it and closed on the point of beginning.

4. Triangulation.—A triangulation survey probably affords the best means for determining the outlines of large rivers, lakes, and other large bodies of water. Triangulation, as applied to hydrographic surveying, consists: (1) in locating distant objects from a measured base; (2) in determining the surface outlines of a river or other body of water by a system of triangles referred to a measured base.

The base line should be measured on fairly level ground in a location convenient for making the angular measurements from its ends. It should be not less than 500 feet long and as much longer as practicable. The ends should be marked with substantial stakes or with stone monuments. The line should be measured carefully with a steel tape.

5. Locating Distant Objects.—For locating points of reference and other distant objects, the angle formed by the intersection of the base line and the line of sight to the object, at each end of the base line, is measured. From the values of these angles and the length and azimuth of the base line, the lengths and azimuths of the lines to the object can be calculated and the object located. Let AB, Fig. 3, represent a base line and C a distant object whose position is to be determined. The base line AB is measured accurately, and the angles ABC and BAC are measured with a transit or

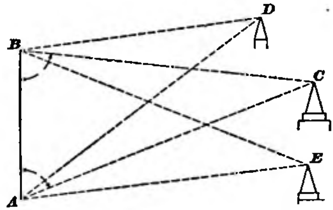


FIG. 3

sextant. Then, in the triangle *A B C*, the side *A B* and the adjacent angles *A B C* and *B A C* are known, from which the sides *A C* and *B C* can be calculated by trigonometry, and the point *C* located. The points *D* and *E* can be located in a similar manner from the same base line.

6. Triangulation of River.—For the survey of a river or other body of water by triangulation, points are selected on both sides for the vertexes of the triangles. Such points are called **triangulation stations.** They should be so located as to give triangles of advantageous form, in which no angle will be less than 30° or greater than 150°. Fig. 4 illustrates the triangulation of a river for the purpose of determining the outline of its shores. Some convenient line, as the line *1–3*, is taken as the base, its length is carefully measured, and its azimuth is either determined or assumed. The angles from *1* and *3* to *2*, and the angle *2*, are carefully measured. Their sum should not differ from 180° by more than 1 minute. The difference between that sum and 180° is distributed equally among the three angles of the triangle, one-third of it being added to, or subtracted from, each angle, as may be necessary to bring the sum to 180°. The same applies to the other triangles, in each of which each angle should be measured directly.

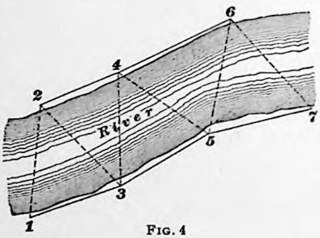


FIG. 4

Knowing the angles of the triangle *1–2–3*, and the length of *1–3*, the lengths of *1–2* and *3–2* are computed by trigonometry. Then, in the triangle *2–3–4*, the angles and the side *2–3* are known, and the other two sides are computed; and so on with the other triangles. At the end of the chain of triangles another line, as *5–7*, whose length has been calculated, is measured, as a check on the work.

The shore line between triangulation stations can be sketched in approximately, or, if it is desired to determine its outline more closely, the more important points can be located with transit and stadia, or by intersections from two triangulation stations when these are so situated as to give satisfactory intersections. If more detailed information is desired, a traverse can be run between adjacent stations and the shore line located by offsets at such intervals as may be desired.

SURVEY OF A SUBMERGED AREA

7. Purpose of Survey.—A hydrographic survey to determine the topography of the bottom of the basin or channel containing a body of water may be made for one or more of the following purposes:

1. To determine what changes it is desirable or necessary to make in the configuration of the channel or basin under consideration.

2. To indicate where material should be removed by dredging or blasting and where it may be deposited for filling, and to measure the quantity of material removed or the extent of the filling.

3. To obtain the information necessary for planning the construction of sea walls, jetties, lighthouses, docks, bridge piers, etc.

4. To construct a map or chart of the channel or basin for navigation purposes.

5. To determine the volume of the body of water, or capacity of the containing basin.

In making the survey of a submerged area, it is first necessary to make an outline survey in order to determine the shore line and locate points of reference. The points of reference are usually on shore and may be located by direct measurement or triangulation, as may be more expedient. In some cases, buoys are anchored in the water and used for reference points; as they are inaccessible, their distances from other points of reference must be determined by computation.

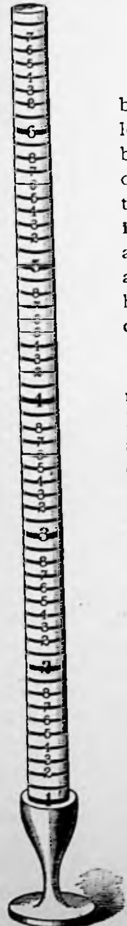


Fig. 5

SOUNDING

8. Soundings.—The shore line having been determined and the reference points located, the next step is to measure the depths, below the water surface, of a sufficient number of points to show the configuration of the bottom; such measurements are called **soundings.** For depths of 18 feet or less, soundings are made with a graduated wooden rod called **a sounding pole.** For greater depths, a line having a weight attached is necessary; this is called a **lead line.**

9. Sounding Pole.—The sounding pole may be of any sound, straight-grained wood. It should be well seasoned to prevent warping, and the bottom end should be provided with a disk-shaped iron shoe, not less than 5 inches in diameter, to prevent the rod from sinking into the soft mud of the bottom.

A good form of sounding pole is illustrated in Fig. 5. Poles of this kind are usually made of white pine finished smooth and round. The length is usually from 15 to 20 feet, and the diameter from 3 to $3\frac{1}{2}$ inches at the lower end and from 2 to $2\frac{1}{2}$ inches at the upper end. The lower end of the pole is formed by an iron shoe that terminates in a disk, as shown.

The pole is painted white and is graduated to feet and tenths, the zero of the graduation being at the bottom of the shoe. Each foot division is marked by a red band about $\frac{3}{8}$ inch wide, and each tenth division by a black band about $\frac{3}{16}$ inch wide; the bands extend entirely around the pole. The graduations are numbered by two sets of figures placed on the pole diametrically opposite each other. The

numbers designating feet are painted in red and those designating tenths in black.

The bottom of the shoe is sometimes hollowed out cup-shaped for the purpose of bringing up samples of the bottom over which the soundings are taken. When samples of the bottom are desired, the cavity is lined with grease or tallow, to which particles of the sand or mud of the bottom will adhere.

10. Lead.—The weight attached to a sounding line is called the **lead**, because it is usually made of that material. It should be long and slender, and should taper slightly toward the upper end, so as to reduce its resistance to being raised in the water. The form shown in Fig. 6 is frequently used. An iron rod *R* has molded around it the lead *L*, which is usually square in cross-section, as shown at *S*, and of sufficient size to give the requisite weight. Small cross-bars attached to the rod prevent the lead from slipping. At the lower end of the rod is attached the cup *C*, which is covered with a leather washer *W* that slides freely on the rod between the cup and the lead. When the lead is lowered to the bottom, the cup sinks far enough into the bottom to fill, and the leather cover prevents the contents from being washed out while the lead is being drawn to the surface. In some cases the cup is omitted and the bottom of the lead hollowed out in conical form. When it is not desired to know the composition of the bottom, a plain lead of nearly cylindrical form, but tapering toward the upper end, will answer the purpose. For still, shallow water, a lead weighing about 5 pounds is satisfactory. A lead weighing 10 pounds is suitable for depths under 40 feet in reasonably quiet water. For greater depths and in strong currents the weight of the lead should be from 15 to 20 pounds.

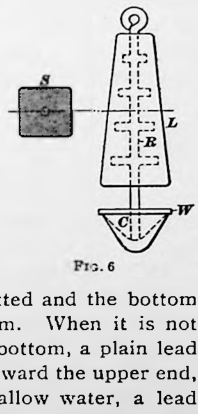


FIG. 6

11. Sounding Line.—Preferably, the sounding line should be of strong, closely plaited linen or twisted hemp.

Sometimes a cotton rope or a wire chain is used, but the use of such materials is not recommended. The line should be of a size suited to the weight of the lead; for ordinary river or lake soundings, about $\frac{3}{8}$ inch in diameter is a good size. It is marked with leather or cloth tags, which are inserted between the strands of the line. For river and harbor surveys, the tags are placed at intervals of 1 foot. At every fifth or tenth interval a conspicuous tag, usually of a bright color, is used. The zero of the graduation is the bottom of the lead.

Before being measured and marked, the line should be thoroughly stretched. This is done by stretching it tightly between two posts or trees, or wrapping it closely around a post or smooth-barked tree, then fastening both ends, wetting thoroughly and allowing it to dry. The slack is then taken up and the operation repeated until the line shows no further slack. Care should be taken not to stretch it too much, as in that case it will shorten in use. The length of the lead line, from the end of the lead to each 10-foot mark, should be tested before and after each day's use, and the results entered in the notebook. The line should preferably be kept under water when not in use; if it is not, it should be soaked in water for $\frac{1}{2}$ hour and then tested for length before the sounding is commenced.

12. Sounding Party and Equipment.—When the soundings are located by means of observations made with instruments stationed on the shore, the sounding party may consist of the recorder, leadsman, and boat crew. If the soundings are located by the stadia method, a stadia rodman is added to the sounding party. When the soundings are located from the boat, the sounding party is usually composed of two observers, a recorder, a leadsman, and the boat crew.

The usual equipment of a sounding party consists of a sounding pole or lead line and two signal flags, one white and one red. The flags are used to signal to the instrumentman on shore when a sounding is being taken, if the soundings are located by an instrument on shore. The white flag

is shown for each sounding except every fifth one, when the red flag is shown. The recorder is provided with a notebook in which to enter depths of soundings, nature of bottom, etc.

13. Making the Soundings.—If the depth of the water does not exceed about 75 feet, the soundings can usually be made while the boat is in motion. When the soundings are made at long intervals and the depth of the water does not exceed about 30 feet, it is usually more advantageous to withdraw the lead from the water after each sounding. In this case the lead is cast far enough ahead of the boat, as each sounding is made, for the line to become vertical when the lead reaches the bottom. If the depth of the water is too great for this method, the soundings can be made at intervals, as the boat moves, without drawing up the lead farther between soundings than is necessary to free it from the bottom.

As the soundings are made, the leadsman calls out the observed depth of each sounding to the recorder, who repeats the depth to prevent mistakes and then enters it in his notebook, together with the time and the number of the sounding. The character of the bottom is observed and noted at such intervals as may be desired, and all changes in the material of the bottom are noted.

GAUGES

14. Tide Gauges.—The bottom depths, as determined by the soundings, are measured from the surface of the water, the elevation of which varies considerably in river and tidal waters. In order to reduce the observed depths to the same surface of reference or to the datum of the survey, it is necessary to know the water level at the time each sounding is made. For this purpose a gauge that will show the height of the water surface should be established at some convenient place. An ordinary graduated board or staff is best for temporary use. This may consist of a board about 6 inches wide, 1 inch thick, and of a length somewhat greater than the variation in the height of the water, painted white and

graduated to feet and tenths in black. Such gauges are used very commonly for this purpose, and are called **staff gauges.**

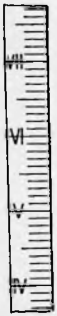


FIG. 7

A simple form of staff gauge is shown in Fig. 7. For facilitating the reading of the gauge, a float, consisting of a small board painted white, may be so placed as to rest on the water surface in front of the gauge. This float moves up or down with the water surface as it rises or falls, and indicates at once the gauge reading.

15. Location of Gauges.—A tide gauge may be attached to a dock, quay wall, pile, stake, tree, or any other stationary object that is in a convenient position and to which the gauge can be secured in a vertical position. Sometimes the gauge is divided into sections and fastened to different objects, as trees, according to their heights, as illustrated in Fig. 8. Each section should slightly overlap the other, so as to afford a continuous gauge reading. In some cases, the gauge may be conveniently set at any suitable inclination and attached to stakes driven firmly in the bank. It may be made in sections and fastened to stakes in such a manner as to conform to the slope of the bank, as illustrated in Fig. 9. Each section should consist of a straight, well-seasoned board from 4 to 6 inches wide and 1 inch thick, and should be painted white. The divisions are determined with the level and should be marked by nails or tacks driven into the face of the board. Such a gauge should be located where the bank is not changing by caving off or filling up, and should be in

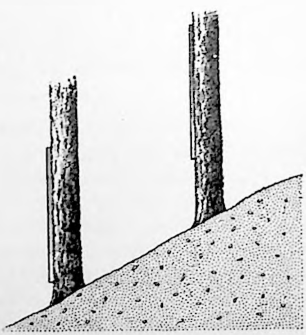


FIG. 8

a location where it will not be disturbed by floating drift at periods of high water. A gauge so placed can be easily observed from the bank at any height of the water.

When a continuous record of the fluctuations of the water surface in tidal waters is desired, a self-registering gauge should be used. This consists essentially of a float that rises and falls with the tide. The float is protected by a perforated box and is so arranged that its motion is recorded by a stylus or pencil on a roll of paper, which passes over a cylinder that is revolved at a uniform speed by clockwork. The path of the pencil on the paper indicates the stage of the water at any given time.

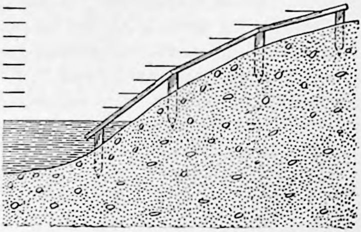


FIG. 9

THE SEXTANT

16. Law of Reflection.—Let AB, Fig. 10, be a plane mirror; PO, a ray of light meeting the mirror at O; OP', the direction taken by the ray after it strikes the mirror; and ON, a line perpendicular to the plane of the mirror. This perpendicular is called the **normal** to the mirror at O. The ray PO coming to the mirror is called the **incident ray,** and the angle NOP that it makes with the normal is called the **angle of incidence.** The ray OP' leaving the mirror is called the **reflected ray,** and the angle NOP' that it makes with the normal is called the **angle of reflection.** It is a general law of physics that the angle of incidence is always equal to the angle of reflection; that is, $NOP = NOP'$. It follows that the angles POA and $P'OB$ are also equal.

FIG. 10

17. Description of Sextant.—The sextant is a hand instrument for measuring angles. By means of it the angle between two lines of sight can be measured by a single operation. The angle between two lines of sight directed to two objects is commonly spoken of as the **angular distance** between the objects. When making the observations, the instrument is held in the hand, and successive angular measurements can be made with great rapidity. It is therefore especially adapted for use in a boat on the water, where the motion renders the use of fixed instruments impracticable.

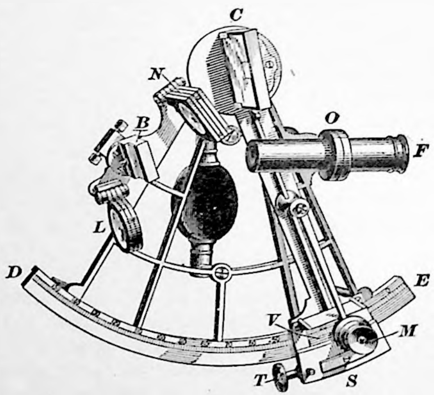


Fig. 11

As the sextant is frequently used in the location of soundings, a description of the instrument and a discussion of its theory are given here.

A sextant, as represented in Fig. 11, consists of a metal frame CDE, and an arm CS, called the **index arm**, which is fitted with a vernier and rotates about the center C of the sextant; to this index arm is also attached the **index mirror** BC, Fig. 12. To the arm CD is fixed the **horizon glass** A, half of the back of which is silvered, while the other half is transparent. The arm CE carries a **telescope** T directed

toward the horizon glass A. Thus, while the telescope is directed to an object H, the rays of light from another body S are reflected first from the mirror BC to the silvered half of the mirror A, and then from this mirror to the telescope in the direction HAT. The observer will thus see both of the bodies H and S in the field of the telescope together.

In order that the ray of light SC may enter the telescope after reflection, the index arm BI must be turned about the pivot C until the mirror BC is brought into the proper position with reference to SC and A. When I is at E, the two mirrors are parallel; and when I has been moved forwards until the rays of light SC, after two reflections, enter the telescope, the arc EI over which the arm BI carrying the mirror BC has been moved will be exactly equal to one-half the angle between SC and HT. Thus, the angle between the rays of light coming from two distant objects H and S may be measured as follows: The observer points the telescope directly toward H and then moves the arm I along the arc ED until the reflected ray $SCAT$ also passes along the line HAT and the image of the second object enters the telescope. The arc EI, which is equal to the angle, called the **index angle**, between the surfaces of the index mirror CB and the horizon glass A, will then be one-half the angle between SC and HA, that is, one-half the apparent angular distance of the bodies H and S.

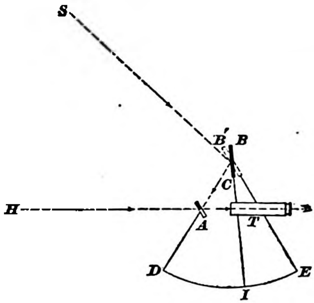


FIG. 12

Since the angle ECI is one-half the true angle between SC and HA, the arc ED has each half degree marked as a whole degree, so that ED, which is an arc of 60°, is divided into 120 equal parts and each part marked 1°. This is done merely to spare the observer the trouble of multiplying the reading by 2.

18. Theory of the Sextant.—That the index angle, or angle between the index mirror BC and the horizon glass A is equal to one-half the angle between SC and HA, Fig. 13, will now be proved. With the index arm in any position, produce the planes of the mirrors to their intersection at O, and produce the perpendiculars to the mirrors, pC and $p'A$, to their intersection at N. Since CE is the zero position of the index arm and CO its given position, the angle ECO is the index angle. Since CE is parallel to AO, $ECO = COA$, and since pC and $p'A$ are perpendicular to CO and AO, respectively, $COA = pNA$. Hence, $pNA = N$ is equal to the index angle.

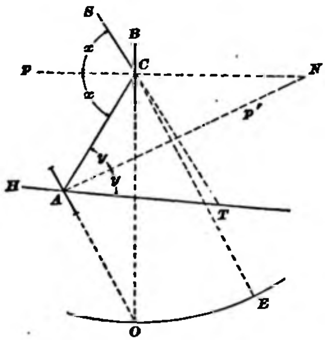


FIG. 13

It will be remembered that the ray of light SC, before reflection, makes the same angle with the perpendicular Cp as the ray of light CA after reflection. Hence, $SCp = pCA$, and similarly $CAp' = p'AT$. Calling each of the former angles x, for brevity, and each of the latter angles y, we have from the triangle ACN,

$$N = x - y$$

The observed angle is $STH = T$. From the triangle ACT,

$$T = 2x - 2y = 2(x - y) = 2N$$

whence,

$$N = \tfrac{1}{2}T$$

That is, the angle between the two mirrors, or the index angle, is equal to one-half the angular distance between the two objects.

The vernier plate V, Fig. 11, is attached to the index bar immediately below the graduations on the limb. The index bar can be clamped to the limb by means of a clamp screw (not shown in the figure), and when so clamped can be

moved very slowly by means of the tangent screw *T*. The magnifying glass *M* is for reading the graduations on the limb and vernier. The two sets of colored-glass shades *N* and *L* are used to prevent the glare of the light from the observed body affecting the eye of the observer; they are attached to the frame by hinges in such manner that the shades *N* can be turned into the path of the reflected ray and the shades *L* into the path of the direct ray.

19. The Vernier.—As stated previously, the divisions on the limb of a sextant correspond to degrees, and each division is subdivided into three, four, or six parts, according to the instrument. These divisions are numbered as shown in Fig. 14, the upper portion of which represents part of the limb of a sextant. The divisions representing degrees on the limb there shown are subdivided into three parts, each part representing a third of a degree, or 20 minutes. By means of the vernier shown in the lower part of the figure, however, the angles can be read to half minutes, or 30 seconds.

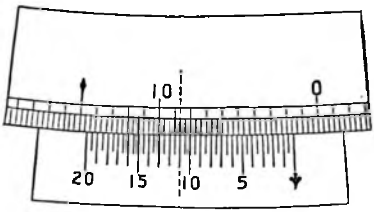


FIG. 14

The vernier of a sextant is substantially the same as that of a transit, except that it is single instead of double; that is, it reads in only one direction, to the left, from the zero point, instead of both to the left and to the right. The first line at the extreme right of the sextant vernier, which is usually designated by a spear-shaped mark, is the zero point or index mark of the vernier. The vernier shown in Fig. 14 is divided into forty equal parts, which together are equal to thirty-nine divisions on the limb. Hence, according to the theory of the vernier (see *Transit Surveying*, Part 1), the least reading of this vernier is $\frac{20}{40}$ minute, or 30 seconds. The numbering of the vernier corresponds to whole

minutes, which are represented by the longer graduations; the shorter graduations represent half minutes. Some sextants are graduated to read to 20 seconds and some to 10 seconds.

The graduations of the limb of a sextant usually extend a few degrees to the right of the zero mark, and it is sometimes convenient to make an observation that requires the vernier to be read when the index mark stands to the right of the zero mark. Such readings are said to be *off the arc.* When readings are made off the arc, the graduations are counted and the vernier is read to the right instead of to the left. The number of degrees and minutes are counted to the right from the zero mark to the first graduation at the left of the index mark; to this is added the number of minutes indicated by the coincident line of the vernier, as counted to the right from the graduation at the extreme left of the vernier. Thus, in Fig. 15, the index mark of the vernier stands to the right of the zero mark of the limb and the reading is off the arc. The angle is read by counting to the right on both the limb and vernier, as just described.

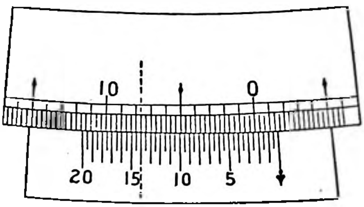


FIG. 15

20. Adjustments of the Sextant.—There are four adjustments of the sextant, as follows:

1. To make the plane of the index glass perpendicular to the plane of the limb.

2. To make the plane of the horizon glass perpendicular to the plane of the limb.

3. To make the line of collimation of the telescope parallel to the plane of the limb.

4. To make the planes of the mirrors parallel when the index reading is zero.

21. To Adjust the Index Glass.—Place the index bar near the middle of the limb; with the eye near the plane of the limb, observe whether the limb as seen directly and its image as reflected in the index glass form a smooth continuous curve; if they do, the glass is perpendicular to the plane of the limb and the adjustment is correct. But if the reflected limb appears to be above that part of the limb seen directly, the glass leans forwards; if it appears to be below, it leans backwards. In either case it is made perpendicular to the plane of the limb by means of the adjusting screws at its base.

22. To Adjust the Horizon Glass.—Look through the telescope and horizon glass toward a star or other well-defined distant object. Move the index bar slowly until the reflected image passes over the image seen directly. If these images coincide, the horizon glass is perpendicular to the plane of the limb. If they do not coincide, the horizon glass is adjusted by an adjusting screw placed under, behind, or beside the glass, according to the construction of the instrument.

23. To Adjust the Telescope.—Place the sextant in a horizontal position on a table or other support, and direct the telescope at some well-defined point or mark about 20 feet away. Place two small blocks of equal height on the limb, one near each extremity. These blocks should be of exactly equal height, so that a line of sight over their tops will be parallel to the plane of the limb, and should be at the same height above the limb as the center of the telescope. Some sextants are provided with two small brass sights that can be placed on the limb for this purpose. Sight over the tops of the two blocks or through the sights, as the case may be, in the direction of the point or mark sighted through the telescope, and note if the line of sight intersects the mark. If it does not, but falls above or below the mark, the telescope is not parallel to the limb. It can be made parallel to the limb by means of the screws in the collar that holds the telescope. This adjustment, however, is not usually made

unless the error is considerable, since a slight lack of parallelism between the line of sight and the plane of the limb does not appreciably affect the angular measurements on the limb.

24. To Adjust the Index—Index Error.—Set the index at zero, look through the telescope toward a star and note whether the direct and reflected images of the star coincide. If they do, the adjustment is correct. If they do not, move the index bar until they do coincide, and clamp it in this position. The reading of the index when in this position is called the **index error.** This error can be corrected by means of screws at the back of the index glass, which cause it to revolve about an axis perpendicular to the plane of the limb. To make the correction, set the index bar at zero and, by turning the screws, revolve the index glass until the two images exactly coincide. This adjustment will usually disturb the previous adjustment of the index glass, and, as a rule, it is not made unless the index error is greater than 3 minutes.

When the index error is less than 3 minutes, it is usually applied as a correction to all observations. If the error is *off* the arc, that is, if the index is to the right of the zero mark, it is additive or plus and should be added to all readings. If the error is *on* the arc, that is, if the index is to the left of the zero mark, the error is subtractive or minus and should be subtracted from all readings.

25. Method of Using the Sextant.—To measure an angle between two objects with a sextant, hold the plane of the limb in the plane of the two objects, look through the telescope toward the less distinct object, and move the index bar until the reflected image of the brighter object comes in contact with the direct image of the less distinct object. Clamp the index bar, and, with the tangent screw, bring the two images exactly together. Note the reading of the vernier and apply the correction for the index error.

In order to have the plane of the limb in the plane of the two objects when the telescope is directed toward the less

distinct object, it may sometimes be necessary to hold the sextant upside down. The measurement of altitudes of celestial bodies with a sextant is explained in *Practical Astronomy*.

EXAMPLE.—The angular distance between two objects, as measured with a sextant, reads on the vernier 35° 36′ 30″; what is the true angular distance if the index error of the sextant is: (*a*) +1′ 20″; (*b*) −1′ 40″?

SOLUTION.—(*a*) Since the vernier reading is 35° 36′ 30″ and the index error is +1′ 20″, the true angular distance is equal to 35° 36′ 30″ + 1′ 20″ = 35° 37′ 50″. Ans.

(*b*) Since in this case the index error is −1′ 40″, the true angular distance is equal to 35° 36′ 30″ − 1′ 40″ = 35° 34′ 50″. Ans.

LOCATING SOUNDINGS

RANGES

26. Preliminary Remarks.—Before starting the sounding work, the stations, triangulation points, and ranges should be carefully located. The work should be so arranged that the soundings may be made and located as rapidly as possible, especially when the area to be sounded is large, or many soundings are to be made. The position of the sun should be considered, so that clear, distinct sights may be had without interference from glare. The observer should be so stationed that while making observations the sun is not directly in his face, but preferably on his back or overhead. If practicable, the order of work should be so arranged that observations toward the west may be taken in the forenoon, and toward the east in the afternoon. In tidal waters, the range of the tide should be considered. If the difference in elevation between high and low tide is very great, sounding work should preferably begin after the tide has fallen to a level about half way between high and low tide, or after half ebb, and cease when it has risen to the same level or at half flood. Usually, however, sounding work can be done at all stages of the tide.

In all sounding operations where simultaneous measurements are to be made, the recorder and the various observers

should have watches set accurately to the same time. Soundings are usually made at regular intervals of time, but there is no fixed rule regarding this. The length of time between successive soundings will depend on the depth of the water, the method of observation, and the distance between adjacent soundings. When the soundings are located from the shore with a transit, the observations are commonly made at intervals of 1 minute. In this case, intermediate soundings may be located by interpolation. When soundings are located from a sounding boat by sextant observations, as many as three observations per minute can be made, since angles can be measured more rapidly with a sextant than with a transit.

27. Sounding Ranges.—Soundings are usually made on well-defined lines or courses whose positions are known. These lines are called **ranges**. They are usually laid out on shore and prolonged across the area of water surface to be sounded. In such cases, two points on each range are selected at which poles or other signals are placed to serve as guides to the sounding party in determining the range. These points should be a considerable distance apart, and should be carefully located in order to accurately establish the direction of the range passing through them. The point near the shore is called the **front range point,** or **front signal,** and that back from the shore is called the **back range point,** or **back signal.** The steersman determines the course of the sounding boat by sighting along the range and keeping the two range points in line.

The ranges should be laid out and arranged with reference to the extent of the area to be sounded and also the method of locating the soundings. In localities where the area to be sounded is comparatively small, the arrangement illustrated in Fig. 16 can be used. Two parallel rows of stakes, as CD and EF, are established. The stakes in each row through which the parallel ranges pass are usually spaced at regular intervals. The length of the interval or the distance between adjacent ranges will depend on the frequency with which the soundings are to be made. The distance between the front

and back range points should be such that well-defined ranges can be established by sighting from the boat to two range points in line.

In Fig. 16, the dotted parallel lines represent the sounding ranges. *A* and *B* are observation stations so situated as to offer a clear view over the area to be sounded, and preferably visible one from the other; the distance between them should be accurately determined.

This system of range lines controls the positions of the soundings, which are made on the range line and distributed over the area to be sounded in such manner as may be desired, thus avoiding unequal distribution, making two or more soundings at or near the same place, or allowing too great an interval between soundings. For such purpose, the range lines are advantageous in nearly all soundings. When a boat is on a range the observer always knows its approximate position. Soundings made on range lines are platted with greater facility than those not made on ranges. The system of range lines is especially adapted for use when the soundings are located from shore by means of angles measured with a transit at one extremity of the base line. One angle for each sounding is measured between the base line and the line of sight to the boat. Before commencing the soundings, the angles made by the range lines with the base line should be measured, and the distance along the base line from the instrument to the intersection of each range line with the base should be determined. In some cases, the soundings are located by means of two angles measured simultaneously from both extremities of the base to the sounding boat; in this case each range is used as a guide for the sounding boat and also as a check on the accuracy of the locations made on it.

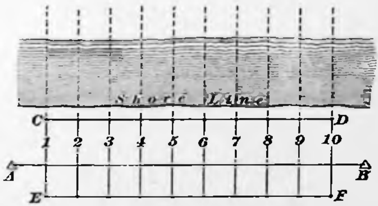


FIG. 16

28. Ranges Marked by Buoys.—If the shore is heavily wooded or rocky and precipitous, it may be impracticable to establish two rows of stakes at a sufficient distance apart to serve as front and back range points. In such a case, buoys may be used for signals to mark the front range points. Each range will then consist of a point on shore and a buoy in the water, as illustrated in Fig. 17. In this figure, *A*, *B*, *C*, and *D* are points or stations on shore, which may be located by direct measurement from point to point, while *1*, *2*, *3*, and *4* are the corresponding buoys. The buoys are located by measuring two angles in each triangle, the angles being read from the shore stations. Thus, the buoy at *1* can be located by measuring the angles *B A 1* and *A B 1*; the buoy

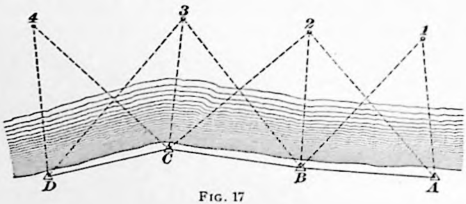


FIG. 17

at 2 by measuring the angles *A B 2* and *B A 2*, or the angles *C B 2* and *B C 2*. The buoys at *3* and *4* are located in a similar manner.

Buoys may be used for points of reference in connection with fixed points on shore. The position of each buoy will vary within small limits according to the stage of the tide, but such variations will not usually be sufficient to cause appreciable error in the platted positions of the soundings.

29. Radial Ranges.—Where topographical conditions will permit, the front range points are located close to the shore line, and some prominent natural object, such as a tall tree, a church spire, a windmill, or the cupola of a building, is selected for a back range signal. In such a case, the distance from the shore line to the back range signal should be such that radial range lines from this point

through the several front range signals will cover the area to be sounded. The distance between adjacent ranges at their extremities should not usually be greater than the average distance between successive soundings on a range. Such an arrangement is illustrated in Fig. 18. In this figure, *AB* is a base line on which the front range points are located; these are represented by small dots. The broken lines numbered *1*, *2*, *3*, etc. are the sounding ranges. The front range points should be at known distances apart, the distance between adjacent range points being carefully measured with a steel tape, and each point marked by a stake. The points *A* and *B* at the extremities of the base line, and

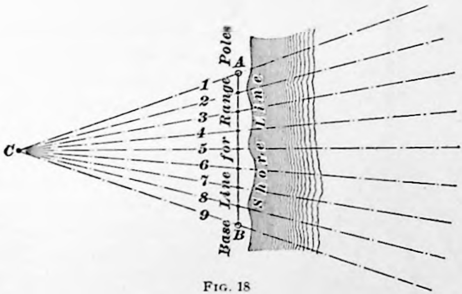


FIG. 18

the back range signal *C*, can be located from other points on the outline survey whose positions are known.

30. Ranges Across Streams.—When soundings are to be made across rivers or streams of considerable magnitude, sounding ranges are usually run across in directions perpendicular to the axis of the stream. Such ranges are marked by range signals placed either on one or both banks, according to the width of the stream.

RANGE SIGNALS

31. Range Poles.—In designating the ranges, it is important to have the front and back range points marked by poles, or other objects, sufficiently conspicuous to be

easily visible from the sounding boat, and designated in such a manner as to distinguish the different ranges, each from the others. Such objects are called **signals**; they may consist of poles or rods, or of natural objects on the shore. When a range point is in the water, the signal marking it usually consists of a buoy, as has been previously stated. In shallow water, signals similar to those used on the shore are often used, being set on, or driven into, the bottom.

Range signals on shore usually consist of poles of suitable height and dimensions. When the sounding ranges are short, ordinary transit sight poles are frequently used for signals. In such cases an assistant holds the rod in position on the stake marking the range point, or it is placed in a vertical position by the stake.

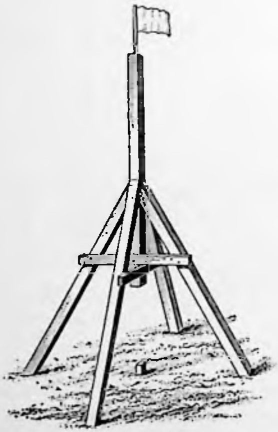


Fig. 19

If the ranges are of considerable length and soundings are to be made at some distance from shore, larger and more conspicuous poles should be used for signals. These may consist of pieces of $4'' \times 4''$ scantling, of suitable lengths. They should be set vertically and may be supported by being firmly driven into the ground, their lower ends being sharpened, or by being placed in holes dug for the purpose. If the ground is hard or rocky, the poles may be supported by braces or by stones piled around their bases. A form of range signal often used is illustrated in Fig. 19. A pole, consisting of a piece of $4'' \times 4''$ scantling of suitable length is supported in a vertical position by four inclined braces. The upper ends of these are nailed to the vertical piece, which is also held in place at the bottom by two horizontal strips of wood extending between the opposite braces, as

shown. The lower end of the vertical pole may be set at any convenient height above the ground; a hole may be bored into its upper end to receive a flagstaff, or the latter may be nailed to the top of the pole. This form of range signal can be set directly over a stake or hub, as shown in the illustration.

Range poles should be whitewashed so as to be conspicuous against the background of the shore. When a number of adjacent ranges are used, they may be designated by attaching colored flags to the poles or signals. In such cases each range is known by some distinctive color or combination of colors. Thus, the flag for range No. *1* may be red; that for range No. *2*, red and white, etc.

FIG. 20

Ranges are sometimes designated by strips of wood, such as laths or barrel staves, nailed to the poles or signals. The strips should be arranged in the form of Roman numerals, as shown in Fig. 20. In such cases the numerals denote the numbers of the ranges; the arrangement shown in Fig. 20 represents range No. *6*. The strips and the pole should be whitewashed so as to be conspicuous.

FIG. 21

32. Targets.—When sounding ranges are to be projected a considerable distance from shore, the range poles or signals should be provided with targets in order to be conspicuous. Such targets should be sufficiently large to be visible from the sounding boat at the most distant point on the range, and of suitable designs or colors to enable the steersman to distinguish readily the different ranges. No rigid rule can be laid down specifying any particular form or design for range targets. They may be made of such forms as are best suited for the purpose, and composed of such materials as are available in each case.

A good form of target for range signals is shown in Fig. 21. The target is lozenge shaped and can be made in the following manner: A strip of wood, about 3 inches wide, 1 inch thick, and 3 feet long, is nailed to the face of the range pole about 3 feet below the top, care being taken to center the strip and to make it perpendicular to the pole. To the framework thus formed, a square piece of white or colored cloth is tacked in such a manner that two diagonally opposite corners of the cloth are at the two extremities of the cross-piece, the two other corners being on the center line of the range pole. Such a target is quickly and economically made and is very effective when the sights are directly in front and the full size of the target is visible.

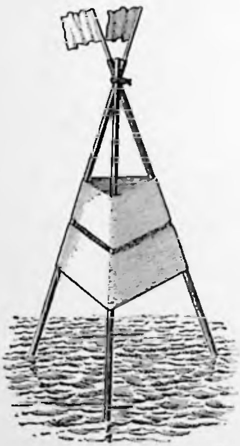


Fig. 22

33. Range Signals in the Water.—When a range point is in the water, the signal marking it usually consists of a buoy; in shallow water, however, stationary range signals are often used. A form of signal used by the United States Coast and Geodetic Survey is shown in Fig. 22. This consists of a tripod about 10 or 12 feet high, each leg being made of a piece of gas pipe about $1\frac{1}{2}$ inches in diameter. The legs are forced firmly into the mud or sand of the bottom, at suitable distances apart, and inclined toward the center. They are lashed together near the top, and flags about $1\frac{1}{2}$ feet square attached to poles just large enough to fit inside the pipes are placed at their upper ends as shown. Two strips of cloth, each about $\frac{1}{2}$ yard wide, are wrapped around the tripod, about half way between its top and bottom. These strips serve as a target.

34. Buoys.—A buoy is a float of wood or other suitable material, or a hollow air-tight vessel, anchored in place by a heavy weight to which it is attached by a rope or chain. Buoys are used to mark certain places or points on the water surface. They are usually employed to designate the limits of a channel or some submerged object in connection with navigation. They are also used as points of reference and for range points in hydrographic surveying, and only such as are suitable for such purposes will be considered here.

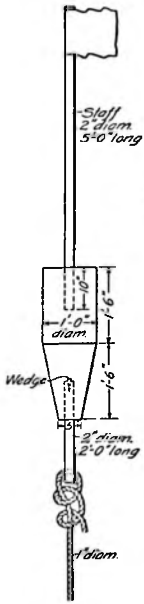


FIG. 23

A form of buoy that has been much used in hydrographic surveying is illustrated in Fig. 23. It consists essentially of a round log of cedar or other light wood, about 1 foot in diameter and 3 feet in length, sawed square at ends. The lower half is trimmed in the shape of a truncated cone, tapering to about 5 inches in diameter at the lower end. A hole about 2 inches in diameter and 9 inches deep is bored into the lower end on the axis of the log, into which a pole 2 inches in diameter is driven. The upper end of the pole is split, and a wedge inserted in the cleft, which is driven up and tightens the pole as it is driven to the end of the socket, thus preventing the pole from pulling out. A hole sufficiently large for the anchor rope to pass through is bored through the pole a few inches above its lower end. The anchor rope is preferably of manila hemp and is about 1 inch in diameter. A good form of knot with which to tie the rope is shown in the figure; it is shown open in order to illustrate the method of forming the knot. This knot is called the **clove hitch.** A hole about 2 inches in diameter and 10 inches deep is bored into the top of the buoy along its axis. This serves as a socket into which is inserted a staff

about 2 inches in diameter and 5 feet long. At the upper end of this staff is fastened a flag about 1 foot square and of suitable design or color.

Another form of buoy is illustrated in Fig. 24. This consists of a log or round piece of light wood, about 3 feet long, sawed square at both ends. This log is trimmed in the shape of a truncated cone, tapering from about 8 or 10 inches in diameter at the upper end to a diameter of about 4 or 5 inches at the lower end. A hole about 2 inches in diameter is bored completely through the log, the center of the hole coinciding with the axis of the log, and a round pole large enough to fit the hole closely is passed through the log and wedged tightly in place. This pole should project about 2 or 3 feet below the bottom of the log or buoy and about 3 or 4 feet above its top. A flag about 1 foot square is fastened to the upper end of the pole as shown. The anchor rope is passed through a hole in the pole near its lower end and tied as described in the preceding article.


FIG. 24


FIG. 25

35. Buoys for Tidal Waters.—The two forms of buoys just described are suitable for use in non-tidal waters and in rivers and streams where the current is not sufficiently strong to drag the top of the buoy under or level with the surface of the water.

Buoys for use in tidal waters should be of sufficient length to be visible at all stages of the tide. The best length for a buoy in a given tidal water will depend on the range of the tide, and is usually greater than that required for buoys in non-tidal waters.

A good form of buoy for use in tidal waters is illustrated in Fig. 25. The length shown is 10 feet, but this may be varied to conform to the range of the tide. The buoy is

made from a log or round piece of light wood and trimmed so as to taper gradually from the middle to each end. It has a diameter of about 1 foot at the middle and tapers to a diameter of from 4 to 6 inches at each end. It is sawed square at the ends, and holes are bored at each end, into which poles are inserted for attaching the flag and anchor rope, as previously described. If preferred, a ring may be fastened to the lower end of the buoy by means of a staple or screw, and used for securing the anchor rope to the buoy.

A more simple form of buoy, consisting of a single stick or log of timber of suitable length and diameter, is often used in tidal waters; the shape is similar to that illustrated in Fig. 25, but the buoy is more uniform in size from top to bottom. Such buoys are commonly called **spar buoys.**

METHODS OF LOCATING SOUNDINGS

36. Methods Employed.—In order to plat soundings correctly on a map or chart, the position of each sounding must be located; that is, its relative position with respect to known points on shore must be determined. Soundings can be located by various methods, depending on local conditions, the object of the survey, and the degree of accuracy required. The following list comprises the best known and most frequently used methods of locating soundings: (1) by time intervals; (2) by one angle measured on shore; (3) by two angles measured simultaneously on shore; (4) by two angles measured in the sounding boat; (5) by transit and stadia; (6) by a fixed line marked by a wire or rope; (7) by the intersection of fixed ranges. These methods will be described in order.

37. By Time Intervals.—When this method is employed, the soundings are made at stated intervals of time while the sounding boat moves at uniform speed along a range or on a course not marked by range signals. The soundings may be made under two conditions, namely: (*a*) The first and last soundings on a range or course are located by observation and all the intermediate soundings are

located by interpolation or time intervals. (*b*) Alternate soundings or those made at convenient intervals are located by observation, and such intermediate soundings as are not observed are located by interpolation. In either case the method of interpolating the intermediate soundings is the same. Knowing the distance between the two end soundings or between two adjacent observed soundings, the time interval between them, and the time interval between consecutive soundings, the position of each intermediate sounding can be determined by proportion as follows:

Let T = time elapsed between two observed soundings on a range or course;
D = distance between these observed soundings;
t = time interval between consecutive intermediate soundings;
d = distance between the intermediate soundings.

Then, since the boat moves at a uniform speed,

$$D : T = d : t$$

from which

$$d = \frac{Dt}{T}$$

Example.—A sounding boat moving at a uniform speed traverses a range 1,800 feet long in 20 minutes, and a sounding is made at each end of the range and at intervals of 1 minute; what is the distance between consecutive soundings?

Solution.—The two end soundings are 1,800 ft. apart $= D$; the elapsed time between them is 20 min. $= T$; and the time interval t between consecutive soundings is 1 min. Substituting these values in the formula gives,

$$d = \frac{1,800 \times 1}{20} = 90 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. A range 500 feet long is traversed at uniform speed in 10 minutes by a sounding boat from which soundings are made at intervals of ½ minute; find the distance between any two consecutive soundings.
Ans. 25 ft.

2. In the preceding example, if the soundings are numbered consecutively 1, 2, 3, etc., from beginning to end of the range, what is the distance between soundings Nos. 5 and 12? Ans. 175 ft.

3. Soundings are made at intervals of 15 seconds or at the rate of 4 per minute, from a boat moving along a course at a uniform speed; the soundings made at the end of each minute are located by observations and the intermediate soundings are interpolated. The observed soundings, when located, are found to be at intervals of 204 feet apart; what is the distance between consecutive soundings? Ans. 51 ft.

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38. By One Angle Measured on Shore.—When this method is used, the boat containing the sounding party traverses a fixed range while an observer on shore measures the angle between a base line and the line of sight to the leadsman at the time a given sounding is made. The ranges are usually parallel to each other and are preferably at right angles to the base line. They should be at known distances apart and the distance from the observation station to each range should be determined by careful measurement along the base line. The base line may have an observation station at each extremity, in which case the observer is stationed at either end, as may be most convenient, and orients his instrument by sighting toward the station at the other end. In Fig. 26 is shown a base line at the ends of which are two observation stations A and B, whose positions have been determined. The ranges numbered *1*, *2*, *3*, etc. are parallel to each other and at right angles to the base line. Each range is designated by front and back range signals whose positions are shown by the small dots in the figure. The

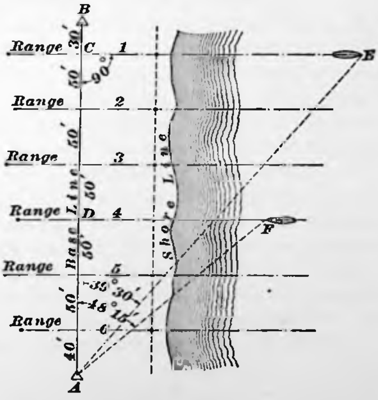

FIG. 26

distances along the base line between the observation stations and the nearest ranges and the distances between adjacent ranges are as shown by the figures.

The field party usually consists of the observer on shore, and in the boat the recorder, the leadsman, the signalman, and the boat crew. In tidal waters, a tide-gauge reader should be added to the shore party; his duties consist in reading the tide gauge every 5 minutes during sounding operations and recording the times and gauge readings. The watches of the observer on shore and the recorder in the boat are set accurately to the same time. The sounding boat traverses the ranges in succession and soundings are made at regular intervals of time, which depend on the depth of the water and on the accuracy required. Usually, from one to four soundings are made per minute. A few seconds before the end of each interval the recorder has the signalman raise his flag, and exactly at the end of the interval the flag is lowered, as a signal to the observer that the sounding has been made. The leadsman calls out the observed depth of each sounding to the recorder, who enters it in his notebook together with the number of the sounding. The exact time of each sounding is recorded. The character of the bottom is observed by the leadsman and noted by the recorder at such intervals as may be required.

The observer is stationed at A with his transit set up over the transit point at that end of the base line. The vernier of the transit is first set at zero and the telescope is directed toward B, the point at the other end of the base line, and the instrument is clamped. The upper plate is then unclamped and the observer turns the instrument in azimuth toward the sounding boat. When making the observations, the observer usually keeps his watch open and lying face uppermost on the upper plate of the transit, for convenience in noting the time. When the signal flag is raised in the boat the observer sights through the telescope toward it, and by turning the upper plate slowly and carefully, keeps the line of sight fixed on the flag. At the instant the flag is lowered the observer ceases to turn the plate. He then looks at his

watch and notes the exact time, then reads the angle on the vernier plate and enters the time and angle in his notebook. The observations are usually made on the flag, as it can be seen more distinctly and at greater distances than a sounding line or pole. On this account the signalman should be stationed near the leadsman, so that the difference between the observed and the true position of each sounding will be small. The upper plate of the transit should remain unclamped while observations are being made on the sounding boat, since there is usually not sufficient time between observations to permit its being clamped, and frequent clamping and unclamping would tend to disturb the position of the transit.

In some instances, there is by each transit, in addition to the observer, a recorder whose duties consist in recording the observed angle for each sounding, and the number of the sounding. The recorder enters in his notebook the observed angle and the number of each sounding, and he also notices that every fifth sounding is a "red" sounding. It is customary for the signalman to raise a red flag for every fifth sounding and a white flag for the intermediate soundings. Whenever the observer sees the red flag displayed by the signalman he calls out "red" to the recorder, who then notes the number of the sounding, which should be a multiple of five, in order to correspond with the number entered by the recorder in the boat. In this way a check is obtained on the numbering of the soundings, and any difference in the numbering by the recorder in the sounding boat and the recorder on shore can be readily detected.

EXAMPLE.—The ranges shown in Fig. 26 are at right angles to the base line; a sounding is made while the sounding boat is at E on range *1*. The observed angle $E A B$ is 39° 30′ and the distance $A C$ along the base line from A to the intersection C of range *1* is 290 feet; find: (*a*) the distance $A E$; (*b*) the distance along the range line from C, its intersection with the base line to the sounding at E.

SOLUTION.—(*a*) Since the ranges are at right angles to the base line, $A E$ is the hypotenuse of the right triangle $A C E$. From trigonometry,

$$A E = \frac{A C}{\cos E A C} = \frac{290}{\cos 39° 30'} = 375.8 \text{ ft. Ans.}$$

(*b*) The distance from E along range *1* to its intersection C with the base line is $E C = A C \tan E A C = 290 \tan 39° 30' = 239.1$ ft., nearly. Ans.

EXAMPLES FOR PRACTICE

1. The distance $A D$ along the base line from A to the intersection of range *4*, Fig. 26, is 140 feet; the angle $F A B$ is 48° 15′; find: (*a*) the distance $A F$; (*b*) the distance along the range from F to its intersection D with the base line. Ans. $\begin{cases} (a) & 210.2 \text{ ft.} \\ (b) & 156.9 \text{ ft.} \end{cases}$

2. A sounding boat is on a range perpendicular to the base line. The angle measured at an observation station between the base line and the line of sight to the boat at the time a given sounding is made is 30° 30′; the distance from the station to the intersection of the base line and range is 230 feet; find: (*a*) the distance from the station to the position of the sounding; (*b*) the distance along the range line from the base line to the position of the sounding. Ans. $\begin{cases} (a) & 266.9 \text{ ft.} \\ (b) & 135.5 \text{ ft.} \end{cases}$

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39. By Two Angles Measured Simultaneously on Shore.—This is one of the commonest methods, and if the work is carefully done it is both convenient and accurate. Two observers are required, each occupying a station whose position with respect to the shore survey has been determined. The observation stations should be so situated as to afford a clear field of view over the area to be surveyed, and when possible should be visible one from another. They may be at the extremities of a base line, whose length has been carefully measured, or at two points whose positions and distance apart have been determined by triangulation. The vernier of each instrument is set to read zero when the telescope is directed toward the other instrument point or toward some common point whose position is known. The field work is entirely similar to that described for the preceding method, and the field party and equipment is the same with the addition of an observer and transit. This method differs from the preceding method, however, in that two angles, instead of one, are measured for each location, the observations being made simultaneously, and the position of the observed sounding is determined by the intersection of the two lines of sight from the two observation stations,

instead of by the intersection of one line of sight with a range line. In using this method it is not necessary to have the sounding ranges parallel or to lay them out at right angles to the base line, although such an arrangement is advantageous in affording a means of checking the accuracy of the angular measurement. For locating soundings over a limited area by this method, the arrangement shown in Fig. 27 is convenient and gives good results. In this figure, A and B are the observation stations at the extremities of a base line AB. The ranges are shown by the parallel lines passing through the small dots indicating the range signals. One position of the sounding boat is shown at C, and the lines of sight to this position from the two observation stations are represented by the dotted lines AC and BC.

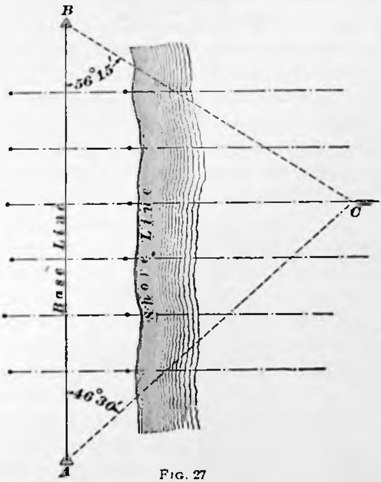


Fig. 27

Example.—A sounding is made while the sounding boat is at the position C, Fig. 27. The angles observed from A and B are 46° 30′ and 56° 15′, respectively, as shown, and the length of the base line AB is 785 feet; find the distances AC and BC, giving values to the nearest foot.

Solution.—The angle $BAC = 46°\,30'$, and $ABC = 56°\,15'$; hence, $ACB = 180° - (46°\,30' + 56°\,15') = 77°\,15'$, and, from trigonometry,

$$AC = \frac{AB \sin ABC}{\sin ACB} = \frac{785 \sin 56°\,15'}{\sin 77°\,15'} = 669 \text{ ft.} \quad \text{Ans.}$$

$$BC = \frac{AB \sin BAC}{\sin ACB} = \frac{785 \sin 46°\,30'}{\sin 77°\,15'} = 584 \text{ ft.} \quad \text{Ans.}$$

40. By Two Angles Measured in the Sounding Boat.—This method, which is used extensively in harbor work, is one of the best general methods of locating soundings. The field party usually consists of two observers, a recorder, a leadsman, and boat crew. In tidal waters a tide-gauge reader is required. The observers, each with a sextant, occupy places in the sounding boat as close to the leadsman as practicable, in order that the observed position of each sounding may be very nearly the same as its true position. At the time a sounding is made, the two observers measure simultaneously the two angles between the lines of sight to three shore objects whose positions have been determined by the shore survey, one line of sight in each observation being directed toward the same object. The objects sighted to should be well defined and prominent, and so located with respect to the area to be sounded as to be readily visible from the sounding boat in all required positions. They should preferably be natural objects, such as church spires, windmills, lighthouses, cupolas, etc., but in case natural objects are not available, signal poles, such as are used for marking ranges, may be used. In making the observations, one observer measures the angular distance between the middle object and the object on the right; and the other observer measures the angular distance between the middle object and the object on the left. This is illustrated in Fig. 28, in which *F* represents the position of the boat at the time of a given sounding, and *A* and *B* are the two angles measured by the two observers between the lines of sight to the three shore objects *E*, *C*, and *D*. The sounding work is conducted as follows:

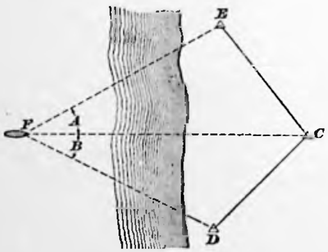


FIG. 28

The sounding boat moves slowly along a range or course, the leadsman making the soundings at the required intervals,

usually two or three per minute. A few seconds before a sounding is to be made the leadsman calls out "ready," when the observers hold their sextants in position to make an observation on the shore objects, each observer moving the vernier arm of his sextant so as to keep the two images in coincidence as the boat moves. At the moment the sounding is made, the leadsman calls out "sound," when each observer reads the angle on his sextant and calls it out to the recorder, who records each angle in its proper column in his notebook. The leadsman calls out the observed depth to the recorder, who also enters it in his notebook, together with the number of the sounding and the time the sounding is made. At required intervals, the leadsman observes the character of the bottom and informs the recorder, who enters it in the proper place in his notebook.

In some cases both angles are measured by one observer with a double sextant; they can also be measured successively by one observer with an ordinary sextant, if the boat is brought to a stop for each sounding. But they are usually measured by two observers, each using an ordinary sextant, in the manner just described. When sextants are used and the angles are so recorded by the recorder that the observer has only to observe and read them, four angles per minute can be observed under ordinary favorable conditions.

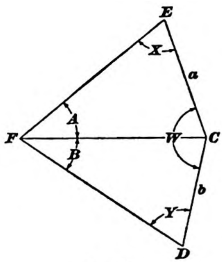


Fig. 29

41. The accurate location of soundings by two sextants involves what is known as the **three-point problem.** This problem can be solved trigonometrically as follows: Let F, Fig. 29, be the position of the boat when a given sounding is made; E, C, and D the three shore objects, whose positions are determined by the angle W and the sides EC and CD, which are designated by a and b, respectively. The angles A and B are the two sextant angles measured in the boat. The problem is to determine the distances FE and FD.

In the triangles EFC and CFD,

$$CF = \frac{a \sin X}{\sin A} = \frac{b \sin Y}{\sin B} \qquad (a)$$

Also, $X + Y + W + A + B = 2 \times 180° = 360°$

whence, $X + Y = 360° - (W + A + B) = S$, say $\qquad (b)$

Therefore, $Y = S - X \qquad (c)$

and $\sin Y = \sin(S - X) = \sin S \cos X - \cos S \sin X$

Substituting this value of $\sin Y$ in (a),

$$\frac{a \sin X}{\sin A} = \frac{b(\sin S \cos X - \cos S \sin X)}{\sin B}$$

Clearing of fractions,

$$a \sin X \sin B = b \sin S \cos X \sin A - b \cos S \sin X \sin A$$

Dividing by $\sin X$, replacing $\frac{\cos X}{\sin X}$ by $\cot X$, and solving for $\cot X$,

$$\cot X = \frac{a \sin B + b \cos S \sin A}{b \sin S \sin A} = \frac{a \sin B}{b \sin S \sin A} + \frac{\cos S}{\sin S}$$

or, writing $\cot S$ for $\frac{\cos S}{\sin S}$,

$$\cot X = \frac{a \sin B}{b \sin S \sin A} + \cot S$$

The value of $\cot Y$ can be determined in a similar manner, or by substituting in equation (c) the value previously found for X.

After determining the values of X and Y, the distances EF and FD can be determined by trigonometry as follows:

$$ECF = 180° - (A + X)$$

$$EF = \frac{a \sin ECF}{\sin A}$$

Similarly, in the triangle FCD,

$$DCF = 180° - (B + Y)$$

$$DF = \frac{b \sin DCF}{\sin B}$$

Having calculated the distances EF and DF for a given sounding, the position of the sounding can be located by drawing arcs with a pair of compasses, with E and D as centers and with radii of lengths EF and DF, respectively. The intersection of the two arcs will be the point F, or the

position of the sounding. In practice, however, it is seldom necessary to calculate the position of soundings located by sextant angles to three fixed points on shore, since the positions of soundings so located can be platted easily without calculation, as will be described further on. But when an important location is made by this method, such as the position of a rock, a reef, a buoy, or a sunken wreck, the location as platted should be checked by calculation.

EXAMPLE.—Given a = 850 feet; b = 760 feet; W = 150°; A = 41° 30′; B = 35° 30′ (Fig. 29); what are the values of the angles X and Y, and of the sides EF and DF?

SOLUTION.—Substituting the given values in equation (*b*), $X + Y = 360° - 227° = 133° = S$; $\cot S = -\cot(180° - S) = -\cot 47°$.

Substituting known values in the formula,

$$\cot X = \frac{850 \sin 35° 30'}{760 \sin 133° \sin 41° 30'} - \cot 47°$$

$$X = 67° 49'$$

whence, $$Y = 133° - 67° 49' = 65° 11'$$

In the triangle FCE,

$$ECF = 180° - (41° 30' + 67° 49') = 70° 41'$$

$$EF = \frac{850 \sin 70° 41'}{\sin 41° 30'} = 1{,}211 \text{ ft.} \quad \text{Ans.}$$

In the triangle DCF,

$$DCF = 180° - (35° 30' + 65° 11') = 79° 19'$$

$$DF = \frac{760 \sin 79° 19'}{\sin 35° 30'} = 1{,}286 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. In Fig. 30, in which F is the position of the sounding boat at the time a given sounding is made, and E, C, D are the three shore points, let a = 1,200; b = 965; W = 146° 30′; A = 28° 15′; and B = 22° 30′; find the angles X and Y and the distances EF and DF.

$$\text{Ans.} \begin{cases} X = 80° 21' \\ Y = 82° 24' \\ EF = 2{,}403 \\ DF = 2{,}437 \end{cases}$$

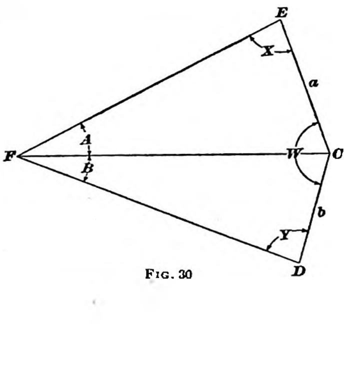


FIG. 30

2. With the same values for a, b, and W, let A = 30°

30′ and $B = 24° 28′$; find the angles X and Y and the distances EF and DF.

$$\text{Ans.} \begin{cases} X = 77° 3′ \\ Y = 81° 29′ \\ EF = 2{,}254 \\ DF = 2{,}240 \end{cases}$$

---

42. By Transit and Stadia.—This is a rapid and efficient method of locating soundings in bodies of calm, smooth water and at distances that do not exceed the limit of good practice for stadia readings. In this method, the positions of the soundings are located with a transit on shore by means of observations taken on the stadia rod held in the sounding boat. Since the stadia rod should be without vertical motion when a reading is taken, it is evident that this method can be used satisfactorily only on smooth water. When this method for locating soundings is used, a complete hydrographic party comprises the observer on shore, with a transit equipped with stadia wires, and the boat party, consisting of the recorder, the leadsman, the stadiaman, and the boat crew. No signalman is required; the recorder acts as signalman. The soundings are not identified by time intervals, but by means of differently colored flags. A red flag is shown for every fifth sounding and a white flag for the intermediate soundings. Two general cases may occur under this method.

1. The transit station may be a point on the sounding range; in this case the azimuth of the range is known and each sounding is located by the distance, corresponding to the observed interval on the stadia rod, as measured along the range.

2. The transit station may be a point whose position has been determined but which is not on a sounding range; in this case the reading of the azimuth angle, as well as the stadia interval, must be observed and recorded for each sounding.

In either case the field work is conducted in the following manner: The sounding boat moves slowly along the range or course while the soundings are being made. The leadsman stands in the bow of the boat and makes the soundings,

calling out the observed depth of each sounding and also the character of the bottom at required intervals. The recorder enters in his notebook the number and the observed depth of each sounding, and the character of the bottom when noted by the leadsman. If the soundings are in tidal waters, the time should also be noted in order to make reductions for the tide heights as given in the notebook of the tide-gauge reader. During the sounding operations, the stadiaman holds the stadia rod vertical and facing the observer. He should be stationed close to the leadsman, in order that the observed positions of the soundings will coincide nearly with their true positions.

The observer keeps the vertical wire of the transit telescope directed to the stadia rod in the boat. If the transit station is on the range on which the soundings are made, the observer merely reads the stadia interval for each sounding and enters it in his notebook, also noting the time and the number of the sounding. If the transit station is not on the sounding range, but is off to one side, the vernier of the transit is first set at zero, the telescope is sighted on some object whose position has been determined, and the instrument is clamped. The upper plate is then unclamped, and the instrument turned in azimuth toward the sounding boat as explained for the second method. Then, for each sounding, the horizontal angle is read and recorded, in addition to the stadia interval, and the number and time of the sounding.

43. By a Fixed Line Marked by a Wire or Rope. This is an accurate method, but it is adapted only to narrow channels. It is often used for measuring cross-sectional areas in a canal or a small stream in connection with the determination of discharge or the measurement of material removed from the channel by dredging or other means. In such cases the wire or rope is stretched from bank to bank between fixed points, as illustrated in Fig. 31, and the soundings are taken at regular intervals along the wire or rope. The points where the soundings are taken are marked

by tin tags or by bits of cloth tied to the wire or rope. When this method is employed in connection with the measurement of dredged material, the stakes $O\ O'$ are carefully located and their positions noted, in order that they may be replaced if disturbed. Soundings are made at

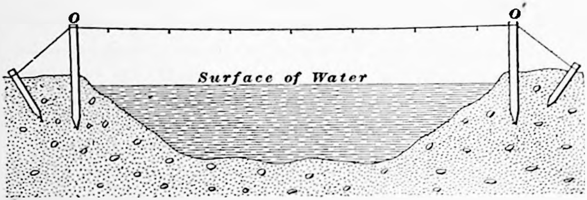


FIG. 31

known intervals along a wire or rope stretched from stake to stake, before and after the dredging operations.

44. By the Intersection of Fixed Ranges.—If a fixed range or section of considerable length is to be sounded a number of times, and the soundings are to be made at the same points each time, the soundings can be located by the

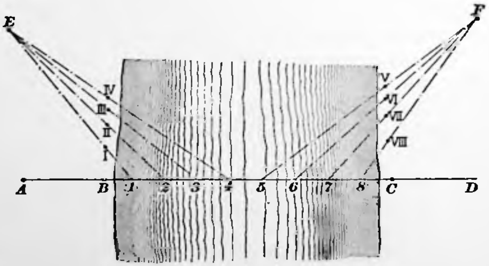


FIG. 32

intersection of a series of ranges with the fixed range or section. Let $A\ D$, Fig. 32, represent a range or section across a river, and *1*, *2*, *3*, *4*, *5*, *6*, *7*, *8* the points where soundings are to be made at successive periods. Range poles are set at A, B, C, and D, fixing the position of the range $A\ D$, unless

those points are marked by natural objects, and poles are also set at *E* and *F*; also at *I*, *II*, *III*, *IV*, etc. The ranges *E–I*, *E–II*, etc. and *F–V*, *F–VI*, etc., produced to their intersection with the range *A D*, will locate the soundings *1*, *2*, etc., and *5*, *6*, etc. The range signals *A*, *B*, *C*, *D*, and the back signals *E* and *F* for the numbered ranges may consist of ordinary range poles, whitewashed, as described in Art. **31.** The front range signals *I*, *II*, *III*, *IV*, *V*, etc., on the numbered ranges, should be designated by Roman numerals indicating the number of each range, as described in Art. **31.**

When this method is used, the sounding party consists of a recorder, a leadsman, and boat crew. The steersman keeps the boat on the fixed range *A D*, and each sounding is made when the leadsman, by sighting toward a numbered signal, finds himself in range with it, and the back signal at the same time in line with the signals designating the fixed range. Thus, when at the position *5*, Fig. 32, the leadsman is in line with the signals *C D*, designating the range *A D*, and also with the signals *V F*, designating range *V*. The boat can be stopped for each sounding if there is little or no current; or if the current is strong, the boat can move slowly, preferably against the current.

45. The Plane of Reference.—In making soundings, the leadsman notes the depth of each sounding below the water surface at the time the sounding is made. Since the elevation of the water surface is constantly changing, especially in tidal waters, it is necessary to select some particular stage or height of the water surface to which the depths of all the soundings are referred. Such a height of the water surface is called a **plane of reference**, and all observed depths are reduced to correspond to depths below this surface. In order to determine the proper reductions to apply to the soundings for different stages of the tide, it is necessary to know the elevation of the water surface at any given time during the sounding work; for this reason it is customary to employ a tide-gauge reader. In tidal waters, the surface of the water at mean low tide is usually taken as the

plane of reference, and the tide gauge should be set with the zero mark at the elevation of mean low tide. In such cases tide-gauge readings should be taken and recorded every 5 minutes during sounding operations, as has been explained. In lakes or reservoirs, where the elevation of the water surface changes but little and very slowly, the lowest recorded stage of the water is usually selected as the plane of reference. In such cases it is customary to take gauge readings twice a day—once in the morning and once in the afternoon during the period of sounding work. In rivers of variable stage, the plane of reference is usually the low-water stage of the river at the locality where the sounding work is done. In some cases, the general datum of the survey is used as a plane of reference. If the river is rising or falling rapidly while the soundings are being made, gauge heights should be read and recorded at intervals of 30 minutes, or even oftener if necessary. If, however, the river is at its normal stage during this period, and is changing but slowly, gauge readings should be taken twice a day, as for lakes or reservoirs.

FORMS FOR SOUNDING NOTES

46. Sounding Book.—For keeping the field notes of sounding work, three forms of field books are used; these are called, respectively, **sounding book**, **tide book**, and **angle book**. The sounding book, the form for which is shown in Form 1, is used by the recorder in the sounding boat. The first three columns contain, respectively, the number, the time, and the observed depth for each sounding; this information is obtained and entered by the recorder while in the sounding boat. The next two columns contain the reduction for tide and the reduced depth for each sounding; these are filled out in the office from the data obtained from the tide book, if the soundings are made in tidal waters. If the soundings are not made in tidal waters, no reductions are needed and these two columns are left blank. In the column headed Remarks should be entered the information obtained by the leadsman relative to the

character of the bottom, and also such information about the sounding ranges, the intervals between successive soundings, etc., as may be desirable. If a lead line is used in making the soundings, any errors in its length should be noted in this column, in order that proper corrections may be made to the observed depths.

FORM 1—SOUNDING BOOK

Soundings off Cape Charles
June 10, 1903

Johnson, Recorder
Kennedy, Leadsman

No.	Time	Soundings Feet	Reduction for Tide	Reduced Soundings Feet	Remarks
1	10:30	4.2			Range 13.
2		5.2			Soundings num-
3	10:31	8.2			bered west from
4		7.2			shore and made at
5	10:32	3.7			$\frac{1}{2}$-minute intervals.
6		3.3			Flag dropped at
7	10:33	3.8			1-minute intervals.
8		4.3			Bottom from sound-
9	10:34	2.8			ing No. 1 to 5, sand;
10		3.3			No. 6 to 8, shells.

47. Tide Book.—The tide book, shown in Form 2, is used by the tide-gauge reader; it contains the readings of the tide gauge at regular intervals of time and the time that each reading is taken. The direction and force of the wind also are usually noted and entered. From these notes, the proper reduction for tide can be obtained for each sounding, and the soundings can all be reduced to the plane of reference.

48. Angle Books.—Forms 3 and 4 show the form for the angle book that is used by the transit or sextant observer to record the angular measurements made in locating the

soundings. In the field he enters in the angle book the time and the observed angle for each sounding to be located. These are entered in the second and third columns, respectively, the first column being left blank until the observer obtains from the sounding book the numbers corresponding to the times of the observed soundings. Each observer is provided with an angle book in which he enters the field notes in the manner described. When the soundings are

FORM 2—TIDE BOOK

Observations of Tides at Cape Charles Gauge
June 10, 1903 J. Mason, Observer

Mean Time of Observation		Reading of Staff Gauge	Wind		Remarks
Hours	Min.	Feet	Direction	Force	
10	30	1.2	N W	Moderate	Gauge fastened to
10	35	1.3			pile at S. E. corner
10	40	1.4			of lighthouse wharf.
10	45	1.5			Tide rising.
10	50	1.6	W		Zero of gauge at
10	55	1.7			mean low water.
11	00	1.8			

located by time intervals and no angular measurements are made, the sounding book constitutes a complete office record of the soundings after the tide-gauge readings are obtained from the tide book and the reductions for the soundings have been entered.

49. Office Record.—When soundings are located by transit or sextant angles, a complete office record is obtained by combining the field notes in the manner shown in Form 5. The notes there shown are those given in Forms 1, 2, 3, and 4.

Form 3—Angle Book

Survey of Channel off Cape Charles, June 10, 1903 Observer No. 1, R. Briggs Young & Sons' Transit No. 1612					
No.	Time	Angle	Object Observed	Station Occupied	Remarks
1	10:30	41° 18′	Signal flag on launch	Transit Sta. A, at S.	Instrument set with vernier at
3	10:31	45° 00′		end of base line	zero when telescope points
5	10:32	49° 00′		on Cape Charles.	to Sta. B. Angles read to
7	10:33	54° 33′			the right from A–B. Sound-
9	10:34	61° 05′			ings on range 13, beginning
					at shore and running west.

Form 4—Angle Book

Survey of Channel off Cape Charles, June 10, 1903 Observer No. 2, J. Smith, Buff & Berger Transit No. 2840					
No.	Time	Angle	Object Observed	Station Occupied	Remarks
1	10:30	73° 05′	Signal flag on launch	Sta. B on E. side of	Vernier at zero when telescope
3	10:31	64° 42′		entrance to bay,	points to Sta. A. Angles
5	10:32	57° 15′		opposite light-	read to left. Soundings on
7	10:33	49° 27′		house.	range 13, beginning near
9	10:34	43° 05′			shore and running west.

FORM 5—OFFICE RECORD

Survey of Channel off Cape Charles, June 10, 1903 C. F. Johnson, Recorder T. Kennedy, Leadsman						Observer No. 1, Briggs, on Sta. A, zeros on Sta. B Observer No. 2, Smith, on Sta. B, zeros on Sta. A		
No.	Time	Soundings Feet	Reduced for Tide	Reduced Soundings Feet	Character of Bottom	Angles and Ranges		Remarks
						No. 1, Range 13	No. 2, Range 13	
1	10:30	4.2	1.2	3	Sand	$41^\circ 18'$	$73^\circ 05'$	
2		5.2	1.2	4				
3	10:31	8.2	1.2	7		$45^\circ 00'$	$64^\circ 42'$	
4		7.2	1.2	6				
5	10:32	3.7	1.2	$2\frac{1}{2}$	Sand	$49^\circ 00'$	$57^\circ 15'$	
6		3.3	1.3	2	Shells			
7	10:33	3.8	1.3	$2\frac{1}{2}$		$54^\circ 33'$	$49^\circ 27'$	
8		4.3	1.3	3	Shells			
9	10:34	2.8	1.3	$1\frac{1}{2}$		$61^\circ 05'$	$43^\circ 05'$	
10		3.3	1.3	2				

FORM 6

Survey of Midland Canal May 2, 1901				Johnson, Inst. Williams, Rod. — Jones, Taylor, Assistants — Gurley Level No. 2916
Sta.	Eleva.	Elevation of Water Surf	Remarks	Soundings
128	52.0	50.0	The water surface is used as the plane of reference.	$\frac{0.0}{2.0}$ $\frac{3.0}{5.0}$ $\frac{6.0}{10.0}$ $\frac{6.5}{13.0}$ $\frac{6.8}{18.0}$ $\frac{6.2}{28.0}$ $\frac{7.4}{38.0}$ $\frac{4.0}{44.0}$ $\frac{0.0}{48.0}$
129	52.5	50.0	Numerator equals the depth, in feet. Denominator equals the dist. from sta.	$\frac{0.9}{3.0}$ $\frac{3.0}{6.0}$ $\frac{6.5}{12.0}$ $\frac{7.0}{15.0}$ $\frac{7.4}{20.0}$ $\frac{6.3}{30.0}$ $\frac{7.0}{39.0}$ $\frac{3.0}{46.0}$ $\frac{0.0}{49.0}$
			Distances measured from left to right.	

50. Form 6.—When soundings are located by means of a wire or rope stretched across the stream, the field notes may be kept in the manner shown in Form 6. The notes there given represent sounding measurements across a canal at Stations 128 and 129 of the shore survey or traverse along the canal bank. The left-hand page of the notebook contains the number of the shore station in the first column, the ground elevation in the second column, and the elevation of the water surface in the third column. In the column headed Remarks is given all necessary information concerning the details of measurements, stage of water, etc. On the right-hand page of the notebook are given the sounding measurements. These are expressed in the form of fractions, the numerator designating the depth, in feet, and the denominator the distance from the shore station for each sounding. Thus, the fraction $\frac{6.0}{10.0}$ represents a depth of 6.0 feet at a distance of 10.0 feet from the shore station. The notes given in Form 6, when platted to scale on cross-section paper, show the cross-section of the canal for each station for which sounding notes are given.

PLATTING THE SOUNDINGS

51. General Methods.—Soundings are platted in various ways, according to the methods by which they are located. When located by ranges or courses, they are platted as follows: Each range or line on which soundings have been made is first platted to scale, in pencil, in the proper position on the map or chart. Then, the distances between the soundings on that range and the distance of each sounding from the end of the range being known, these distances are scaled on the pencil line and the position of each sounding as thus located is marked by a dot.

When the soundings are located by means of two transits on shore, as described in the third method, the base line is platted to scale in its proper position on the map and from each of its ends pencil lines are drawn making angles with the base equal to the angles measured in locating the soundings. The lines thus drawn represent the lines of

sight from the ends of the base to the positions of the soundings, and their directions are determined by the measured angles given in the notes. The intersection of corresponding lines drawn from the opposite ends of the base, representing the lines of sight of two simultaneous observations to a given sounding, will locate the sounding on the map.

In Fig. 33, *A B* represents a given base line as platted. The dotted lines *A–1*, *A–2*, *A–3*, and *B–1*, *B–2*, *B–3*, represent the directions of lines of sight to the soundings *1*, *2*, *3*. These lines are laid off at the observed angles as recorded in the notes. The intersections of the corresponding lines drawn from opposite ends of the base line give the locations of the respective soundings.

Soundings that have been located by this method can also be platted in the following manner: The distances from

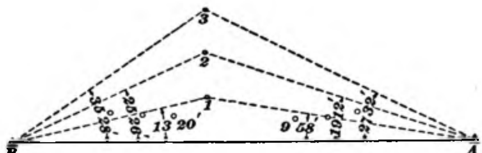
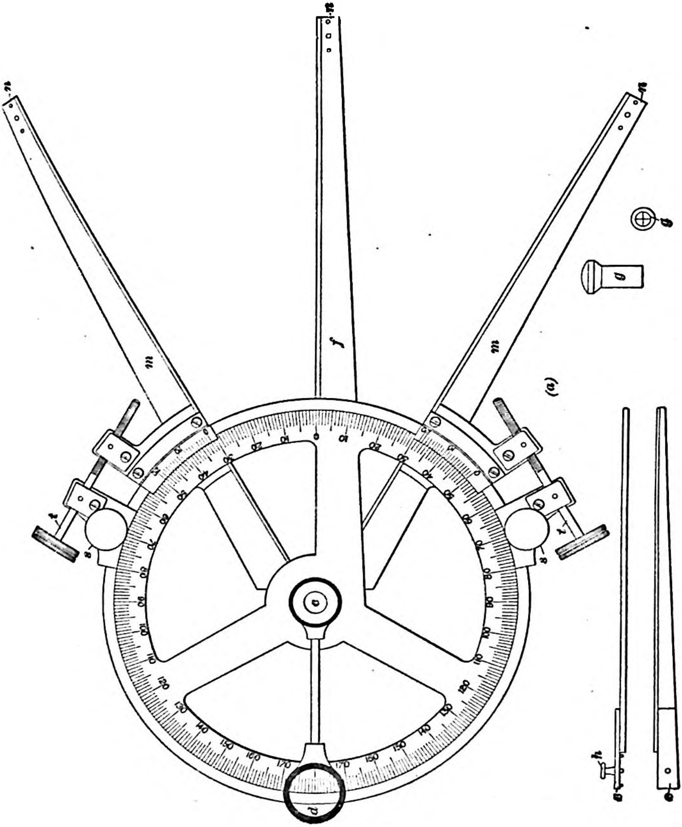


Fig. 33

each sounding to the respective extremities of the base line are calculated from the field notes. Then, the base line having been platted to scale in its proper position on the map, from its extremities as centers and with radii respectively equal to the two corresponding distances calculated for a given sounding, the arcs of two circles are drawn lightly in pencil with a pair of compasses. The intersection of the two arcs is the location of the sounding.

This method of platting is not nearly so expeditious as the method by the intersection of straight lines drawn from the instrument points, since the former method involves the calculation of two distances for each sounding, whereas the latter requires no calculation. It is sometimes valuable as a check, however, or to apply as a test in case of doubt regarding the position of a sounding.



(a) (b)

Fig. 34

52. The Three-Arm Protractor.—If the soundings have been located by sextant observations, as described in the fourth method, the most convenient way to plat them is by the use of what is called a **three-arm protractor**.

This instrument, which is sometimes called a **station pointer**, consists of a graduated metal circle, to which is attached a fixed arm *f*, Fig. 34, and two movable arms *m*, *m*. The movable arms revolve around a central point, which is the center of the graduated circle. One edge of the fixed arm and the inner edges of the movable arms extend outwards radially in line with the central point, and are beveled, as shown. The circle is divided into 360 degrees, with the zero point opposite the beveled edge of the fixed arm, which is also known as the **zero arm**, so that the prolongation of this beveled edge passes through the zero mark on the circle and also through the center *c* of the instrument. Each movable arm is provided with a vernier, as shown in the figure, and also with a clamp screw *s* and tangent screw *t*. A magnifying glass *d* is pivoted and hinged to the center of the circle and swings parallel to the graduations.

Each instrument is usually provided with three interchangeable centers, which are cylindrical in form, as shown at *g*. Each center fits snugly into a cylindrical opening *c* in the center of the instrument. One center has a glass bottom, with two etched lines intersecting at the central point; another center has a transparent horn bottom with a small hole at the central point, through which a pencil point can be inserted; and the third center is provided with a spring needle point for pricking the central point into the drawing paper.

Three-arm protractors are made of several sizes. The graduated circle is usually from 5 to $6\frac{1}{4}$ inches in diameter, and the arms are from 15 to 18 inches in length. Extensions for lengthening the protractor arms are furnished with each instrument. Each extension, as shown at *e*, *e*, Fig. 34, is provided with a splice to which are attached three studs that fit into corresponding holes at the end *n* of the protractor arm. After fitting the studs in place, the extension is secured

to the protractor arm by tightening the thumbscrew *h*. The extensions are used when soundings are to be platted that are beyond the reach of the regular protractor arms.

Before using a three-arm protractor, it is a good plan to carefully test the alinement and centering of the arms. To do this, place the protractor on the drawing board and draw lines along the straight edges of the three arms, then remove the protractor and prolong the lines inwards, noticing whether the three lines intersect in a common point. The operation should be repeated several times with the arms in different positions. If the three lines intersect in a common point for all positions of the arms, they may be considered to be truly centered.

53. The three-arm protractor is used almost exclusively for the purpose of platting soundings that have been located by sextant angles from the sounding boat. The way of using it is as follows: The movable arms of the protractor are set at the marks on the graduated circle designating the two sextant angles for any given sounding, and are firmly clamped. The instrument is then placed on the chart in such a position that the beveled edges of the three arms will pass through the platted positions of the three fixed points. This is done by placing the instrument on the paper with the beveled edge of the fixed or zero arm passing through the middle point, and sliding it around on the paper until the beveled edges of the two clamped arms also pass through the two respective outside points. The center of the instrument will then represent the position of the sounding. This point is marked by a pencil dot if a horn center is used, or pricked on the chart if a needle-point center is in the protractor at the time.

54. The Tracing-Paper Method.—When no three-arm protractor is available, soundings that have been located by sextant observations can be platted by means of a piece of tracing paper. Three lines are drawn on tracing paper in such positions as to intersect at a common point and include the two angles measured for any given sounding, the middle

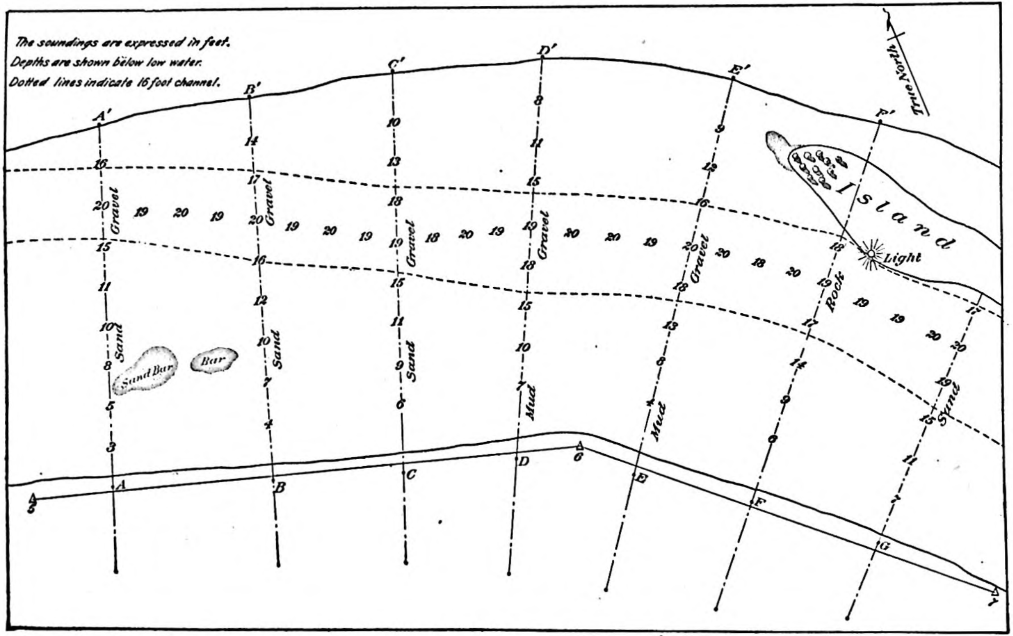
line forming a side of each angle. Then, to locate the sounding, the tracing paper is placed on the map in such a position that the three lines will pass through the platted positions of the three fixed points. The intersection of the three lines will then be the position of the sounding, which is pricked through the tracing on the map or chart.

HYDROGRAPHIC MAPS AND CHARTS

55. Maps or charts of hydrographic surveys should be drawn in accordance with the principles stated in *Mapping*, Parts 1 and 2. An outline map of a river or lake survey should show the lines and angles of the outline survey, and the triangulation stations, if any. It should also show the shore line and such details of the adjacent topography as may be considered necessary.

A complete hydrographic map of a river, lake, or reservoir should show, in addition to the outline of the water surface and the adjacent topography, the form or contour of the river bed or of the submerged portion of the containing valley or basin. In order to do this effectively, lines of equal depth should be drawn; these lines show the contours of the submerged area and correspond to contour lines on a topographical map. They are located and drawn on a hydrographic map in the following manner: The soundings are platted, the position of each sounding is indicated on the map by a small dot, and the depth of each sounding is written directly over its location on the map. The lines of equal depth or the subaqueous contours are then located and drawn according to the method described in *Mapping*, Part 2, for platting contours. The contour interval will vary according to the importance of the survey, the frequency of the soundings, and the object for which the survey is made.

56. Navigation Charts.—A navigation chart of a river, lake, harbor, or other navigable body of water should show, in addition to the shore line and the adjacent



The soundings are expressed in feet.
Depths are shown below low water.
Dotted lines indicate 16 foot channel.
True North
Island
Light
Sand Bar
Bar
Gravel
Sand
Mud
Rock

FIG. 35

topography, the position of the navigable channel and of all rocks, sand bars, reefs, sunken wrecks, or other obstacles to navigation. In a chart of a navigable river, it is customary to show both banks and such of the adjacent topography as is desired; also the positions of islands and of such obstacles to navigation as may exist. Contour lines are drawn showing the navigable channel and the outlines of sand bars and shoals. The depths of other parts of the stream or body of water are written in the positions where the soundings are made. The depths are usually expressed in feet, and are always so expressed in shallow water. In some cases fractional half feet are used, but not smaller fractions. Soundings in the sea where the depths exceed 18 feet are usually expressed in fathoms, though there is no uniform practice with regard to the depth.

Fig. 35 represents a chart of a portion of a navigable river. The line of the survey is drawn in a light full line, and the angle stations are designated by numbers. The depths of soundings, in feet, are written in figures at the places where the soundings were made, and the character of the bottom is written under the figures expressing the depths. The limits of the navigable channel, which has a minimum depth of 16 feet, are shown by the dotted lines. The ranges on which the soundings were made are shown in the figure for purposes of illustration, but this is not customary in practice. In the field work of locating the soundings, the sounding ranges were laid out across the axis of the stream and range signals were established on both banks to fix the position of each range. The soundings were located by angles measured with a transit on shore, as described in Art. 38. The boat containing the sounding party started on the south side of the river and moved along range *A* to the north bank; the soundings were made at the required intervals as the boat progressed. After traversing range *A*, the boat proceeded to the north end of range *B*, and then moved southwards along that range while soundings were taken at regular intervals, until the depth of sounding indicated that the deepest part of the channel had been reached. Then, in

order to sound the deepest part of the channel, the boat was headed in a direction approximately at right angles to the range *B*, and propelled at a uniform speed toward the range *A*, soundings being made at regular intervals of time until the boat reached that range. The time taken to traverse the distance between the two ranges was noted, as was also the number of intermediate soundings and the time interval between soundings. Then the distance between the two ranges, the time taken by the boat to traverse this distance, the number of intermediate soundings, and the time interval between successive soundings all being known, the locations of the intermediate soundings were interpolated in the manner described in Art. **36.**

After the channel soundings between ranges *A* and *B* were made, the boat returned to range *B*, and sounding work on that range was resumed from the place indicated by signal from the transitman on shore. After range *B* was sounded, the boat proceeded to range *C* and moved northward along that range to the deepest part of the channel, then along the deep channel westwards to range *B*, soundings being made at regular intervals in the manner just described. The boat then returned to range *C* and proceeded northwards until the entire range had been sounded. It then proceeded to range *D* and sounded southwards along that range to the south bank, and so on back and forth across the river until all the ranges were sounded. Successive ranges were traversed in opposite directions and side trips were made for the channel soundings between each two adjacent ranges. For locating soundings made on ranges *A* and *B*, the observer was at the instrument point designated as Station 5, with the vernier of his transit at zero when the telescope was directed toward Station 6. After range *B* was sounded the transit was moved to Station 6 and set up over that station, from which soundings made on the remaining ranges were located. For locating soundings on ranges *C* and *D*, the vernier was set at zero when the telescope pointed to Station 5; and for locating soundings on ranges *E* and *F*, the vernier was set at zero when the telescope pointed to Station 7.

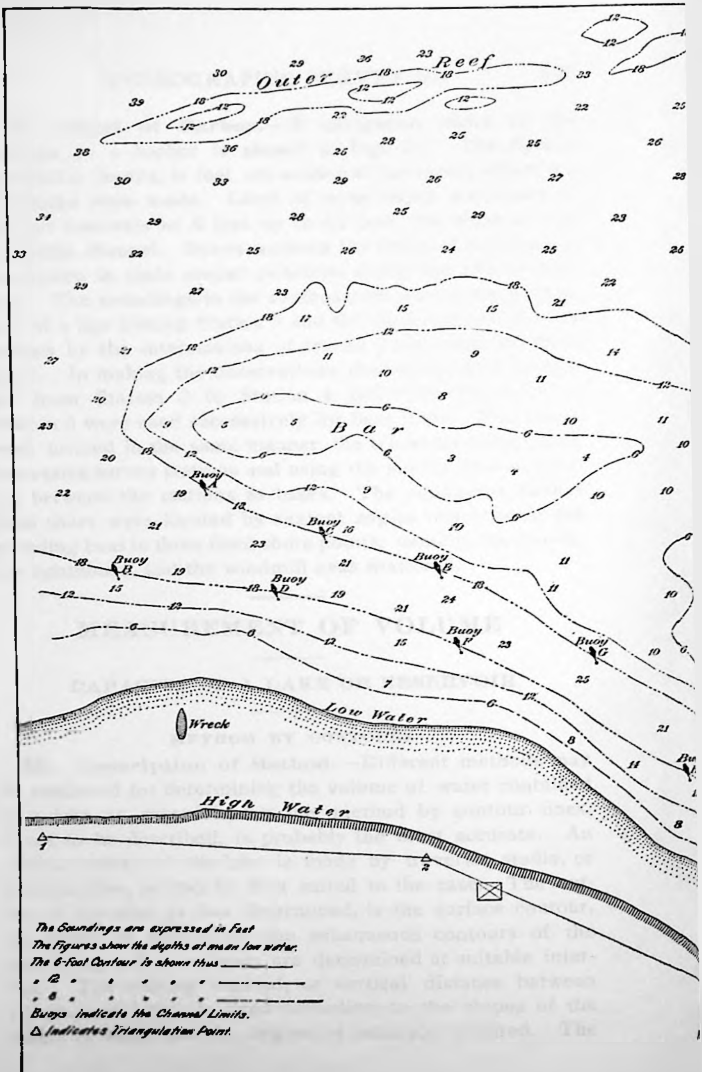
57. Chart of Harbor.—A navigation chart of the entrance to a harbor is shown in Fig. 36. The figures expressing depths, in feet, are written at the places where the soundings were made. Lines of equal depth are drawn at vertical intervals of 6 feet up to 18 feet, the depth of the navigable channel. Buoys marking the limits of the channel are shown in their proper positions along the 18-foot contour. The soundings in the southeastern part of the harbor, east of a line joining Station 3 and the buoy marked *H* were located by the intersections of transit lines observed from shore. In making the observations, the survey lines extending from Station 3 to Station 4 and from Station 4 to Station 5 were used successively for base lines. The buoys were located in the same manner, the transitmen occupying successive survey stations and using the survey lines extending between the stations as bases. The soundings farther from shore were located by sextant angles observed in the sounding boat to three fixed shore points; namely, the church, the lighthouse, and the windmill near Station 3.

MEASUREMENT OF VOLUME

CAPACITY OF A LAKE OR RESERVOIR

METHOD BY CONTOURS

58. Description of Method.—Different methods may be employed for determining the volume of water contained in a lake or reservoir, but the method by contour lines, about to be described, is probably the most accurate. An outline survey of the lake is made by traverse, stadia, or triangulation, as may be best suited to the case. The outline of the lake, as thus determined, is the surface contour. By means of soundings, the subaqueous contours of the containing valley or basin are determined at suitable intervals. The contour interval, or vertical distance between adjacent contours, is fixed according to the slopes of the valley or basin and the degree of accuracy required. The


Outer Reef
Bar
Buoy A
Buoy B
Buoy C
Buoy D
Buoy E
Buoy F
Buoy G
Wreck
Low Water
High Water
The Soundings are expressed in Feet
The Figures show the depths at mean low water.
The 6-Foot Contour is shown thus
Buoys indicate the Channel Limits.
△ Indicates Triangulation Point.


Point of Rock
True North
Pinnacle Rock
Church
Low Water
High Water
Wharf
Buoy
Buoy
Lighthouse

§ 20 20933

. 36

notes thus obtained are then platted, and a map is made showing the outline of the water surface and the several contour lines.

The solid figure included between any two adjacent contours will resemble a prismoid, whose parallel end surfaces are the surfaces enclosed by the respective contour lines, and whose perpendicular length or height is the contour interval. The area of the water surface and the area enclosed by each contour line can be determined from the plat by any of the methods for finding the areas of irregular figures described in *Trigonometry*, Part 2. When the areas enclosed by the various contours, which form the end areas of the several prismoids, are known, the volume of each prismoid can be found approximately by multiplying one-half the sum of its end areas by its height. The sum of the volumes of the several prismoids will be the volume of water in the lake. This is what is known as the **end-area method** of calculating volumes.

59. Volume by End-Area Method.—Suppose that Fig. 37 represents the plat of a lake whose capacity is

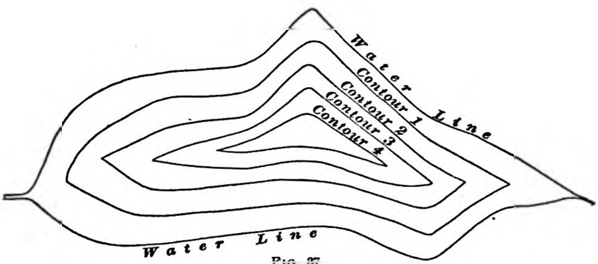


FIG. 37

required. Let A_0, A_1, A_2, A_3, and A_4 denote the areas bounded by the water-line and by the contours *1*, *2*, *3*, and *4*, respectively; let v_{0-1}, v_{1-2}, v_{2-3}, and v_{3-4} denote, respectively, the volumes of the prismoids included between the water surface and the first contour, and between the successive contours; also, let h denote the contour interval, and V the

total volume. By the method of average end areas, the approximate volumes of the several prismoids are

$$v_{0-1} = \frac{A_0 + A_1}{2} \times h$$

$$v_{1-2} = \frac{A_1 + A_2}{2} \times h$$

$$v_{2-3} = \frac{A_2 + A_3}{2} \times h$$

$$v_{3-4} = \frac{A_3 + A_4}{2} \times h$$

The total volume of the lake or reservoir is equal to the sum of the volumes of the several prismoids as expressed by the preceding equations, or

$$V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4} = h\left(\frac{A_0}{2} + A_1 + A_2 + A_3 + \frac{A_4}{2}\right)$$

In order to express this as a general formula applicable to any number of contours, it may be written in the form

$$V = h\left(\frac{A_0}{2} + \Sigma A_m + \frac{A_n}{2}\right)$$

In this formula,

A_0 = area included by surface contour;
A_n = area included by lowest contour;
ΣA_m = sum of areas included by the intermediate contours.

EXAMPLE.—Suppose that in Fig. 37 the contour interval is 5 feet and that the areas enclosed by the several contours are as follows: A_0 = 13,350 square feet, A_1 = 8,100 square feet, A_2 = 4,280 square feet, A_3 = 1,925 square feet, and A_4 = 520 square feet; find the volume of water in the lake, in cubic feet, by the end-area method.

SOLUTION.—By substituting the given values in the formula, the volume of water is found to be

$$V = 5 \times \left(\frac{13,350}{2} + 8,100 + 4,280 + 1,925 + \frac{520}{2}\right) = 106,200 \text{ cu. ft. Ans.}$$

60. Volume by Prismoidal Formula.—If the volumes of the prismoids are calculated by the prismoidal formula, two adjacent prismoids are taken as one prismoid whose height is equal to twice the contour interval. The area included by the contour that lies between the two prismoids

taken is considered the middle area of the prismoid and so used in the formula. By thus combining the first two prismoids of Fig. 37 and applying the prismoidal formula given in *Geometry*, Part 2, the expression for their volume is

$$v_{0-1} + v_{1-2} = \frac{2h}{6}(A_0 + 4A_1 + A_2)$$

The volume of the last two prismoids is

$$v_{2-3} + v_{3-4} = \frac{2h}{6}(A_2 + 4A_3 + A_4)$$

By adding these two expressions, the total volume of the lake is found to be

$$V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4}$$
$$= \frac{h}{3}(A_0 + 4A_1 + 2A_2 + 4A_3 + A_4)$$

In order to express this as a general formula applicable to any number of contours, it may be written in the form

$$V = \frac{h}{3}(A_0 + 4\Sigma A_1 + 2\Sigma A_2 + A_n)$$

In this formula,

A_0 = area included by surface contour;
A_n = area included by lowest contour;
ΣA_1 = sum of areas included by intermediate contours whose subscripts are odd numbers;
ΣA_2 = sum of areas included by intermediate contours whose subscripts are even numbers.

EXAMPLE.—Suppose that in Fig. 37 all values are the same as in the example solved in Art. 59; namely, h = 5 feet, A_0 = 13,350 square feet, A_1 = 8,100 square feet, A_2 = 4,280 square feet, A_3 = 1,925 square feet, and A_4 = 520 square feet; what is the volume of water in the lake, in cubic feet, as determined by the prismoidal formula?

SOLUTION.—By substituting the given values in the formula,

$$V = \frac{5}{3}(13{,}350 + 4 \times 8{,}100 + 2 \times 4{,}280 + 4 \times 1{,}925 + 520)$$
$$= 104{,}217 \text{ cu. ft. Ans.}$$

61. Construction for Interpolating Contour.—It is evident that the prismoidal formula can be applied to the prismoids in pairs, as just described, only when there is an even number of prismoids. When there is an odd number

of prismoids, the last prismoid may be computed separately by the method of average end areas, or by interpolating a middle contour on the contour map, calculating the area included by it, and then applying the prismoidal formula. The middle contour can be interpolated as follows: The two end contours are platted to the same scale, preferably on cross-section paper, the smaller inside the larger, making the two figures concentric as nearly as practicable. A third contour is then drawn in such position that each point will be midway between the corresponding points of the inner and outer figures. This interpolated contour can be sketched in largely by the eye, aided to such an extent as may be desired by measurements on lines drawn radially from a point approximately in the center of the figure. The area of the surface included by this interpolated contour may be taken as the middle area of the prismoid. This area can be easily determined by means of the cross-section paper or by the planimeter.

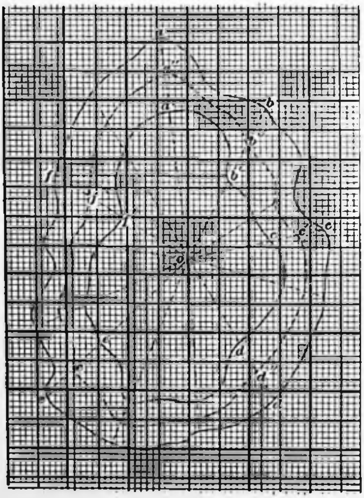


FIG. 38

Thus, in Fig. 38, $a\,b\,c\,d\,e\,f$ and $a'\,b'\,c'\,d'\,e'\,f'$ represent the contours including, respectively, the upper and the lower base of a prismoid, as platted on cross-section paper. The contour represented by the dotted line $a''\,b''\,c''\,d''\,e''\,f''$ lies midway between the boundaries of the two bases. This contour

is constructed or interpolated in the following manner: The point o is chosen as the center and the radial lines $o\,a$, $o\,b$, $o\,c$, etc. drawn to the outer contour; a', b', c', etc. are the points where these lines cross the inner contour. By measurement, the point a'' is located on the radial line $o\,a$ midway between a and a'; in like manner, b'' is located midway between b and b'; c'' is located midway between c and c', etc. The contour is then sketched through the points a'', b'', c'', d'', e'', and f'', as represented by the dotted line. The area of the surface enclosed by this interpolated contour is then determined by counting the squares of the cross-section paper, and is taken as the middle area of the prismoid.

EXAMPLES FOR PRACTICE

1. Suppose that the areas bounded by the water-line of a lake and by contours 1, 2, 3, 4, and 5 are as follows: A_0 = 15,450 square feet, A_1 = 10,240 square feet, A_2 = 8,360 square feet, A_3 = 7,730 square feet, A_4 = 6,890 square feet, and A_5 = 5,240 square feet. If the contour interval is 10 feet, calculate the volume of water in the lake, in cubic feet, by the end-area method. Ans. 435,650 cu. ft.

2. Suppose that the areas bounded by the water-line of a lake and by contours 1, 2, 3, 4, 5, and 6 are as follows: A_0 = 14,320 square feet, A_1 = 10,280 square feet, A_2 = 9,360 square feet, A_3 = 7,480 square feet, A_4 = 5,780 square feet, A_5 = 4,760 square feet, and A_6 = 3,250 square feet. If the contour interval is 5 feet, calculate the volume of water in the lake by the prismoidal formula. Ans. 229,880 cu. ft.

METHOD BY PARALLEL CROSS-SECTIONS

62. Description of Method.—The following is also a good method for determining approximately the capacity of a lake or reservoir: A survey is made to determine the outline of the water surface, which is platted accurately to scale. Then, at selected points, parallel ranges are laid out across the lake, dividing its surface into trapezoids, as illustrated in Fig. 39. If the shores of the lake are irregular, the ranges are so located that straight lines connecting the points where the adjacent ranges intersect the shore line will be as much inside as outside of the water-line. By the

aid of the plat this can usually be done with a reasonable degree of accuracy. The small irregular areas included between the straight line and the water-line will then approximately balance, and it will be sufficiently accurate to consider the lake boundary as straight between adjacent ranges. The ranges having been located, soundings are made along them and the cross-section of the lake is determined on each range. The cross-sections are platted, as shown in Fig. 39, and the area of each cross-section is computed.

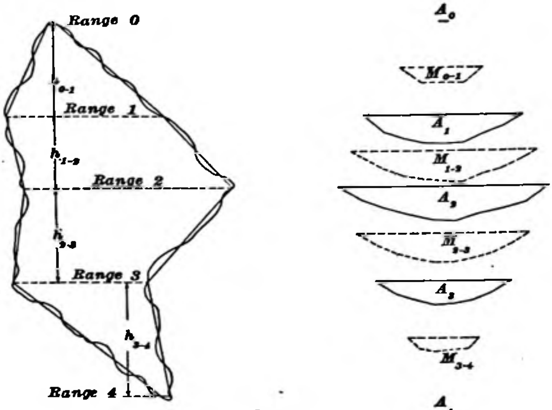


FIG. 39

The lake basin is thus divided into prismoids, whose bases are the cross-sections, and whose altitudes are the perpendicular distances between adjacent ranges, and the capacity of the lake is equal to the sum of the volumes of the prismoids.

63. Volume by End-Area Method.—The approximate capacity of the lake or reservoir can be calculated by the method of average end areas in the following manner: Let A_0, A_1, A_2, A_3, and A_4 denote, respectively, the areas of the cross-sections on the parallel ranges designated in Fig. 39 as *Range 0*, *Range 1*, *Range 2*, *Range 3*, and *Range 4*, respectively. Also, let h_{0-1}, h_{1-2}, h_{2-3}, and h_{3-4} denote, respectively,

the perpendicular distances between the adjacent parallel ranges *1*, *2*, *3*, and *4*, as shown in the figure, and let v_{0-1}, v_{1-2}, v_{2-3}, and v_{3-4} denote the volumes of the corresponding prismoids. The end range *0* is merely a short straight line that represents the end of the lake and corresponds somewhat to the cutting edge of a wedge in the prismoid included between it and the cross-section on the adjacent range *1*. The same is true of the end range *4* with respect to the prismoid included between it and the cross-section on range *3*. The cross-sections on *0* and *4* thus consist of a straight line merely, and each of the areas A_0 and A_4 is zero. These areas should therefore be taken at zero in computing the volumes of the two end prismoids. By the end-area method, the expressions for the volumes of the several prismoids are

$$v_{0-1} = \frac{A_0 + A_1}{2} \times h_{0-1}$$

$$v_{1-2} = \frac{A_1 + A_2}{2} \times h_{1-2}$$

$$v_{2-3} = \frac{A_2 + A_3}{2} \times h_{2-3}$$

$$v_{3-4} = \frac{A_3 + A_4}{2} \times h_{3-4}$$

and the total volume of the lake is

$$V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4} = \tfrac{1}{2}\left[(A_0 + A_1)h_{0-1} + (A_1 + A_2)h_{1-2} + (A_2 + A_3)h_{2-3} + (A_3 + A_4)h_{3-4}\right]$$

This formula applies to Fig. 39 or to any lake or reservoir for which the measurements are made on five parallel ranges; its application to such a case will be clearly understood. In order to make the formula applicable to measurements made on any number of ranges, it may be written in the form

$$V = \tfrac{1}{2}\left[(A_0 + A_1)h_{0-1} + (A_1 + A_2)h_{1-2} \ldots (A_m + A_n)h_{m-n}\right]$$

In this formula, A_n denotes the area of the cross-section on the last range, and A_m that on the next to the last range.

EXAMPLE.—Suppose that the areas of the several cross-sections of the lake shown in Fig. 39, as measured on the ranges, are: $A_0 = 0$,

A_1 = 4,256 square feet, A_2 = 6,322 square feet, A_3 = 3,130 square feet, and $A_4 = 0$; also, that the perpendicular distances between ranges are: h_{0-1} = 250 feet, h_{1-2} = 192 feet, h_{2-3} = 256 feet, and h_{3-4} = 310 feet; what is the capacity of the lake, in cubic feet, as calculated by the end-area method?

SOLUTION.—By substituting the given values in the formula, the operations, in detail, are as follows:

$$
\begin{array}{lr}
(A_0 + A_1)h_{0-1} = (0 + 4{,}256) \times 250 \;.\;.\; = & 1\,0\,6\,4\,0\,0\,0 \\
(A_1 + A_2)h_{1-2} = (4{,}256 + 6{,}322) \times 192 = & 2\,0\,3\,0\,9\,7\,6 \\
(A_2 + A_3)h_{2-3} = (6{,}322 + 3{,}130) \times 256 = & 2\,4\,1\,9\,7\,1\,2 \\
(A_3 + A_4)h_{3-4} = (3{,}130 + 0) \times 310 \;.\;.\; = & \underline{9\,7\,0\,3\,0\,0} \\
 & 2)\underline{6\,4\,8\,4\,9\,8\,8} \\
V = & 3\,2\,4\,2\,4\,9\,4 \text{ cu. ft. Ans.}
\end{array}
$$

64. Interpolating Middle Cross-Section.—When the volume of a lake or reservoir is determined by measuring cross-sections on parallel ranges, the ranges cannot usually be located advantageously at uniform intervals, but must be located in such positions as will determine most accurately the form of the lake or reservoir. Consequently, two adjacent prismoids cannot be considered as one prismoid whose length is equal to the aggregate length of the two, and the intervening section considered as the middle section in applying the prismoidal formula, as in the preceding method. For, since the two prismoids are not of the same length, the intervening section is not the middle section. Hence, when the prismoidal formula is applied to this method, the middle area of each prismoid is determined by platting its two end cross-sections together on cross-section paper and interpolating a middle cross-section. The area of the interpolated cross-section is then determined and taken as the middle area of the prismoid. The method of interpolating the middle cross-section is similar to that for interpolating the middle contour explained in Art. **61**, but is even more simple.

Let $L\,M\,N$ and $O\,P\,Q$, Fig. 40, represent the bottom profiles of the measured cross-sections on two adjacent ranges. A third line $R\,S\,T$ is drawn in such position that each point is midway between corresponding points in the other two lines. In most cases, it will be sufficiently accurate to locate

points on the interpolated line midway between corresponding points that are located by soundings on the other two lines. Thus, the point S is located midway between M and P, which are points that have been located by soundings. The point R is located in the horizontal surface line midway between O and L, and the point T in the surface line midway between N and Q. The surface lines of the three cross-sections coincide between L and N. The area of the interpolated cross-section can now be determined by counting the squares of the cross-section paper, and this is taken as the middle area of the prismoid whose end areas are the areas of the cross-sections LMN and OPQ. The middle area of the two end prismoids is determined in a similar manner. In this case the end cross-section is a straight line, and points

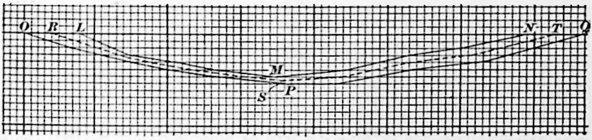


FIG. 40

on the interpolated line are located midway between the points that have been located by soundings and corresponding points on the straight line, at proportional distances from the ends of the line. When the volume of a lake or reservoir is determined by measuring cross-sections on parallel ranges and applying the prismoidal formula, the middle area of each prismoid is determined in the manner described.

65. Volume by Prismoidal Formula.—When the area of the middle cross-section has been determined, the volume of each prismoid can be determined by applying the prismoidal formula in the usual manner. The sum of the volumes of the several prismoids is the volume of water contained in the lake or reservoir.

In Fig. 39, let M_{0-1}, M_{1-2}, M_{2-3}, and M_{3-4} denote the middle areas of the prismoids whose altitudes are h_{0-1}, h_{1-2}, h_{2-3}, and h_{3-4}, respectively. Then, by applying the prismoidal

formula, the volumes of the several prismoids are found to be as follows:

$$v_{0-1} = \frac{h_{0-1}}{6}(A_0 + 4M_{0-1} + A_1)$$

$$v_{1-2} = \frac{h_{1-2}}{6}(A_1 + 4M_{1-2} + A_2)$$

$$v_{2-3} = \frac{h_{2-3}}{6}(A_2 + 4M_{2-3} + A_3)$$

$$v_{3-4} = \frac{h_{3-4}}{6}(A_3 + 4M_{3-4} + A_4)$$

Then, $V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4}$,

or $V = \frac{1}{6}[(A_0 + 4M_{0-1} + A_1)h_{0-1} + (A_1 + 4M_{1-2} + A_2)h_{1-2} + (A_2 + 4M_{2-3} + A_3)h_{2-3} + (A_3 + 4M_{3-4} + A_4)h_{3-4}]$

In order to express this as a general formula applicable to any number of cross-sections, it may be written in the form

$$V = \frac{1}{6}[(A_0 + 4M_{0-1} + A_1)h_{0-1} + (A_1 + 4M_{1-2} + A_2)h_{1-2} + \ldots (A_m + 4M_{m-n} + A_n)h_{m-n}]$$

In this formula,

A_n = area of last section;

A_m = area of next to last section;

M_{m-n} = area of middle section;

h_{m-n} = perpendicular distance between A_m and A_n.

EXAMPLE.—Suppose that all values are the same as in the example solved in Art. 63; namely, $A_0 = 0$, $A_1 = 4{,}256$ square feet, $A_2 = 6{,}322$ square feet, $A_3 = 3{,}130$ square feet, $A_4 = 0$, $h_{0-1} = 250$ feet, $h_{1-2} = 192$ feet, $h_{2-3} = 256$ feet, and $h_{3-4} = 310$ feet; and suppose, also, that the areas of the interpolated middle sections are: $M_{0-1} = 1{,}107$ square feet, $M_{1-2} = 5{,}498$ square feet, $M_{2-3} = 4{,}536$ square feet, and $M_{3-4} = 863$ square feet; what is the capacity of the lake, in cubic feet, as calculated by the prismoidal formula?

SOLUTION.—By substituting the given values in the formula, the operations, in detail, are as follows:

$(A_0 + 4M_{0-1} + A_1)h_{0-1} = (0 + 4 \times 1{,}107 + 4{,}256) \times 250$ . . . =	2171000
$(A_1 + 4M_{1-2} + A_2)h_{1-2} = (4{,}256 + 4 \times 5{,}498 + 6{,}322) \times 192$ . . . =	6253440
$(A_2 + 4M_{2-3} + A_3)h_{2-3} = (6{,}322 + 4 \times 4{,}536 + 3{,}130) \times 256$ . . . =	7064576
$(A_3 + 4M_{3-4} + A_4)h_{3-4} = (3{,}130 + 4 \times 863 + 0) \times 310$ . . . . . . =	2040420
	6)17529436
$V =$	2921573 cu. ft. Ans.

EXAMPLES FOR PRACTICE

1. In Fig. 41, which represents a lake, BC, DE, and FG are parallel ranges on which soundings have been taken. The depths of

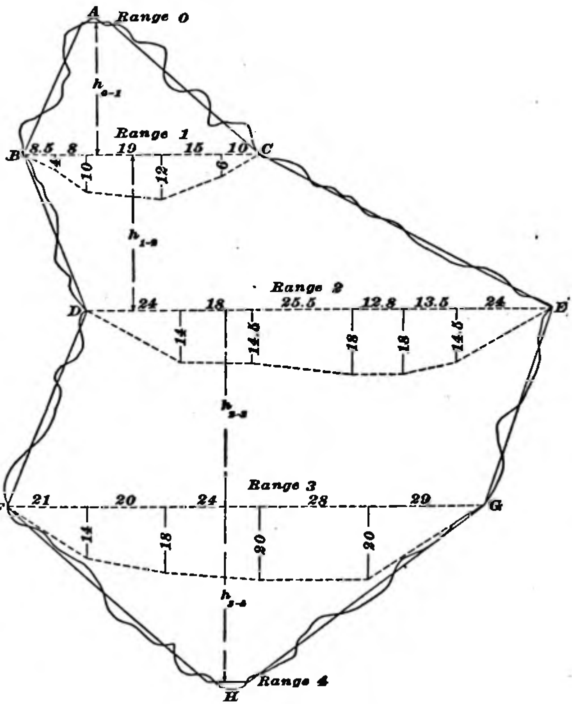


FIG. 41

the soundings and the distances between soundings are indicated in the figure. Using these values, calculate the areas A_1, A_{11}, and A_3 of the cross-sections on the ranges.

Ans. $\begin{cases} A_1 = 447.0 \text{ sq. ft.} \\ A_2 = 1{,}462.7 \text{ sq. ft.} \\ A_3 = 1{,}773.0 \text{ sq. ft.} \end{cases}$

2. If, in Fig. 41, the distances between adjacent ranges are $h_{0-1} = 35$ feet, $h_{1-2} = 42$ feet, $h_{2-3} = 54$ feet, and $h_{3-4} = 48$ feet, what is the capacity of the lake as determined by the method of average end areas? Ans. 177,840 cu. ft.

3. Calling Range *0*, 8 feet long and Range *4*, 10 feet long, plat the areas of the cross-sections in Fig. 41 on cross-section paper and determine the interpolated middle areas.

$$\text{Ans.} \begin{cases} M_{0-1} = 131.4 \text{ sq. ft.} \\ M_{1-2} = 856.3 \text{ sq. ft.} \\ M_{2-3} = 1{,}626.7 \text{ sq. ft.} \\ M_{3-4} = 487.3 \text{ sq. ft.} \end{cases}$$

4. As determined by the prismoidal formula, what is the capacity of the lake represented in Fig. 41? Ans. 160,480 cu. ft.

CAPACITY OF A VALLEY OR BASIN FOR WATER STORAGE

66. By Contours.—If close results are desired, it is best to make a complete topographical survey of the area to be flooded and construct a contour map of the area, employing the methods described in *Topographic Surveying*. The general method is as follows: The location of the dam for impounding the water having been selected, the elevation of the spillway or overflow is decided on; this determines the height of the water in the basin. The **spillway** is that part of the dam over which the waste water is allowed to flow, and is usually somewhat lower than the crest of the dam. The impounded water will rise to a height corresponding to the elevation of the spillway and will form a pond or lake whose boundary will be a contour line extending around the border of the basin. This line, whose position thus defines the limits of the area overflowed by the water, is called the **flow line**. In Fig. 42 the flow line, which is at the elevation of the spillway, is one contour interval lower than the crest of the dam.

After the survey has been made, the flow line, the successive contours, and the outline of the projected dam are platted, as shown in Fig. 42. The planes of the contours, including that of the flow line, will intersect the face of the dam in a series of horizontal lines, as shown. A line joining the ends of these horizontal lines on the inner face of the dam, at the points where they meet the sides of the valley,

will indicate the inner outline of the base of the dam, or the inner toe of the slope, as shown in Fig. 42. A similar line will indicate the outer toe of the slope. Only the contour lines and flow lines need be considered, however, in determining the capacity of the basin. The areas enclosed by the flow line and the several contour lines are either calculated or measured with a planimeter, and the capacity of the reservoir is determined by the method of average end areas or by the prismoidal formula, as explained in preceding articles.

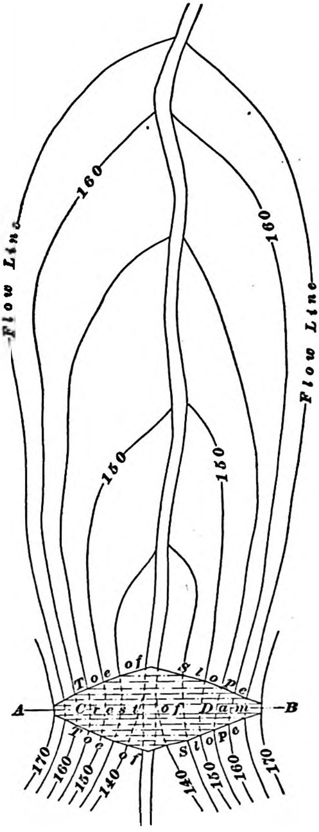


FIG. 42

EXAMPLE.—Suppose that in Fig. 42 the contour interval is 5 feet and that the areas enclosed by the several contours are as follows: $A_0 = 9{,}475$ square feet, $A_1 = 7{,}415$ square feet, $A_2 = 4{,}175$ square feet, $A_3 = 1{,}810$ square feet, and $A_4 = 685$ square feet; what is the capacity of the reservoir, in cubic feet, as determined: (*a*) by the end-area method? (*b*) by the prismoidal formula?

SOLUTION.—(*a*) Substituting the given values in the formula in Art. 59,

$$V = 5\left(\frac{9{,}475}{2} + 7{,}415 + 4{,}175 + 1{,}810 + \frac{685}{2}\right) = 92{,}400 \text{ cu. ft. Ans.}$$

(*b*) Substituting the given values in the formula in Art. 60,

$$V = \tfrac{5}{3}[9{,}475 + 4(7{,}415 + 1{,}810) + 2 \times 4{,}175 + 685] = 92{,}350 \text{ cu. ft.}$$

EXAMPLES FOR PRACTICE

1. From a survey made of a reservoir to determine its capacity, the areas enclosed by the flow line and the four successive contours are found to be as follows: $A_0 = 4{,}095$ square feet, $A_1 = 3{,}156$ square feet, $A_2 = 2{,}369$ square feet, $A_3 = 1{,}854$ square feet, and $A_4 = 1{,}044$ square feet; if the contour interval is 4 feet, what is the capacity of the reservoir as determined by the method of average end areas?
Ans. 39,794 cu. ft.

2. What is the capacity of the reservoir referred to in the preceding example, as determined by the prismoidal formula? Ans. 39,889 cu. ft.

---

67. By Parallel Cross-Sections.—The capacity of a valley or basin for water storage can also be determined approximately as follows: A site having been selected for a dam, the elevation of the spillway is fixed and a survey is made to determine the location of the flow line on the ground. A plat of the survey is made showing the flow line and the outline of the projected dam. Suitable locations are then selected for a series of parallel cross-lines joining points on the flow line, situated on opposite sides of the valley. These cross-lines are located in such positions as to divide the area enclosed by the flow line into trapezoids. The cross-lines are located in such positions that straight lines joining the ends of adjacent cross-lines will either coincide with the flow line or lie equally on both sides of it. This can usually be done easily by the aid of the plat even when the flow line is quite irregular.

The locations for the parallel cross-lines having been determined from the plat, the lines are located and measured on the ground, and levels are taken over them, thus determining the cross-section of the valley within the flow line on each of the parallel cross-lines. The profile of this cross-section is platted, preferably on cross-section paper, and a straight line is drawn joining the points where the profile intersects the flow line. This straight line is horizontal and corresponds to what will be the water surface when the basin or reservoir is full of water to the flow line, and the cross-section thus formed represents what will be the cross-section

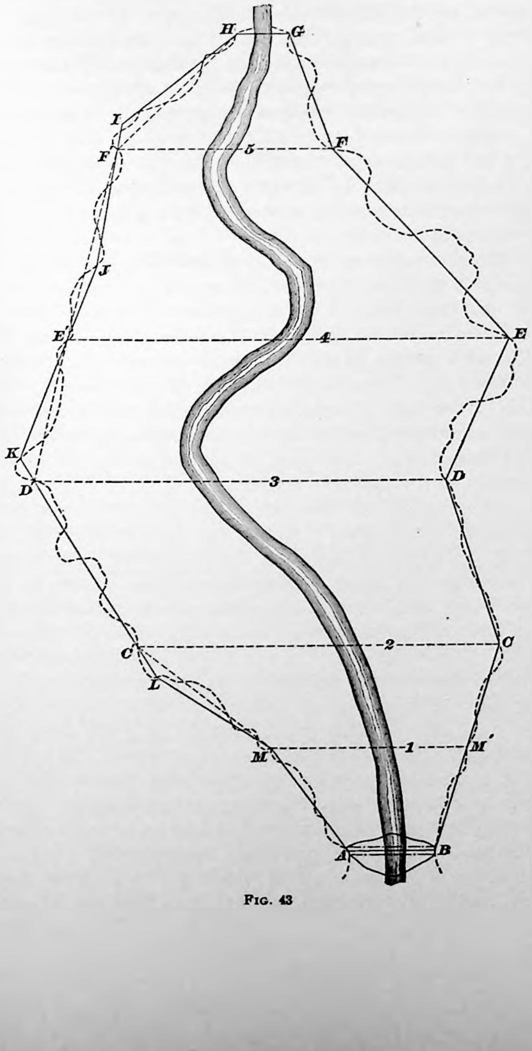


FIG. 43

of the water. The several cross-sections thus determined correspond to the cross-sections of a lake as obtained by soundings. The perpendicular distances between adjacent cross-sections are measured on the ground, calculated from the outline survey or scaled from the plat. The area of each cross-section is found, and the volume of each prismoid and total volume of the lake are determined by the end-area method as described in Art. **63**, or by the prismoidal formula, as in Art. **65**. In the latter case, the middle section may be interpolated, as described in Art. **64**, or measured on the ground.

Let Fig. 43 represent the plat of a closed survey around the limits of the reservoir, following the approximate position of the flow line. Such a position is obtained for the survey line by following along the side of the valley and locating each station of the survey at or near the elevation of the flow line. The irregular line is the flow line. AB is the axis of the dam, and the dotted lines *1*, *2*, *3*, etc. are the parallel cross-lines that divide the area enclosed by the flow line into trapezoids. One end of each cross-line is located at a station of the outline survey, as at C, D, E, F, or M, but the opposite end of the line will not usually fall at a station, but at some intermediate point on the line, as at C', D', E', F', or M'. If straight lines are drawn joining such ends of the cross-lines as lie at intermediate points on the survey line with the ends of the adjacent cross-lines, the lines so drawn and the corresponding parts of the survey line will form small triangles, some of which will lie inside, and some outside, of the original outline survey. Thus, the line $D'E'$ forms one side of the small triangle $D'E'K$; the line $E'F'$ forms one side of the small triangle $E'JF'$, etc. The effect of such triangles is usually small, however, and in most cases it is possible to locate the cross-lines in such positions that straight lines joining the corresponding ends of adjacent cross-lines will approximate the flow line sufficiently closely, and the small triangles formed may be neglected.

The lines BC, CD, DE, EF, and FG are sides of the survey, and it is seen from the plat that they cut off small

irregular areas that lie between the survey line and the flow line on both sides of the latter, and that these areas will approximately balance; that is, the areas lying on the outside of the survey line will be approximately equal to those lying inside of it. Similarly, if the points H and F', F' and E', E' and D', etc. are joined by straight lines, these lines will cut off approximately equal areas on both sides of the flow line. The trapezoid $F' H G F$ will then contain approximately the same area as the irregular figure $F' I H G F$ bounded by the flow line; and similarly, each trapezoid included between any two adjacent parallel cross-lines will be approximately equal to the figure included between the same two cross-lines and limited by the flow line. The valley or basin is thus divided into prismoids whose bases are the cross-sections measured on the parallel cross-lines, and whose altitudes are the perpendicular distances between adjacent cross-lines. From these the volume of each prismoid can be calculated as explained in Art. 65. The capacity of the valley or basin is the sum of the volumes of the several prismoids.

EXAMPLES FOR PRACTICE

1. The areas of the several cross-sections of a valley intended for water storage are found to be: $A_0 = 0$, $A_1 = 396$ square feet, $A_2 = 678$ square feet, $A_3 = 910$ square feet, $A_4 = 720$ square feet, and $A_5 = 586$ square feet. The perpendicular distances between the cross-sections are $h_{0-1} = 40$ feet, $h_{1-2} = 45$ feet, $h_{2-3} = 50$ feet, $h_{3-4} = 32$ feet, and $h_{4-5} = 36$ feet. What will be the capacity of the reservoir, in cubic feet, as determined by the end-area method? Ans. 121,373 cu. ft.

2. Assuming the middle areas to be: $M_{0-1} = 280$ square feet, $M_{1-2} = 410$ square feet, $M_{2-3} = 802$ square feet, $M_{3-4} = 830$ square feet, and $M_{4-5} = 670$ square feet, calculate, by the prismoidal formula, the capacity of the reservoir referred to in the preceding example.
Ans. 120,744 cu. ft.

UNITED STATES LAND SURVEYS

(PART 1)

SYSTEM OF ORIGINAL SURVEYS

PRELIMINARY

1. Land surveying consists in locating, measuring, and marking on the ground the boundary lines of tracts of land. This is done for the purpose of determining the position, form, and extent, and usually, also, the area, of each tract surveyed; the operation as a whole is called a **land survey.** Land surveys are of two classes; namely, *original surveys*, and *resurveys*.

2. Original surveys are made to determine the boundaries and areas of tracts that have not previously been surveyed. When this is done for the purposes of sale, and the land has been transferred in accordance therewith, the boundary lines thus fixed are unchangeable, except by the mutual consent of the interested parties, even though errors were made in the survey. The chief requirements in original surveys are:

(*a*) *Careful, accurate work* with the proper instruments, and correct computations to determine where the boundaries ought to be and the areas included by them.

(*b*) *Monuments* to mark the boundaries on the ground in the most permanent manner practicable.

(*c*) *A full and complete record* of the survey, giving the lengths and directions of the boundary lines and accurate descriptions of the location and character of the monuments

and other marks that define the position of the boundary lines on the ground. This record is known as the **field notes,** and is of especial value and importance as evidence by which to find or to relocate the boundary lines.

When original surveys are made by private owners, as, for example, the surveys for the sale of acreage tracts or town lots, they are made on any system to suit the circumstances of the case or the ideas and plans of the owners. When they are made by government authority, as, for example, the United States land surveys, they are based on a definite system provided by law.

3. Resurveys.—The determination of the true bearings and distances between the successive corners along the line of an original survey is commonly called a **retracement.** A retracement and survey made for the purpose of relocating and remarking the corners and lines of the original survey, when such corners and lines are wholly or partly lost or in dispute, is called a **resurvey.** Though their meanings are not exactly the same, the terms *resurvey* and *retracement* are often used interchangeably.

GENERAL OUTLINE OF SYSTEM

4. Authority for the Public-Land Surveys.—The surveys of the public lands of the United States are authorized by various acts of Congress. All executive duties appertaining to the surveying and sale of such public lands, or in any wise relating thereto, are performed by the Commissioner of the General Land Office, under the direction of the Secretary of the Interior. The General Land Office is connected with and under the authority of the Department of the Interior. Each large tract or region that by reason of its natural features or political boundaries is more or less separate from the other portions of the country, or that for any reason it is desirable or convenient to survey separately, is set apart as a separate **surveying district.** In many cases, a surveying district comprises one state, though in some cases it consists of parts of different states. The

surveys of the public lands in each surveying district are in charge of a surveyor general, who is appointed by the President of the United States, but the surveys are performed by skilful surveyors engaged as deputies by the surveyor general, and who are known as **United States deputy surveyors.** Under the United States system of land surveys, the public lands are divided by the deputy surveyors into tracts, approximately 1 mile square, called **sections,** and their further subdivision into half sections, quarter sections, half-quarter, and quarter-quarter sections is provided for by law.

5. Basis and Divisions of System.—All the surveys in each surveying district are referred to an **initial point.** This is located and marked accurately, and its true position in latitude and longitude is determined astronomically and recorded. From the initial point, a line, called a **base line,** is run east and west on a true parallel of latitude, and a true meridian, called the **principal meridian,** is run north and south. These two lines, perpendicular to each other, are taken as lines of reference for all the surveys of the region. Using them as a basis, the land is laid out into blocks approximately 24 miles square by means of **standard parallels,** which are lines running east and west, parallel to the base line and at intervals of 24 miles; and **guide meridians,** which are lines running north and south on true meridians at intervals of 24 miles.

Monuments are set along these lines at intervals of 1 mile, or 80 chains, for the corners of sections, and at the intermediate half miles for the corners of half and quarter sections. Such monuments are commonly known as **corners.** The blocks formed by the standard parallels and guide meridians are divided into **townships** of as nearly 6 miles square as the form of the earth will permit, by means of parallels of latitude and meridian lines, and each township is further divided into 36 **sections** containing as nearly as may be 640 acres each. A monument that is set to mark the corner of a section is called a **section corner,** and a monument set on the line between two section corners to mark

the corner of a half or quarter section is called a **quarter-section corner or quarter post.**

Corners set at the end of every mile and half mile on base lines and standard parallels, when those lines are run, are called **standard corners.** Other corners set on the same lines, where the subdivision lines close on them, are called **closing corners.** Thus, the corner of Sections 31 and 32, on a standard parallel, is a standard corner, while the corner of Sections 5 and 6, on the same line, is a closing corner. Previous to 1846, closing corners were set on the north and west boundaries of every township, but they are now restricted to base lines and standard parallels.

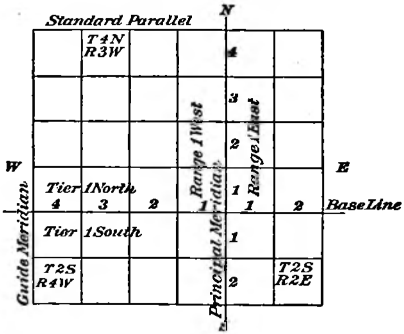


FIG. 1

6. Numbering of Townships.—According to the United States Manual of Surveying Instructions, any series of contiguous townships or sections lying north or south of each other constitutes a **range,** and the north-and-south line bounding a range of townships is called a **range line.** A series of contiguous townships or sections lying east and west of each other constitutes a **tier.** The east and west lines dividing a range into townships are called **township lines.**

Tiers of townships are numbered to the north and south from the base line, commencing with number 1 for the

townships adjacent to the base line, and ranges of townships are numbered to the east and west from the principal meridian, beginning with number 1 for the townships adjacent to the principal meridian, as shown in Fig. 1. By this means the location of a township is readily known from its description. Thus, Township 2 South, Range 4 West (usually abbreviated in the field notes to T. 2 S, R. 4 W) indicates that the township is in the second tier south of the base line and in the fourth range west of the principal meridian.

7. Numbering of Sections.—Sections are numbered successively from east to west and west to east across the township. Commencing with Section 1 in the northeast corner of the township, the sections are numbered in regular order westwards through the north tier of sections to Section 6 in the northwest corner, then through the next tier to the south they are numbered eastwards successively to Section 12 in the most eastern range of sections, and so on, alternately westwards and eastwards across the township, to Section 36 in the southeast corner of the township. This system of numbering is illustrated in Fig. 2.

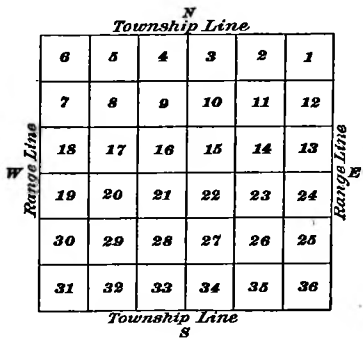


Fig. 2

Sections or legal subdivisions of sections that for any reason have not their regular legal complement of land, as when a portion of their area is taken up by waters or reservations, are termed **fractional sections**. In some cases, whole sections are occupied by lakes, rivers, etc. In such cases the remaining sections, whole or fractional, receive the same numbers that they would have received if all the sections in the township were complete.

8. Convergency of Meridians: Excess and Deficiency.—Since the range lines are required to be true meridians, the townships cannot be precisely 6 miles square, but are necessarily of a slightly trapezoidal form, owing to the fact that true meridians are not parallel to each other, but converge toward the poles of the earth. This is known as the **convergency of meridians.** This convergency of the meridians varies with the latitude. In a township 6 miles square, as measured on the parallels, that is, on the township lines, it is equal to 41.9 links in latitude 30°, while in latitude 49° it is equal to 83.5 links. Hence, the law provides that "in all cases where the exterior lines of the townships to be subdivided into sections and half sections shall exceed or shall not extend 6 miles, the excess or deficiency shall be added to or deducted from the western or northern ranges of sections or half sections in the township, according as the error may be in running lines from east to west or from south to north."

This produces fractional lots in those parts of Sections 1, 2, 3, 4, 5, and 6, adjoining the north boundaries of the townships, and in those parts of Sections 6, 7, 18, 19, 30, and 31 adjoining their west boundaries. The areas of these and all other fractional lots are marked on the official plats of the surveys, and the land is sold accordingly. The areas of the regular sections and lots in the township are not marked on the plats, but are sold as containing 640 acres, or aliquot parts thereof, although in a majority of the surveys it is the exception and not the rule that the sections do contain exactly 640 acres.

9. Initial Point.—In executing the United States land surveys in any surveying district, an initial point for the surveys of that district is first established. This is located astronomically under special instructions from the General Land Office; that is, in such position and by such methods as the General Land Office may direct, and its latitude and longitude are determined accurately.

An initial point should have a conspicuous location, visible from distant points on lines; it should be perpetuated

by an indestructible monument, preferably a copper bolt firmly set in a rock ledge; and it should be witnessed by rock bearings, without relying on anything perishable.

10. Classification of Lines.—The initial point having been established, the lines of public-land surveys are extended therefrom. They are classified as follows: (1) base lines and standard parallels; (2) principal and guide meridians; (3) township exteriors; (4) subdivision and meander lines. Only the base line and principal meridian can pass through the initial point.

11. Base Line.—From the initial point, the base line is extended east and west on a true parallel of latitude, by the use of transit or solar instruments. The transit is used for the alinement of all important lines.

The direction of a base line conforms to a parallel of latitude and is controlled by true meridians; consequently, the correct determination of true meridians by observations on Polaris at elongation is a matter of prime importance. Since the base line is at every point perpendicular to the true meridian at that point, it is not a straight line, but is necessarily slightly curved.

In laying out a base line or parallel of latitude, certain reference lines, called tangents and secants, having a known position and relation to the required parallel of latitude, are prolonged as straight lines, and the positions of points on the parallel of latitude are determined by offset measurements from these reference lines.

The proper township, section, and quarter-section corners are established at lawful intervals along the base line, and meander corners are established at the intersection of the line with all meanderable streams, lakes, or bayous.

12. Method of Prolonging Lines.—In prolonging these reference lines, two backsights and two foresights are taken at each setting of the instrument. In one method, the horizontal limb is revolved 180° in azimuth between the observations, taking the mean of observations. Another method, called *double backsights and foresights*, is still more

exact, and therefore preferable, since it not only insures a straight line, if the transit is leveled, but also detects the least error of collimation. In this method, the transit points are placed at short intervals along the line, and each backsight is taken by setting the vertical cross-wire of the telescope on two transit points at some distance apart back on the line. Then, if the transit point over which the instrument is set has not been fixed truly in line, the two transit points observed cannot be brought in line with the cross-wire at the same time, and the error will be detected at once. If the two transit points observed can be bisected with the cross-wire at the same time, the telescope is plunged forwards and one or two new points are set in advance. The transit is then moved forwards and set up over one of the new points and again backsighted on two or more transit points, in order to check its position. Thus, suppose that the instrument is set up at *I*, Fig. 3, and backsighted on the transit points *A* and *B*. If the point *I* is not truly in line and the cross-wire is made to bisect the point *B*, the line of sight will not bisect the point *A*, but will pass to one side of it, as indicated by the dotted line *B A′*. The error can be corrected by taking the instrument back to the point *B* and testing the forward sight, or by taking it to the point *A* and prolonging the line *A B* by a foresight. Or, when at the point *I*, the transit can be moved over until by trial it is found to be accurately in line with the points *A* and *B*, as indicated by the line *B I′*. The line can then be prolonged forwards as before.

FIG. 3

Where a solar apparatus is used in connection with a transit, the deputy surveyor tests the instrument, whenever practicable, by comparing its indications with a meridian determined by observations of Polaris, and in all cases where error is discovered he makes the necessary corrections of his line before proceeding with the survey. All operations must be fully described in the field notes.

In order to detect errors and insure accuracy in measurement, two sets of chainmen are employed; one to note distances to intermediate points and to locate topographical features, the other to act as a check. Each measures 40 chains, and in case the difference is inconsiderable, the corner is placed midway between the ending points of the two measurements; but if the discrepancy exceeds 8 links on even ground, or 25 links on mountainous ground, the true distance is found by careful rechaining by one party or both.

13. Principal Meridian.—This line must be a true meridian and be extended from the initial point, either north or south, or in both directions, as the conditions may require, by the use of transit or solar instruments. The methods used for determination of directions, and the precautions to be observed to secure accuracy in measurement, are the same as for the base line.

14. Standard Parallels.—These were formerly called **correction lines.** The standard parallels are extended east and west from the principal meridian, at intervals of 24 miles north and south of the base line, in the manner prescribed for running said base line. In the earlier surveys, standard parallels as well as guide meridians were placed at various distances up to 60 or more miles apart. This was done under special instructions given by the surveyor general of the district at the time. It is no longer permitted.

15. Guide Meridians.—These lines are extended as true meridians north from the base line or standard parallels, at intervals of 24 miles east and west from the principal meridian, as measured on the base line or standard parallel, in the manner prescribed for running the principal meridian. When conditions require that such guide meridians be run south from the base or correction lines, they are initiated at established corners on such lines, marked as closing corners.

16. Township Exteriors.—Whenever practicable, the township exteriors, in a block of land 24 miles square bounded by standard lines, are surveyed successively

through the block. Beginning with those of the southwestern township, the exterior boundaries of townships belonging to the west range are first surveyed in succession through the range, from south to north, and in a similar manner the other three ranges are surveyed in regular sequence. The township boundaries extending north and south formed by range lines, are called **meridional boundaries,** and those extending east and west, that is, the township lines, are designated as **latitudinal boundaries.**

The meridional boundaries of townships have precedence in the order of survey and are run from south to north on true meridians, with permanent corners at lawful distances.

The latitudinal boundaries are run from east to west on trial lines, called **random lines,** and are corrected back on true lines. When a random line intersects the line to which it is run at some point other than the true corner, the distance from the point of intersection to the true corner is called the **falling** of the random line. The falling of a random line north or south of the township corner to be closed on is measured, and, with the resulting true return course, recorded in the field notes.

When running random lines from east to west, temporary corners are set at intervals of 40 chains, and permanent corners established on the true line as corrected back, thereby throwing the excess or deficiency against the west boundary of the township.

SUBDIVISION OF TOWNSHIPS

17. Meridional Section Lines.—The exterior boundaries of a full township having been established, the lines subdividing the township into sections are run from the corners that have been set on the boundary lines of the township. The north-and-south section lines extending through the township are called **meridional section lines,** and the east-and-west section lines are called **latitudinal section lines.**

The meridional section lines are made parallel to the range line forming the east boundary of the township by applying

to the bearing of the latter a small correction, dependent on the latitude, taken from Table I.* This table gives, to the nearest whole minute, the convergency of two meridians 6 miles long and from 1 to 5 miles apart.

TABLE I

CORRECTIONS FOR CONVERGENCY WITHIN A TOWNSHIP

Latitude Degrees	Corrections, in Minutes of Arc, to be Applied to Bearing of Range Lines at a Distance of				
	1 Mile	2 Miles	3 Miles	4 Miles	5 Miles
30 to 35 . . .	1	1	2	2	3
35 to 40 . . .	1	1	2	3	3
40 to 45 . . .	1	2	2	3	4
45 to 50 . . .	1	2	3	4	5
50 to 55 . . .	1	2	3	5	6
55 to 60 . . .	1	3	4	5	7
60 to 65 . . .	2	3	5	7	8
65 to 70 . . .	2	4	6	8	10

By applying the corrections given in Table I according to the following rules, the bearing of each of the five meridional section lines in a township can easily be determined from the bearing of the range line forming the east boundary of the township:

Rule I.—*If the bearing of the range line is due north, a tabular correction will be equal to the bearing of the corresponding meridional section line west of north.*

Rule II.—*If the bearing of the range line is west of north, the sum of this and a tabular correction will be the bearing of the corresponding meridional section line.*

*Tables I, II, and III, and the list of abbreviations in Art. 32, descriptions of corners in Art. 33, specimen of field notes in Art. 38, and many authoritative statements throughout this Section are either wholly or partly from the United States Manual of Surveying Instructions for 1902.

Rule III.—*If the bearing of the range line is east of north, the difference between this and a tabular correction will be the bearing of the corresponding meridional section line; this bearing will be east of north when the tabular correction is less than the bearing of the range line, and west of north when the correction exceeds the bearing.*

EXAMPLE.—In latitude 47°, the range line forming the east boundary of a township bears N 0° 2′ E; what should be the bearing of each of the five meridional section lines in the township?

SOLUTION.—By reference to Table I, it is found that for a latitude between 45° and 50° the correction is 1 min. for each mile of distance from the range line. Hence, by applying rule III, the bearings of the five meridional section lines are found to be as follows:

Bearing of first line, from the corner for Sections 35 and 36, N 0° 1′ E. Ans.

Bearing of second line, from the corner for Sections 34 and 35, true north. Ans.

Bearing of third line, from the corner for Sections 33 and 34, N 0° 1′ W. Ans.

Bearing of fourth line, from the corner for Sections 32 and 33, N 0° 2′ W. Ans.

Bearing of fifth line, from the corner for Sections 31 and 32, N 0° 3′ W. Ans.

EXAMPLES FOR PRACTICE

1. In latitude 42° north, the range line forming the east boundary of a township bears N 0° 2′ W; what should be the bearing of: (*a*) the first meridional section line west of this range line? (*b*) the second? (*c*) the third? (*d*) the fourth? and (*e*) the fifth?

Ans. (*a*) N 0° 3′ W
(*b*) N 0° 4′ W
(*c*) N 0° 4′ W
(*d*) N 0° 5′ W
(*e*) N 0° 6′ W

2. In latitude 45° north, the bearing of the range line forming the east boundary of a township is due north; what should be the bearing of: (*a*) the first meridional section line west of the range line? (*b*) the second? (*c*) the third? (*d*) the fourth? and (*e*) the fifth?

Ans. (*a*) N 0° 1′ W
(*b*) N 0° 2′ W
(*c*) N 0° 3′ W
(*d*) N 0° 4′ W
(*e*) N 0° 5′ W

3. In latitude 35° north, the bearing of the range line forming the east boundary of a township is N 0° 1′ E; what should be the bearing

of: (*a*) the first meridional section line west of this range line? (*b*) the second? (*c*) the third? (*d*) the fourth? and (*e*) the fifth?

Ans. (*a*) North
(*b*) North
(*c*) N 0° 1′ W
(*d*) N 0° 2′ W
(*e*) N 0° 2′ W

4. In latitude 40° north, the bearing of the range line forming the east boundary of a township is N 0° 1′ W; what should be the bearing of: (*a*) the first meridional section line west of this range line? (*b*) the second? (*c*) the third? (*d*) the fourth? and (*e*) the fifth?

Ans. (*a*) N 0° 2′ W
(*b*) N 0° 3′ W
(*c*) N 0° 3′ W
(*d*) N 0° 4′ W
(*e*) N 0° 5′ W

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18. Method of Subdividing Townships.—At or near the southeast corner of the township, a true meridian is determined by observing Polaris or by solar observations, and the deputy surveyor's instrument is tested thereon; then from said corner the first mile of the east and south boundaries are retraced, if the survey of the exteriors and the subdivisions have been provided for in separate contracts; but if the survey of the exterior and subdivisional lines are included in the same contract, these retracements are omitted. All disagreements of bearings or measurements are stated in the field notes.

After testing his instrument on the true meridian, the deputy surveyor commences at the corner to Sections 35 and 36, on the south boundary of the township, and runs a line parallel to the range line, establishing at 40 chains the quarter-section corner between Sections 35 and 36, and at 80 chains the corner for Sections 25, 26, 35, and 36.

From the last-named corner, a random line is run eastward, without blazing, parallel to the south boundary of Section 36, to an intersection with the east boundary of the township, placing a post for temporary quarter-section corner at a distance of 40 chains from the point of beginning. If the random line intersects the township boundary exactly at the corner for Sections 25 and 36, it is blazed back and established as the true line, the permanent quarter-section

corner being established thereon, midway between the initial and terminal section corners.

If the random line intersects the township boundary to the north or south of the corner, the falling, or distance of the point of intersection north or south of the corner, is measured, and from the data thus obtained, the true return course is calculated and the true line blazed and established, and on this line the position of the quarter-section corner is determined and established midway between the section corners. Then from the corner for Sections 25, 26, 35, and 36, the west and north boundaries of Sections 25, 24, 13, and 12 are established in the same manner as those of Section 36, with the exception that the random line for the north boundary of each of these sections is run parallel to the established south boundary of the same section, instead of the south boundary of Section 36; that is, the random line between Sections 24 and 25 is run parallel to the established south boundary of Section 25, etc.

Then, from the last established section corner, that is, the corner of Sections 1, 2, 11, and 12, the line between Sections 1 and 2 is projected northwards on a random line, parallel to the east boundary of the township, to its intersection with the north boundary of the township, setting a post for temporary quarter-section corner at 40 chains. If the random intersects said north boundary exactly at the section corner previously set on said boundary, it is blazed back and established permanently in its original position, allowing the fractional measurement to remain permanently in that portion of the line between said corner and the north boundary of the township.

If, however, the random line intersects the north boundary of the township to the east or west of the section corner previously set on that line, the consequent falling is measured, and from the data thus obtained the true return course is calculated and the true line established. The permanent quarter-section corner is placed on this line at 40 chains from the initial corner of the random line, thereby throwing the fractional measurement in that portion of the line lying

between the quarter-section corner and the north boundary of the township.

When the north boundary of a township is a base line or standard parallel, however, the line between Sections 1 and 2 is run parallel to the range line as a true line, the quarter-section corner is placed on this line at 40 chains from the initial corner, and a closing corner is established at the point of intersection with the base or standard line. The distance from the said closing corner, to the nearest standard corner on the base or standard line, is measured and noted as a connection line. This distance is the **closing distance.** The line that intersects the base line or standard parallel is a **closing line.**

Each successive range of sections progressing to the west is surveyed in a similar manner until the fifth range is attained; then, from the section corners established on the west boundary or the fifth range of sections, random lines are projected to their intersection with the west boundary of the township, and the true return lines established as in the survey of the first, or most eastern, range of sections, with the exception that on the true lines thus established the quarter-section corners are established at 40 chains from the initial corners of the random lines, the fractional measurements being thereby thrown into those portions of the lines situated between the quarter-section corners and the west boundary of the township.

On both meridional and latitudinal section lines, quarter-section corners are established at points equidistant from the corresponding section corners, except on the lines closing on the north and west boundaries of the township, and in those situations the quarter-section corners are established at precisely 40 chains to the north or west (as the case may be) of the respective section corners from which those lines respectively start, by which procedure the excess or deficiency in the measurements is thrown on the extreme tier or range of quarter sections, as the case may be.

19. An Exception—Witness Corners.—The deputy is not required to complete the survey of the first range of

sections from south to north before commencing the survey of the second or any subsequent range of sections, but the corner on which any random line closes must have been previously established by running the line that determines its position, except as follows: Where it is impracticable to establish such section corner in the regular manner, it is established by running the latitudinal section line as a true line, with a true bearing, determined as directed for random lines, setting the quarter-section corner at 40 chains and the section corner at 80 chains.

When the proper point for the establishment of a township or section corner is inaccessible, and a corner can be erected on each of the two lines that approach the township or section corner, at distances not exceeding 20 chains therefrom, such a corner is established on each line. Corners thus established are called **witness corners.** The witness corner is marked as conspicuously as a section corner, and bearing trees are used wherever possible.

20. Meandering.—At every point where a standard, township, or section line intersects the bank of a navigable stream or any meanderable shore, a corner, called a **meander corner,** is established at the time the line is run. The meander corners thus established on the survey, lines that intersect a shore line are connected by a traverse line that follows the sinuosities of the shore. Such traverse lines are for the purpose of determining the form of the shore; they are called **meander lines,** and the process of running them is called **meandering.** All streams 3 chains or more wide are meandered on both banks. Navigable bayous, lakes, and ponds having an area of more than 25 acres are also meandered. In meandering, the deputy begins at a meander corner and, following the bank as nearly as practicable along the high water-line, takes the bearing and measures the length of each course, closing on the next meander corner.

A meander line is always a compass traverse. The courses are taken at the nearest quarter degree and are recorded as compass courses by their angle from the true

meridian, and not as the transit angles of a deflection traverse by the angular deviation of each course from the preceding course. The lengths of the courses are taken in full chains or multiples of 10 links for convenience of computation, using odd links only in the closing course. These meander lines are run merely for the purpose of defining the shore lines and ascertaining the approximate area of land in the subdivisions made fractional by the body or bodies of water. They follow the high water-line of the shore as nearly as practicable, but do not follow all the little sinuosities of the bank. A meander line is not a boundary line;* the high water-line of the bank is the boundary line.

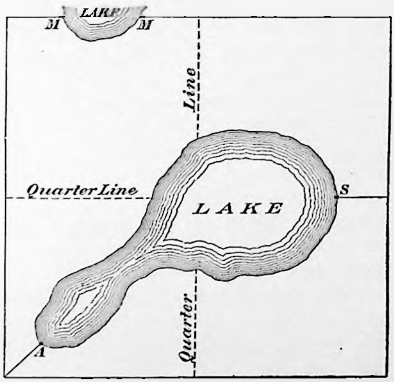

FIG. 4

21. Meander Corners.—The meander corners set on regular division lines of the government survey, that is, on standard, township, or section lines, are called **regular meander corners.** It is sometimes the case, however, that a lake or deep pond lies entirely within the boundaries of a section, so that its shore is not intersected by any regular division line of the government survey. In order to meander such a lake or pond, two lines are run to it from the two nearest survey corners on different sides of the lake, and the lengths and bearings of these lines are measured and recorded. Meander corners are established where the lines intersect the margin of the lake or pond, and the meander

*By the supreme court of Minnesota, however, a meander line has been held to be a boundary line in a case where a body of water never existed at substantially the place indicated on the official plat.

lines are then extended around the lake or pond from the points thus established. If the line thus run to the lake or pond from a survey corner coincides with a legal subdivision line of the section, that is, coincides with a quarter-section line, a half-quarter line, or a quarter-quarter line, the meander corner set on the line is called a **special meander corner.** But if the line run to the lake or pond does not coincide with a legal subdivision line of the section, the meander corner set on it is called an **auxiliary meander corner.** Fig. 4 shows a fractional section with regular meander corners at M and M, a special meander corner on the quarter-section line at S, and an auxiliary meander corner at A, near the section corner.

22. **Objects and Data to Be Noted.**—The following is a brief summary of the objects and data that, if intersected by the survey line or situated in its vicinity, are to be noted in the field book as they are encountered during the progress of the survey:

The precise course and length of every line run, noting all necessary offsets therefrom, with the reason for making them, and method employed.

The kind and diameter of all bearing trees, with the course and distance of the same from their respective corners; and the precise relative position of witness corners to the true corners.

The kind of materials of which corners are constructed.

Trees on line. The name, diameter, and distance on line to all trees that it intersects.

Intersections of the line with, and descriptions of, land objects and water objects, such as hills, ravines, settlers' claims, rivers, lakes, watercourses, swamps, etc.

Descriptions of the character of the land's surface, the soil, timber, prairie, bottom lands, springs, improvements, roads and trails, mineral deposits, rapids, cataracts, precipices, caves, etc.

Natural curiosities, fossils, ancient works of art, such as mounds, ditches, and objects of like nature.

The magnetic declination is incidentally noted at all points of the lines being surveyed, where any material change in the same indicates the probable presence of iron ores, and the position of such points is identified in the field notes.

23. Limits for Closing.—Every random line having a nominal length of 1 mile is required to close within 50 links of the objective point, and the length of the line must be within 50 links of its theoretical length. For random lines of greater length, the corresponding errors of closure must not exceed 50 links per mile of line run. Otherwise the line must be rerun, and the error found and corrected.

North-and-south section lines, except those of fractional sections next to the township boundary, must be 80 chains long. In those of fractional sections, the actual length must be within 150 links of the theoretical length.

The north boundary and the south boundary of any section, except in a fractional range, must be within 50 links of equal length.

Meanders between any two successive meander corners must close by traverse within $\frac{8}{8}$ link for each chain of the meander line.

CONVERGENCY OF MERIDIANS

24. Amount of Convergency of Meridians.—In latitude 30° north, the angle of convergency between meridians 1 mile apart is 30 seconds, very closely, so that in this latitude the meridians at the opposite sides of a township incline to each other at an angle of 3 minutes, very closely. In latitude 44° north, the convergency is very closely 50 seconds per longitudinal mile, or 5 minutes per township. The convergency of meridians can be determined approximately but very closely by means of the two following principles, which, though not rigidly exact,* are closely approximate:

Principle I.—*The angle of convergency between two meridians is equal to their difference in longitude multiplied by the sine of the latitude.*

*Because the earth is not a true sphere.

Principle II.—*The lengths of the parallels of latitude intercepted between any two meridians are proportional to the cosines of the latitudes.*

These principles are sufficiently accurate for the purposes of the surveyor when the meridians considered are not more than 24 miles apart. At the equator, the length of 1° of longitude is equal to 69.164 statute miles, and the cosine of the latitude is unity. Hence, the second principle can be stated in the following form, which for some purposes is more convenient:

The length of an arc of 1° of longitude at any given latitude, expressed in miles, is equal to 69.164 times the cosine of the latitude.

Let c° = angle of convergency, in degrees;
c' = angle of convergency, in minutes;
L = difference in longitude, in same unit as c;
l = latitude for which convergency is calculated;
m = distance, in miles, corresponding to difference in longitude.

Then, according to the principle first stated

$$c^\circ = L^\circ \sin l \qquad (a)$$

and according to the second principle,

$$m = 69.164\, L^\circ \cos l$$

$$L^\circ = \frac{m}{69.164 \cos l} \qquad (b)$$

Substituting this value of L° in (a),

$$c^\circ = \frac{m \sin l}{69.164 \cos l} = \frac{m \tan l}{69.164} \qquad (1)$$

The value of an angle expressed in minutes is 60 times its value expressed in degrees; hence, $c' = 60\, c^\circ$ and by multiplying both numbers of the preceding formula by 60 and substituting c' for $60\, c^\circ$, it becomes

$$c' = \frac{m \tan l}{1.1527} = .8675\, m \tan l \qquad (2)$$

Formulas 1 and 2 are solved easily by means of logarithms, and for this purpose formula 1 may be written in the form

$$\log c^\circ = \log m + \log \tan l + \bar{2}.16012 \qquad (3)$$

Likewise formula 2 may be written in the form

$$\log c' = \log m + \log \tan l + \bar{1}.93828 \qquad (4)$$

EXAMPLE 1.—What is the angle of convergency between two meridians 6 miles apart in latitude 30° north?

SOLUTION.—The natural tangent of 30° is .57735, and by substituting this and the value of m in formula 2, the angle of convergency is found to be

$$c' = .8675 \times 6 \times .57735 = 3.005' = 3', \text{ practically. Ans.}$$

EXAMPLE 2.—What is the angle of convergency between two meridians 6 miles apart in latitude 44° north?

SOLUTION.—Log 6 = .77815 and log tan 44° = $\bar{1}.98484$. The substitution of these values in formula 4 gives

$$\log c' = .77815 + \bar{1}.98484 + \bar{1}.93828 = .70127$$

which is the logarithm of 5.0265′ = 5′ 2″, nearly. Ans.

25. Difference in Length of Northern and Southern Township Boundaries.—Owing to the fact that the earth is not a true sphere, but is somewhat flattened at the poles, the difference in latitude subtended by an arc 6 miles in length extending north and south is not exactly the same at all latitudes, but varies from about 5.227 minutes in latitude 30° to 5.193 minutes in latitude 70°. For most purposes of the surveyor, however, it may be taken at 5.2 minutes for all ordinary latitudes. In what follows, this value will be used; the resulting error will be so small as not to affect the results materially.

Let l_s = latitude of southern boundary of township;
l_n = latitude of northern boundary of township;
d_s = length of southern boundary of township;
d_n = length of northern boundary of township.

Then, from principle II of the preceding article,

$$d_s : d_n = \cos l_s : \cos l_n$$

from which

$$d_n = d_s \frac{\cos l_n}{\cos l_s} \qquad (1)$$

For ordinary purposes it is sufficiently accurate to write $l_n = l_s + 5.2'$.

Formula 1 is easily solved by means of logarithms, for which purpose it may be written

$$\log d_n = \log d_s + \log \cos l_n - \log \cos l_s \qquad (2)$$

EXAMPLE.—The southern boundary of a township is in latitude 35° north and its length is just 480 chains. What should be the length of the northern boundary of the township?

SOLUTION.—Log 480 = 2.68124, log cos 35° = $\bar{1}.91336$, and log cos 35° 5.2′ = $\bar{1}.91290$. The substitution of these values in formula 2, gives

$$\log d_n = 2.68124 + \bar{1}.91290 - \bar{1}.91336 = 2.68078$$

whence $d_n = 479.49$. Ans.

EXAMPLES FOR PRACTICE

1. What is the angle of convergency between two meridians 6 miles apart in latitude 33° north? Ans. 3′ 23″

2. What is the angle of convergency between two meridians 6 miles apart in latitude 47° north? Use logarithms. Ans. 5′ 35″

3. The southern boundary of a township is on a correction line in latitude 45° north and its length is therefore 480 chains; what should be the length of the northern boundary of the township? Ans. 479.27 ch.

4. The southern boundary of a township is on a correction line in latitude 40° north; what should be the length of its northern boundary? Ans. 479.4 ch.

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26. Table of Convergency of Meridians.—Table II gives the amount of convergency of meridians for each degree of latitude between latitudes 30° and 70°, inclusive, and is used in determining the theoretical length of township and section lines.

The second column of the table contains the convergency of two meridians 6 miles long and 6 miles apart and having their south ends at the latitude stated in the first column. This convergency is the difference in the distances between the two meridians as measured on the parallels of latitude through their north and south extremities. When the latitude of the parallel that passes through the south end of the

TABLE II

CONVERGENCY OF MERIDIANS 6 MILES LONG AND 6 MILES APART, AND OTHER RELEVANT DATA, BETWEEN LATITUDES 30° AND 70° NORTH

Latitude Degrees	Convergency: On the Parallel Links	Convergency: Angle Min.	Convergency: Angle Sec.	Difference of Longitude per Range: Minutes of Arc	Difference of Longitude per Range: Seconds of Arc	Difference of Longitude per Range: Seconds of Time	Difference of Latitude, in Minutes of Arc, for 1 Mile in Arc	Difference of Latitude, in Minutes of Arc, for 1 Tp. in Arc
30	41.9	3	0	6	0.36	24.02		
31	43.6	3	7	6	4.02	24.27		
32	45.4	3	15	6	7.93	24.53	.871′	5.225′
33	47.2	3	23	6	12.00	24.80		
34	49.1	3	30	6	16.31	25.09		
35	50.9	3	38	6	20.95	25.40		
36	52.7	3	46	6	25.60	25.71		
37	54.7	3	55	6	30.59	26.04	.870	5.221
38	56.8	4	4	6	35.81	26.39		
39	58.8	4	13	6	41.34	26.76		
40	60.9	4	22	6	47.13	27.14		
41	63.1	4	31	6	53.22	27.55		
42	65.4	4	41	6	59.62	27.97	.869	5.217
43	67.7	4	51	7	6.27	28.42		
44	70.1	5	1	7	13.44	28.90		
45	72.6	5	12	7	20.93	29.39		
46	75.2	5	23	7	28.81	29.92		
47	77.8	5	34	7	37.10	30.47	.869	5.212
48	80.6	5	46	7	45.79	31.05		
49	83.5	5	59	7	55.12	31.67		
50	86.4	6	12	8	4.83	32.32		
51	89.6	6	25	8	15.17	33.01		
52	92.8	6	39	8	26.13	33.74	.868	5.207
53	96.2	6	54	8	37.75	34.52		
54	99.8	7	9	8	50.07	35.34		
55	103.5	7	25	9	3.18	36.22		
56	107.5	7	42	9	17.12	37.14		
57	111.6	8	0	9	31.97	38.13	.867	5.202
58	116.0	8	19	9	47.83	39.19		
59	120.6	8	38	10	4.78	40.32		
60	125.5	8	59	10	22.94	41.52		
61	130.8	9	22	10	42.42	42.83		
62	136.3	9	46	11	3.38	44.22	.866	5.198
63	142.2	10	11	11	25.97	45.73		
64	148.6	10	38	11	50.37	47.36		
65	155.0	11	8	12	16.82	49.12		
66	162.8	11	39	12	45.55	51.04		
67	170.7	12	13	13	16.88	53.12	.866	5.195
68	179.3	12	51	13	51.15	55.41		
69	188.7	13	31	14	28.77	57.92		
70	199.1	14	15	15	10.26	60.68	.866	5.193

meridians and forms the south boundary of the township of which the meridians form the meridional boundaries is the same as a tabular latitude given in the first column, the corresponding convergency given in the second column will be the convergency required. For latitudes intermediate between the tabular latitudes, the convergency is obtained by interpolation.

The third column of the table contains the angle of convergency between meridians 6 miles apart and at the various latitudes.

For the various latitudes given in the first column, the difference in longitude between the east and west meridional boundary lines of a township is given in minutes and seconds of arc in the fourth column, and in the fifth column the same value is given in seconds of time.

The difference of latitude for 1 mile in arc, that is, for a distance of 1 mile measured along a true meridian, at the various latitudes, is given in the sixth column, and the difference in latitude for one township in arc, that is, for a distance of 6 miles measured along a true meridian at the various latitudes, is given in the last column.

For the purpose of computing the convergency of the section lines within boundaries of a regular township, the township may be regarded as a plane figure, generally a trapezoid, and its boundaries may be considered to be straight lines. The method of using the table in computing the convergency will be understood from the following example:

EXAMPLE.—In latitude 42°, the length of the south boundary of T. 3 S, R. 9 W is 5 miles 79 chains and 83 links (written 5 mi. 79 ch. 83 li., or 5 mi. 79.83 ch.). (*a*) What is the theoretical length of the north boundary? (*b*) What is the greatest and what the least length permitted for the random line for that boundary? (*c*) What is the theoretical length of the line between Sections 7 and 18? (*d*) What are the limiting lengths for the random of that line?

SOLUTION.—(*a*) By referring to the second column of Table II, we find that in latitude 42° the convergency of meridians 6 mi. long and 6 mi. apart, as measured on the parallel, is 65.4 li., or .654 ch. By subtracting this from the length of the south boundary, the theoretical

length of the north boundary is found to be equal to 5 mi. 79 ch. 83 li. – 65.4 li. = 5 mi. 79 ch. 17.6 li. = 5 mi. 79.176 ch. Ans.

(*b*) As the measured length of the random line must be within 3 ch. of its theoretical length, the greatest length permitted for the random line is 5 mi. 79 ch. 17.6 li. + 3 ch. = 6 mi. 2 ch. 17.6 li., and its least permitted length is 5 mi. 79 ch. 17.6 li. – 3 ch. = 5 mi. 76 ch. 17.6 li. Ans.

(*c*) According to Art. **8**, the excess or deficiency in the length of the latitudinal township boundaries is thrown into the west range of sections and, consequently, the length of the south boundary of Section 31 is equal to 5 mi. 79 ch. 83 li. – 5 mi. = 79 ch. 83 li. According to Art. **8**, the convergency is theoretically confined to the west range of sections, and since the line between Sections 7 and 18 is two-thirds of the distance from the south to the north boundary, its length will be equal to the length of the south line of Section 31 minus two-thirds of the total convergency, or 79 ch. 83 li. – $\frac{2}{3}$ × 65.4 li. = 79 ch. 39.4 li. Ans.

(*d*) Since, according to Art. **23**, the random for a section line must measure within 50 li. of the theoretical length of the line, its greatest permitted length is equal to 79 ch. 39.4 li. + 50 li. = 79 ch. 89.4 li., and its least permitted length is equal to 79 ch. 39.4 li. – 50 li. = 78 ch. 89.4 li. Ans.

NOTE.—It will be well to notice here that the least permitted length of a section line is just 1 chain less than its greatest permitted length. Hence, having found its greatest permitted length, it is merely necessary to subtract 1 chain from this in order to determine its least permitted length.

EXAMPLES FOR PRACTICE

1. The southern boundary of a township is on a correction line in latitude 40° north. (*a*) What should be the length of the northern boundary of the township? (*b*) What is the least length permitted for the random of the northern boundary of this township? (*c*) What is the greatest permitted length? (*d*) What is the theoretical length of the line between Sections 30 and 31?

Ans. (*a*) 479.4 ch.; (*b*) 476.4 ch.; (*c*) 482.4 ch.; (*d*) 79.9 ch.

2. The southern boundary of a township is in latitude 48° north and its length is 6 miles. (*a*) What should be the length of the northern boundary of the township? (*b*) What is the theoretical length of the line between Sections 19 and 30? (*c*) What is the least length permitted for the random of this line? (*d*) What is the greatest permitted length?

Ans. (*a*) 5 m. 79 ch. 19 li.; (*b*) 79.73 ch.; (*c*) 79.23 ch.; (*d*) 80.23 ch.

3. (*a*) What should be the length of the northern boundary of a township whose southern boundary is in latitude 38° north and has a length of 480 chains? (*b*) What are the limiting lengths permitted for the random of the northern boundary of this township? (*c*) What is the theoretical length of the line between Sections 18 and 19? (*d*) What are the limiting lengths for the random of this line?

Ans. (*a*) 479.43 ch.
(*b*) 476.43 and 482.43 ch.
(*c*) 79.72 ch.
(*d*) 79.22 and 80.22 ch.

4. (*a*) What should be the length of the northern boundary of a township whose southern boundary is in latitude 35° north and has a length of 480 chains? (*b*) What are the limiting lengths permitted for the random of the northern boundary of this township? (*c*) What is the theoretical length of the line between Sections 6 and 7? (*d*) What are the limiting lengths for the random of this line?

Ans. (*a*) 479.49 ch.
(*b*) 476.49 and 482.49 ch.
(*c*) 79.58 ch.
(*d*) 79.08 and 80.08 ch.

INSTRUMENTS USED

27. Instruments for Alinement.—For running lines of the United States land surveys, Burt's improved solar compass or a transit of approved construction, with or without solar attachment,* may be used. No other instrument is now permitted. The use of the magnetic needle for running these lines is strictly prohibited. The direction of all lines must be determined independently of the needle. Deputy surveyors using instruments with solar apparatus are required to make observations on Polaris at the beginning of every survey and whenever necessary to test the solar apparatus. Those using transits without solar attachment are required to obtain the meridian by observations of Polaris every clear night. Deputy surveyors are required to examine the adjustments of their instruments and take the latitudes daily, the weather permitting, while running all lines of the public surveys. All instruments used must be tested at least once a year, and oftener if necessary, on the true meridian established by the direction of the surveyor

* The solar attachment is fully described in *Practical Astronomy*, which forms part of this Course.

general of the district, and all defective instruments must be put in a proper condition for accurate work.

28. Instruments for Measuring Distances.—The instruments used for measuring the lengths of the lines of the United States land surveys are the surveyors' chain and marking pins, described in *Chain Surveying.* The use of the engineers' chain is not permitted in the United States land surveys. Distances of height or depth, however, may be given in feet and inches.

Each deputy surveyor must be provided with a standard steel chain or tape, precisely adjusted to the standard measures kept by the surveyor general. This is not used in the field work, but is kept for the purpose of frequently comparing and testing the field chains or tapes..

CORNERS OF THE PUBLIC-LAND SURVEYS

CORNER MONUMENTS

29. The establishment of corners is the consummation of the field work of land surveys. If this is not done in a distinct and permanent manner, the main object of the survey will not be attained. The United States Manual of Surveying Instructions is furnished to every deputy surveyor engaged in the public-land surveys. This gives complete and detailed instructions regarding how to mark corners under all circumstances likely to occur. The corners established in these surveys are township, section, quarter-section, meander, and witness corners. These corners are defined by planting monuments of the most enduring materials available and by marking and noting the directions and distances from the corners to the most permanent objects in the vicinity. A monument consists of what is called the *corner* and its *accessories.*

The **corner** may be an iron post, rod, or pipe, a cross cut on a rocky ledge, a marked stone of suitable size and form, or in case none of these can be obtained, it may be a post of durable timber.

30. The **accessories** to a corner are for the purpose of identifying or **witnessing** the corner, in order that its position can afterwards be determined definitely and unmistakably. They consist of prominent and durable objects, such as rocks, trees, etc., properly marked, whose bearings and distances from the corner are determined and recorded together with their descriptions. An object whose bearing and distance from a corner are determined and recorded is called a **bearing object**, and if a tree, it is called a **bearing tree**. The accessories to a corner may be any of the following, which are named in the order of their value and desirability:

Bearing objects, such as notable cliffs, rocks, boulders, etc., marked with a cross, the letters B. O. and a section number.

Memorials buried 12 to 24 inches deep at the corner, such as glass or stoneware, marked stones, cast iron, charcoal, or a charred stake.

Pits of proper size and arrangement.

Mound of stones at proper position and distance from the corner.

Bearing trees, blazed and marked as required.

Stake in pit, with letters and figures necessary.

Mounds of earth, which in many regions are the least durable and least useful of all accessories.

In establishing corners, the first preference is given to durable stones, then posts, and lastly mounds with stake in pit. The selection of the particular construction to be adopted for any corner is left to the deputy surveyor, who, in the selection of the corner, is instructed to assign the greatest weight to the durability of the materials and permanency of the monuments. Posts are not to extend more than 12 inches above ground, and if more than 3 feet long, the extra length is to be put below the surface.

31. Marking Lines Between Corners.—The marking of trees and brush along the lines of the public-land surveys is required by law. Trees that are intersected by the line

are marked by two chops or notches cut on the sides facing the line, without any other marks. Such trees are called **sight trees** or **line trees**. Fig. 5 illustrates the manner of marking line trees. The chops or blazes are made on each side of the tree where the line intersects it. A sufficient number of other trees are marked, if found within 50 links of the line, so that the line can be readily traced. Where a tree thus blazed stands very near the line, the blazes are opposite each other, coinciding in direction with the line, and where the tree is farther from the line, the blazes are placed nearer each other on the side of the tree toward the line; the farther the tree is from the line the

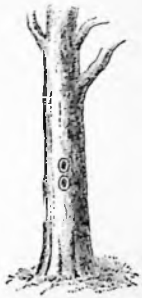


FIG. 5

FIG. 6

FIG. 7

nearer the blazes approach each other. Fig. 6 illustrates the method of marking a tree that stands near the line, as seen from the line at a point opposite the tree. Fig. 7 illustrates the method of marking a tree that stands at some distance from the line, as seen from a point in the line opposite the tree.

Trees are blazed through the bark into the wood, so as to make a permanent mark. Such a blaze endures and is recognizable as long as the tree stands. The blazing of trees is not omitted where trees 2 inches or more in diameter are found within 50 links of the line. Lines are also marked by cutting away such small brush as interferes

with correct sighting of instruments. Random lines are not blazed, but bushes and limbs may be lopped and stakes may be set on the random line at every 10 chains to enable the surveyor on his return to establish the true line. These stakes are removed when the true line is marked.

DESCRIPTIONS OF CORNERS

32. Abbreviations Allowed.—In the descriptions of corners in the field notes and records, the dimensions of stones, posts, and pits are expressed in the order of their length, breadth, and thickness, as for instance "a stone $23 \times 10 \times 8$ in." To describe a mound, the material, diameter of base, and altitude are given, as "mound of earth 4 ft. base, 2½ ft. high." "The following contractions are authorized to be used in the preparation of field notes, transcripts, inspection reports, and similar records, and no others should be introduced:*

A.	acres	elong. . .	elongation
a. m. . .	forenoon	frac. . . .	fractional
A. M. C.	aux. meander corner	ft.	foot, feet
asc. . . .	ascend	G. M. . .	guide meridian
astron. . .	astronomical	h., hrs. . .	hour, hours
bdy. . . .	boundary	ins. . . .	inches
bdrs. . . .	boundaries	lat. . . .	latitude
bet. . . .	between	L. C. . .	lower culmination
B. O. . .	bearing object	lks. . .	links
B. T. . .	bearing tree	l. m. t. . .	local mean time
C. C. . .	closing corner	long. . .	longitude
chs. . . .	chains	m.	minutes
cor., cors.	corner, corners	mag. . . .	magnetic
corr. . . .	correction	M. C. . .	meander corner
decl. . . .	declination	mer. . . .	meridian
dep. . . .	departure	mkd. . .	marked
desc. . . .	descend	N.	north
dia. . . .	diameter	NE. . . .	northeast
diff. . . .	difference	NW. . . .	northwest
dist. . . .	distance	obs. . . .	observe
D. S. . .	deputy surveyor	obsn. . .	observation
E.	east	p. m. . .	afternoon

*United States Manual of Surveying Instructions.

Pol.	Polaris
Pr. Mer.	principal meridian
Pt. of Tr.	point of triangulation
¼ sec.	quarter section
R., Rs.	range, ranges
red.	reduce, reduction
S.	south
S. C.	standard corner
SE.	southeast
sec., secs.	section, sections
S. M. C.	special meander corner
sq.	square
St. Par.	standard parallel
SW.	southwest
T., or Tp.	township
Ts., or Tps.	townships
temp.	temporary
U. C.	upper culmination
var.	variation
W.	west
W. C.	witness corner
w. corr.	watch correction
W. P.	witness point
w. t.	watch time"

The abbreviations given in this list are required to be used in the field notes and similar records of an official character relating to the surveys of the public lands of the United States. The same abbreviations are not necessarily used in the field notes and records of private surveys, though it is well for the private surveyor to use them in order to be familiar with them. These abbreviations will be often used in this text, except for the words *chains*, *hours*, *inches*, and *links*, for which the abbreviations *ch.*, *hr.*, *in.*, and *li.*, respectively, will be used, in order to be uniform with the forms of these abbreviations used in other parts of this Course.

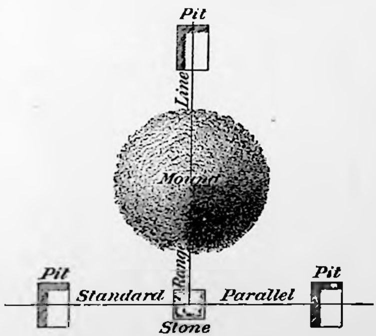


FIG. 8

33. Authorized Forms and Descriptions of Corners.—There are thirteen classes of corners with eight variations of construction and markings in each class. The following are examples of the descriptions and arrangements of corners—one in each class:

1. *Standard Township Corner: Stone, With Pits and Mound of Earth.*—Set a — stone, —×—×— in., — in. in the ground, for standard corner of Tps. 13 N, Rs. 21 and 22 E, marked S. C. on N; with six grooves on N, E, and W faces; dig pits, 30 × 24 × 12 in., crosswise on each line, E and W, 4 ft., and N of stone, 8 ft. distance; and raise a mound of earth, 5 ft. base, 2½ ft. high, N of corner.

Fig. 8 illustrates the standard township corner here described.

2. *Closing Township Corner: Stone, With Mound of Stone.* Set a — stone, —×—×— in., — in. in the ground, for

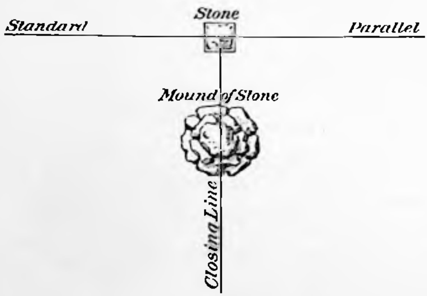


Fig. 9

closing corner of Tps. 4 N, Rs. 2 and 3 W, marked C. C. on S; with six grooves on S, E, and W faces; and raise a mound of stone, 2 ft. base, 1½ ft. high, S of corner. Pits impracticable.

Mound of stone will consist of not less than four stones, and will be at least 1½ ft. high, with 2 ft. base. Fig. 9 illustrates the closing township corner just described.

3. *Corner Common to Four Townships: Stone, With Bearing Trees.*—Set a — stone, —×—×— in., — in. in the ground, for corner of Tps. 2 and 3 N, Rs. 2 and 3 W, marked with six notches on each edge, from which

A —, — in. diameter, bears N —° E, — li. distance, marked T 3 N R 2 W S 31 B T.

A —, — in. diameter, bears S —° E, — li. distance, marked T 2 N R 2 W S 6 B T.

A —, — in. diameter, bears S —° W, — li. distance, marked T 2 N R 3 W S 1 B T.

A —, — in. diameter, bears N —° W, — li. distance, marked T 3 N R 3 W S 36 B T.

All bearing trees will be marked with the township, range, and section in which they stand. Fig. 10 illustrates the corner common to four townships as described above.

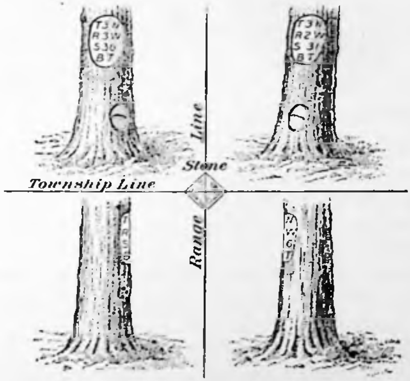


FIG. 10

4. *Corner Common to Two Townships Only: Post, With Pits and Mound of Earth.*—Set a — post, 3 ft. long, 4 in. sq., with marked stone (charred stake or quart of charcoal), 24 in. in the ground, for corner of Tp. 2 N, R. 5 W, and Tp. 3 N, R. 6 W, on N boundary Tp. 2 N, R. 6 W, marked

T 2 N R 5 W S 6 on SE, and

T 3 N R 6 W S 36 on NW face, with six notches on N and W edges; dig pits 30 × 24 × 12 in., on each line, E and W, 4 ft., and N of post, 8 ft. distance; and raise a mound of earth, 5 ft. base, 2½ ft. high, N of corner.

Fig. 11 illustrates the corner common to two townships only, as just described.

5. *Corner for One Township Only: Post, With Bearing Tree.*—Set a — post, 3 ft. long, 4 in. sq., 24 in. in the ground, for SW corner of Tp. 3 N, R. 6 W, marked

T 3 N R 6 W S 31 on NE,

S 1 on SE,

T 2 N R 7 W S 1 on SW, and

S 1 on NW face, with six notches on N and E edges; from which

A —, — in. diameter, bears N —° E, li. distance, marked

T 3 N R 6 W S 31 B T.

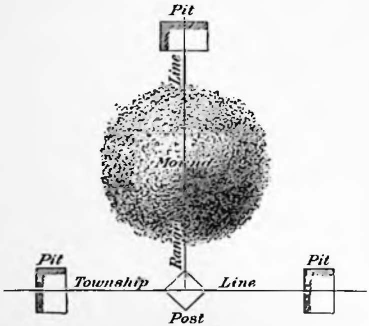


Fig. 11

Fig. 12 illustrates the corner for one township only, as just described.

6. *Standard Section Corner: Mound of Earth, With Deposit, and Stake in Pit.*—Deposit a marked stone (charred stake or quart of charcoal), 12 in. in the ground, for standard corner of Sections 33 and 34; dig pits, 24 × 18 × 12 in., crosswise on each line, N, E, and W of corner, 5 ft. distance; and raise a mound of earth, 4 ft. base, 2 ft. high, over deposit.

In E pit drive a — stake, 2 ft. long, 2 in. sq., 12 in. in the ground, marked

S C T 13 N R 22 E on N,

S 34 on E, and

S 33 on W face; with three grooves on E and W faces.

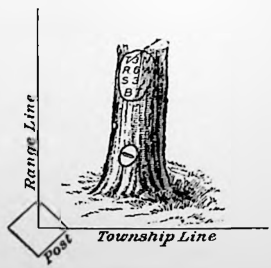


Fig. 12

Fig. 13 illustrates the standard section corner described in the preceding notes.

7. *Closing Section Corner: Tree Corner, With Pits and Mound of Earth.*—A —, — in. diameter, for closing corner of Sections 1 and 2, I mark

C C T 4 N R 3 W on S,

S 1 on E, and

S 2 on W side, with one notch on E, and five notches on W side, dig pits, 18 × 18 × 12 in. S, E, and W of corner, 5 ft. distance; and raise a mound of earth around tree.

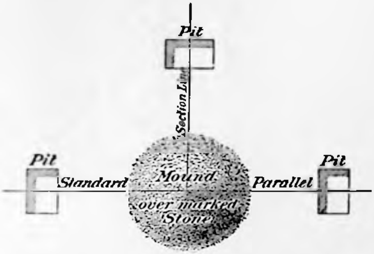


Fig. 13

Fig. 14 illustrates the closing corner just described.

8. *Corner Common to Four Sections: Tree Corner, With Bearing Trees.*—A —, — in. diameter, for corner of Sections 5, 6, 7, and 8, I mark

T 2 N S 5 on NE,

R 2 W S 8 on SE,

S 7 on SW, and

S 6 on NW side, with five notches on S and E sides; from which

A —, — in. diameter bears N —° E, — li. distance, marked

T 2 N R 2 W S 5 B T. A —, — in. diameter, bears S —° E, — li. distance marked T 2 N R 2 W S 8 B T.

Fig. 14

A —, — in. diameter, bears S —° W, — li. distance, marked T 2 N R 2 W S 7 B T.

A —, — in. diameter, bears N —° W, — li. distance marked T 2 N R 2 W S 6 B T.

Fig. 15 represents the tree corner with four bearing trees,

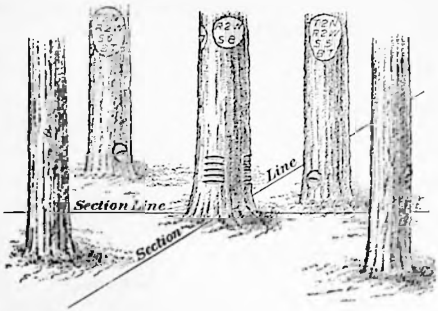


FIG. 15

the corner being common to four sections. Bearing trees are always marked on the side facing the corner, and, consequently, the marks are to be seen plainly on only two of the bearing trees in the figure.

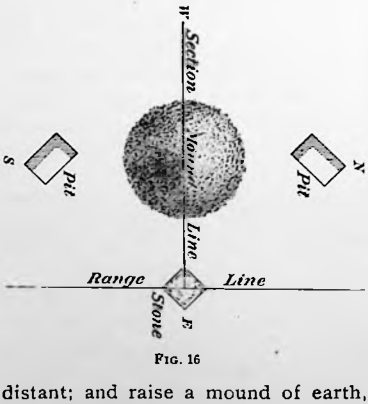


FIG. 16

9. *Section Corner Common to Two Sections Only: Stone, With Pits and Mound of Earth.*—Set a — stone, —×—×— in., — in. in the ground for corner of Sections 25 and 36 marked with five notches on N, and one notch on S edge; dig pits 24 × 24 × 12 in., in each section, 6 ft. distant; and raise a mound of earth, 4 ft. base, 2 ft. high, W of corner.

Fig. 16 shows the section corner common to two sections only, described in the preceding notes. Such a corner when on the range line has notches on its opposite edges, coinciding with the range line, to indicate the distances, in miles, to the NE and SE corners of the township.

10. *Section Corner Referring to One Section Only: Stone With Mound of Stone.* Set a — stone, —×—×— in. in the ground, for SW corner of Section 12, marked with one notch on E edge; and raise a mound of stone, 2 ft. base, 1½ ft. high, NE of corner.

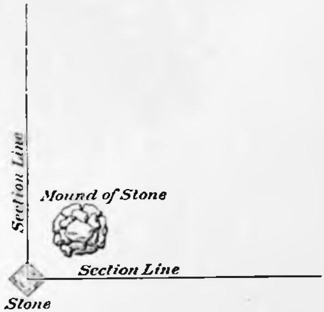


FIG. 17

Fig. 17 shows the stone corner to one section only as described in the preceding notes.

11. *Quarter Section Corner: Stone, With Bearing Trees.* Set a — stone, —×—×— in., — in. in the ground, for ¼ section corner marked ¼ on W face; from which

A —, — in. diameter, bears N —° E, — li. distance, marked ¼ S 16 B T.

A —, — in. diameter, bears N —° W, — li. distance, marked ¼ S 17 B T.

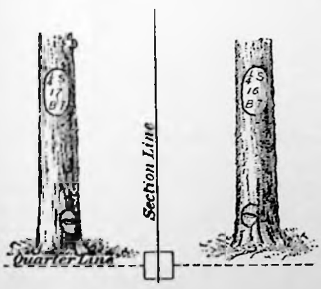


FIG. 18

Fig. 18 represents the quarter-section corner described in the notes.

12. *Meander Corners: Post, With Pit and Mound of Earth.*—Set a — post, 3 ft. long, 4 in. sq., with marked stone (charred stake or quart of charcoal), 24 in. in the

ground for meander corner of fractional Sections 19 and 20, marked

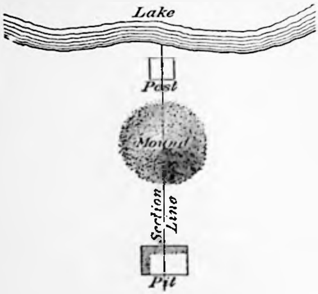


Fig. 19

M C on N,
T 15 N on S,
R 20 E S 20 on E, and
S 19 on W face; dig a pit, 36 × 36 × 12 in., 8 ft. S of post; and raise a mound of earth, 4 ft. base, 2 ft. high, S of corner.

Fig. 19 represents the meander corner just described.

Corners on Reservation or Other Boundaries Not Conforming to the Rectangular System

Corner Monument of Stone, With Deposit

Deposit a marked stone (charred stake, quart of charcoal, or vial with record* enclosed), 12 in. in the ground, for the SW corner of the Nez Perces Indian Reservation; and build a monument of stone, 3 ft. sq. at base, 2 ft. sq. on top, 3 ft. high, over deposit; marked

SW cor N P I R on NE,†
P L ‡ — ‖ M — ‖ ch. on SE,
P L — § on SW, and
P L on NW face.

34. Size, Position, and Distance of Pits and Mounds.—The two following tables summarize briefly the rules relating to the requirements for pits and mounds with respect to their size, position, and distance from the corner.

* The record will consist of a brief description of the corner, with the date of its construction.

† The markings will be cut into large stones, inserted in the middle of the lowest course on each side of the monument.

‡ The letters P. L. indicate public land, unsurveyed.

‖ The proper number of miles and chains, from the initial point, will be stated.

§ The year in which the monument is established will be placed in the blank.

TABLE III

REQUIREMENTS AS TO SIZE AND POSITION OF PITS

Kind of Corner	Size at Tree Corner Inches	Size at Other Corners Inches	Position From Corner
Standard township corner	24×18×12	30×24×12	Across N, E, and W lines
Closing township corner	24×18×12	30×24×12	Across E, W, and S lines
Corner of 4 townships	24×18×12	24×24×12	On lines N, E, S, and W
Corner of 2 townships	24×18×12	30×24×12	On each line
Corner of 1 township	30×24×12	36×36×12	On each line
Standard section corner	18×18×12	24×18×12	Across E, W, and N lines
Closing section corner	18×18×12	24×18×12	Across E, W, and S lines
Corner of 4 sections	18×18×12	18×18×12	In each section NE, etc.
Corner of 2 sections	18×18×12	24×24×12	In both sections
Corner of 1 section	24×24×12	36×36×12	In the section
Quarter-section corner	18×18×12	18×18×12	On line each side
Meander corner	36×36×12	36×36×12	On line, rear of corner
On reservation line	36×36×12	36×36×12	See Manual

TABLE IV

DISTANCE OF PITS AND REQUIREMENTS AS TO MOUNDS

Kind of Corner	Distance of Pits at			Mouuds		
	Post Corner	Mound of Earth Corner Feet	Tree Corner Feet	Size in Feet		Position From Corner
				Stone	Earth	
Standard township corner	E and W 4 feet, N 8 feet .	5	5	$2\times1\frac{1}{2}$	$5\times2\frac{1}{2}$	N
Closing township corner .	E and W 4 feet, S 8 feet .	5	5	$2\times1\frac{1}{2}$	$5\times2\frac{1}{2}$	S
Corner of 4 townships . .	N, E, and W 4 feet, S 8 feet	5	5	$2\times1\frac{1}{2}$	$5\times2\frac{1}{2}$	S
Corner of 2 townships . .	E and W 4 feet, N 8 feet .	5	5	$2\times1\frac{1}{2}$	$5\times2\frac{1}{2}$	Various
Corner of 1 township . .	8 feet	5	5	$2\times1\frac{1}{2}$	$5\times2\frac{1}{2}$	Various
Standard section corner .	E and W 3 feet, N 7 feet .	5	4	$2\times1\frac{1}{2}$	4×2	N
Closing section corner . .	E and W 3 feet, S 7 feet .	4	5	$2\times1\frac{1}{2}$	4×2	S
Corner of 4 sections . . .	$5\frac{1}{2}$ feet	4	5	$2\times1\frac{1}{2}$	4×2	W
Corner of 2 sections . . .	6 feet	4	5	$2\times1\frac{1}{2}$	4×2	W
Corner of 1 section . . .	8 feet	5	5	$2\times1\frac{1}{2}$	4×2	Various
Quarter-section corner . .	3 feet	4	4	$2\times1\frac{1}{2}$	$3\frac{1}{2}\times1\frac{1}{2}$	Various
Meander corner	8 feet	5	8	$2\times1\frac{1}{2}$	4×2	With pit
On reservation line . . .	4 feet	5	5	3×2	$5\times2\frac{1}{2}$	Various

35. Bearing Trees.—When used for standard corners, bearing trees are selected on the north side of the line; when used for closing corners, they are selected on the south side of the line. When the trees can be found within 300 links of the corner, one tree is marked for a corner that relates to only one section, two trees are marked for a corner of two sections only, or for a quarter-section corner or meander corner, and four trees are marked for every corner common to four townships or sections and on reservation lines.

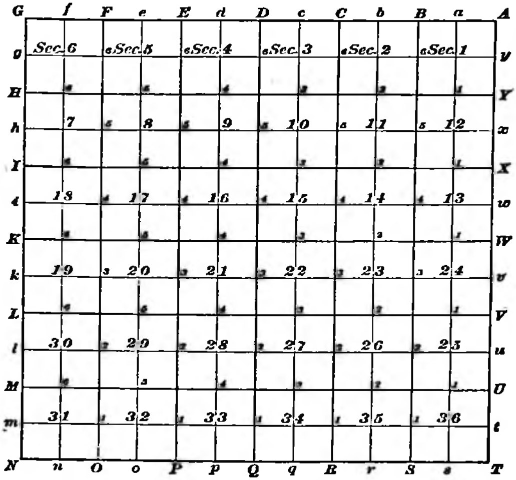


Fig. 20

When only a few bearing trees or rocks are accessible, they are used though even 10 or more chains distant. Bearing trees and objects must have their bearings given from the true meridian and the distance measured from the center of the corner to the center of the bearing tree. The following rule is now in force relative to marks on bearing trees:

Rule.—*Place all letters and figures on that part of the tree which would probably remain as the stump, and make one plain*

blaze high on the same side to attract notice in case of snow or dense undergrowth.

36. System of Designating Corners.—A system of notation that is used by the United States Land Department for designating the different corners in a township is shown in Fig. 20. By the use of this system, field notes can be abridged and rendered more concise and explicit, with correspondingly less liability to error both in recording and reading them. The diagram shown in the figure represents the plat of a township in which the different corners are designated by a conventional system of letters and figures, as shown.*

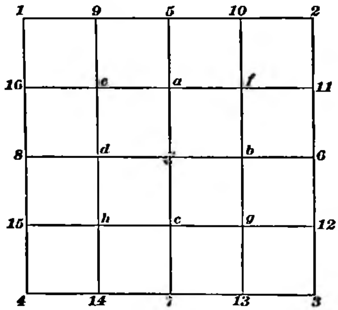


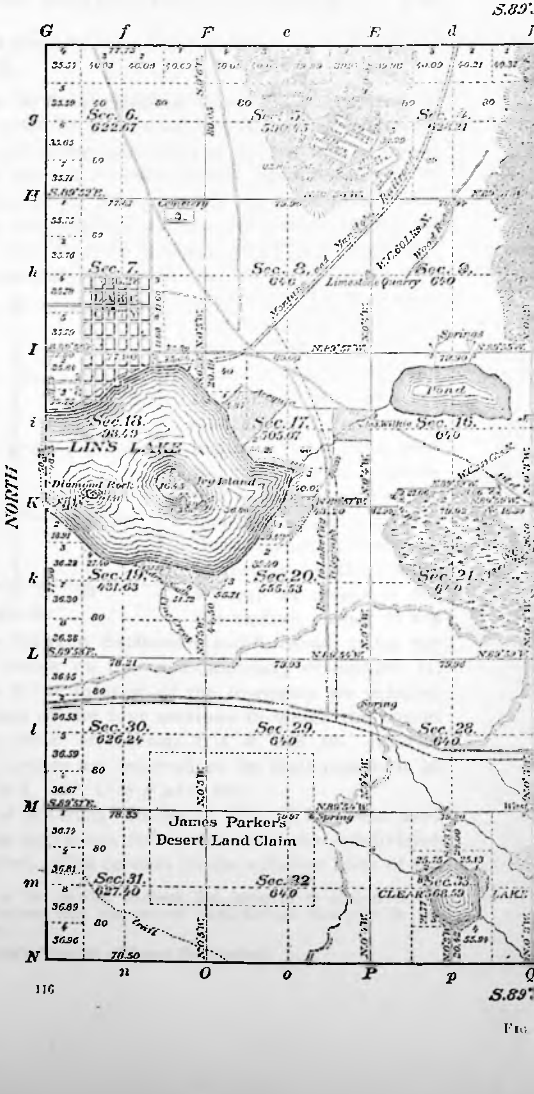
FIG. 21

All corners in the township boundaries are designated by letters, capital letters being used for the section corners and small letters for the quarter-section corners. Such corners are referred to merely by the proper letters, as *corner E* for the section corner in the north boundary between Sections 4 and 5, *corner m* for the quarter-section corner on the west boundary of Section 31. Section corners in the interior of the township are referred to by the numbers of the four sections to which the corner is common, as *corner of Sections 3, 4, 9, and 10.* Interior quarter-section corners are referred to by their positions on the lines, as *V to L at 4, O to F at 5*, etc.

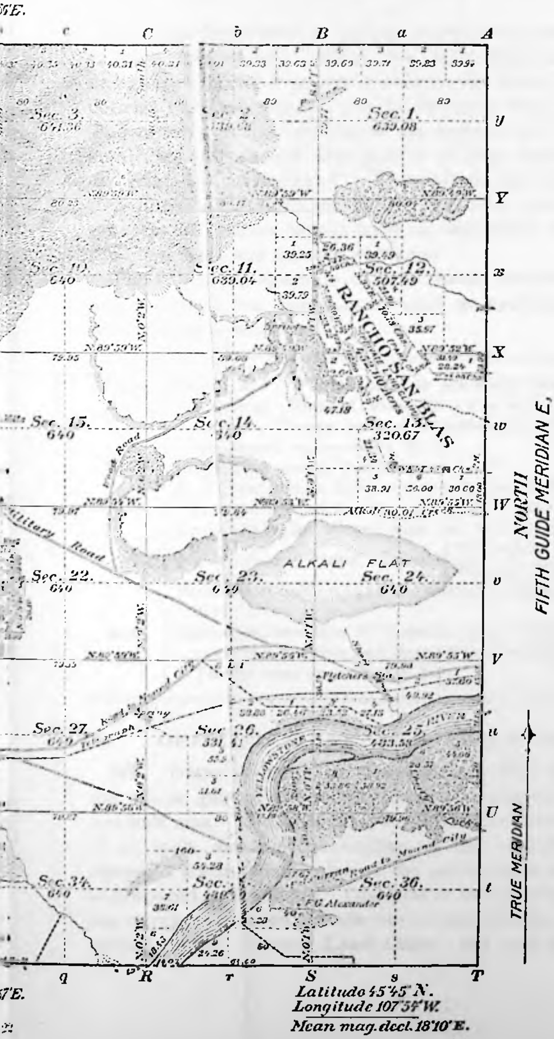
The system of notation† shown in Fig. 21 has been used by many private surveyors for designating the subdivision corners of sections. The corners on the exterior lines of the

*In the plats of the earlier surveys the letter *A* is placed in the upper left-hand corner and the letters pass around the plat in the opposite direction.

†From Hodgman's Manual of Land Surveying.


S.89°
G
f
F
c
E
d
Sec. 6.
622.67
Sec. 5.
Sec. 4.
Sec. 7.
Sec. 8.
640
Sec. 9.
640
Limestone Quarry
Northern and Manitoba Railroad
Wood Road
Cemetery
Springs
Pond
Sec. 18.
Sec. 17.
505.07
Sec. 16.
640
LIN'S LAKE
Diamond Rock
Ivy Island
Sec. 19.
431.63
Sec. 20.
555.53
Sec. 21.
Spring
Sec. 30.
626.24
Sec. 29.
640
Sec. 28.
640
James Parker's
Desert Land Claim
Sec. 31.
627.40
Sec. 32
640
Sec. 33
CLEAR
LAKE
Trail
NORTH
H
h
I
i
K
k
L
l
M
m
N
n
O
o
P
p
Q
S.89°

Fig


A
B
C
a
Sec. 1.
639.08
Sec. 2.
Sec. 3.
Sec. 10.
640
Sec. 11.
639.04
Sec. 12.
507.49
RANCHO SAN BLAS
Sec. 13.
320.67
Sec. 14.
640
Sec. 15.
640
Military Road
ALKALI FLAT
Sec. 22.
640
Sec. 23.
Sec. 24.
640
Sec. 25.
483.58
Sec. 26.
Sec. 27.
YELLOWSTONE
Sec. 34.
640
Sec. 36.
640
Y
X
W
V
U
q
R
r
S
s
T
NORTH
FIFTH GUIDE MERIDIAN E.
TRUE MERIDIAN
Latitude 45°45' N.
Longitude 107°54' W.
Mean mag. decl. 18°10' E.

§ 21

section are numbered in definite order and according to their relative importance. Thus, the four section corners are designated by the first four numbers, the four quarter-section corners on the section lines by the next four numbers, etc. The corners in the interior of the section are designated by small letters, except the center of the section, which is designated by a capital *C*. The corners on the quarter-section lines are designated by the first four small letters, the other small letters being used to designate the corners at the centers of the quarter sections.

The following example of field notes illustrates the manner in which the notation is used in keeping notes when subdividing a section:

Chains	
	Began at 7. Found stake in place and both bearing trees standing. Planted stone 25 × 8 × 6 in. marked + for corner. Thence north on random, dec. 2° 30′ E, setting temporary stakes every 10 ch.
80.22	Intersected section line 26 li. west of 5.
	At 5 found rotten stake at correct point, S 28° W 66 li. from stump of W oak bearing tree of U. S. Survey.
	Drove stake for corner and put broken earthenware and glass around it and marked Wh. oak 12 in. N 66° E 42 li. Wh. oak 18 in. N 34 W 63 li.
	From 5 ran east on random, setting temporary stakes every 10 ch.
39.92	Intersected sec. line 12 li. north of 2. Found earthen post in correct position and bearing trees of resurvey standing. Thence west on corrected line.

OFFICIAL PLAT AND FIELD NOTES

37. Official Plat of Township.—Fig. 22 represents the official plat of the subdivision of a township, reduced to one-half size. The scale used for the official plats is 40 chains to an inch, making the outline of the township 12 inches square. Each official plat of the subdivision of a township is prepared in triplicate. One plat is retained in the office of the surveyor general of the surveying district, one is transmitted to the General Land Office, and one is filed in the

United States land office for the district. When the counties of any state become organized, copies of the official plats of the townships in each county are usually filed in the office of the recorder of deeds for the county, or in the office of the county surveyor, and are open to public inspection.

The township represented in Fig. 22 is township No. 15 north, range No. 20 east (T. 15 N, R. 20 E) of the principal meridian, state of Montana. The notes of the survey of the west and north boundaries of Section 36 and the west boundary of Section 25 of this township are given in Art. 38. A careful study of this plat will be instructive.

On the plat are shown the bearings of all section lines, the lengths of all east-and-west section lines and of the north-and-south section lines in the north tier of sections, the bearings and lengths of all fractional subdivisions; all natural features, artificial features, and improvements, such as lakes, streams, mountains, wooded lands, swamps, roads, railroads, telegraph lines, irrigating ditches, irrigated areas, settlers' claims, ranches, and towns; and such other information regarding the country surveyed as may be of value. All section lines whose lengths are not given have the regular length of 80 chains.

The township is in range 20 east, and, consequently, its east boundary is formed by the fifth guide meridian east of the principal meridian. The respective bearings of the north-and-south section lines, passing to the westward through the township, vary by 1 minute, according to the principle explained in Art. 17.

The north-and-south section line between Sections 21 and 22 and the east-and-west section line between Sections 21 and 16 are offset in order to pass around an impassable swamp. The method of offsetting the lines is clearly shown.

At about the center of Section 33 is a lake lying wholly within the section. The two positions of the north-and-south quarter line are extended from the quarter-section corners on the north and the south boundaries of the section to the respective shores of the lake, and special meander corners are set at the points where these lines intersect the shore of

the lake, and these corners are connected by meander lines delineating the form of the lake. The meander corner on Diamond Rock in Section 18 is an auxiliary meander corner.

It will be noticed that wherever a section line or other survey line crosses a stream, the width of the stream, in links, is marked on the plat.

The letters around the outer edge of the diagram are for the purpose of designating the different section and quarter-section corners on the township boundaries, according to a system that will be explained in the next article.

38. **Field Notes.**—The field notes of the public-land surveys are the record of the deputy surveyor's work. They are the evidence written at the time and place, as the work is performed, and which, if correctly recorded, furnish, after the monuments themselves, the best and most reliable evidence of the location of the boundaries. The field notes of the United States Surveys contain in a condensed form a minute record of everything done by the deputy surveyor and his assistants relative to running, measuring, and marking lines and establishing corners, and the observations made by them relative to the topography of the country surveyed, and such other data as are required to be recorded by them. These notes are usually spoken of as the *original field notes*. They are not necessarily the entries made in the field in the deputy's pocket notebooks or tablets, but the copy from such tablets fully written out in ink for the permanent record of the work. The following specimen of field notes shows the general style and manner of keeping the notes, and also the method of running the west and north boundaries of Section 36, and the west boundary of Section 25. The notes given are those of the respective boundaries of the two sections mentioned, in the township shown by the plat of Fig. 22.

SPECIMEN OF FIELD NOTES

Subdivision of T. 15 N, R. 20 E

Chains	
	I commence at the cor. of secs. 1, 2, 35, and 36, on the S bdy. of the Tp., which is a sandstone, 6 × 8 × 5 in. above ground, firmly set, and marked and witnessed as described by the surveyor general.
	Thence I run
	N 0° 01′ W, bet. secs. 35 and 36.
	Over level bottom land.
4.50	Wire fence, bears E and W.
20.00	Enter scattering cottonwood timber, bears E and W. F. G. Alexander's house bears N 28° W.
29.30	Leave scattering cottonwoods, bearing E and W; enter road, bears N.
30.00	SE cor. of F. G. Alexander's field; thence along west side of road.
39.50	To crossroads, bears E to Mound City; N to Lake City. F. G. Alexander's house bears S 40° W. The ¼ sec. cor. point will fall in road; therefore,
	Set a cedar post, 3 ft. long, 3 in. sq., with quart of charcoal, 24 in. in the ground, for witness cor. to ¼ sec. cor., marked W. C. ¼ S 35 on W and 36 on E face; dig pits, 18 × 18 × 12 in. N and S of post, 3 ft. dist.; and raise a mound of earth, 3½ ft. base, 1½ ft. high, W of cor.
40.00	Point for ¼ sec. cor. in road.
	Deposit a marked stone 24 in. in the ground, for ¼ sec. cor. The SE cor. of Pat. Curran's field bears W, 5 li. dist.
40.50	Set a limestone, 15 × 8 × 6 in., 10 in. in the ground, for witness cor. to ¼ sec. cor., marked W. C. ¼ S on W face; dig pits, 18×18×12 in. N and S of stone, 3 ft. dist.; and raise a mound of earth, 3½ ft. base, 1½ ft. high, W of cor.
	Thence along E side of field.
50.50	NE cor. of Pat. Curran's field, bears W 4 li. dist.
51.50	Leave road; which turns to N 70° W, leads to ferry on Yellowstone River; thence to Lake City.
57.50	Enter dense cottonwood and willow undergrowth, bears N 54° E and S 54° W.
72.50	Leave undergrowth, enter scattering timber, bears N 60° E and S 60° W.
80.00	Set a locust post, 3 ft. long, 4 in. sq., 24 in. in the ground, for cor. of secs. 25, 26, 35, and 36, marked T 15 N S 25 on NE, R 20 E S 36 on SE,

Specimen of Field Notes—(Continued)

Subdivision of T. 15 N, R. 20 E

Chains	
	S 35 on SW, and
	S 26 on NW face; with 1 notch on S and E faces; from which
	An ash, 13 in. diam., bears N 22° E, 26 li. dist., marked T 15 N R 20 E S 25 B T.
	A sycamore, 23 in. diam., bears S 71¼° E, 37 li. dist., marked T 15 N R 20 E S 36 B T.
	A walnut, 17 in. diam., bears S 64° W, 41 li. dist., marked T 15 N R 20 E S 35 B T.
	A cottonwood, 13 in. diam., bears N 21¼° W, 36 li. dist., marked T 15 N R 20 E S 26 B T.
	Last 20 ch. of this mile subject to overflow, 2 to 4 ft. deep.
	Land, level bottom.
	Soil, alluvial; 1st rate. No stones were obtainable.
	Timber, scattering cottonwood, sycamore, ash, and walnut; undergrowth, cottonwood and willow.
	Dense undergrowth, 15 ch.
	S 89° 57′ E, on a random line bet. secs. 25 and 36.
40.00	Set temp. ¼ sec. cor.
79.96	Intersect E bdy. of Tp. 3 li. N of cor. of secs. 25, 30, 31, and 36, which is a sandstone, 5 × 8 × 5 in. above ground, marked and witnessed as described by the surveyor general
	Thence I run
	N 89° 56′ W, on a true line bet. secs. 25 and 36.
	Over level bottom land, through scattering timber.
13.00	Leave scattering timber, bears N and S.
18.60	Cherry Creek, 12 li. wide; clear water, 1 ft. deep; gentle current, sandy bottom; course N.
20.50	Enter heavy timber, bears N and S.
32.50	Leave heavy timber, bears NW and SE.
39.98	Deposit a quart of charcoal, 12 in. in the ground, for ¼ sec. cor.; dig pits, 18 × 18 × 12 in. E and W of cor. 4 ft. dist.; and raise a mound of earth, 3½ ft. base, 1½ ft. high, over deposit. In E pit drive a cedar stake 2 ft. long, 2 in. sq., 12 in. in the ground, marked ¼ S 25 on N and 36 on S face.
46.50	Enter heavy timber, bears N and S.
76.00	Leave heavy, enter scattering timber, bears N 25° E and S 25° W.

SPECIMEN OF FIELD NOTES—(Continued)

Subdivision of T. 15 N, R. 20 E

Chains	
79.96	The cor. of secs. 25, 26, 35, and 36.
	Land nearly level; mostly subject to overflow 2 to 5 ft. deep.
	Heavily timbered land, 41.5 ch.
	N 0° 1′ W, bet. secs. 25 and 26.
	Over level bottom land, through scattering timber.
25.36	Right bank of Yellowstone River.
	Set a locust post, 3 ft. long, 4 in. sq., 24 in. in the ground, for meander cor. of frac. secs. 25 and 26, marked M C on N, T 15 N on S, R 20 E, S 25 on E, and S 26 on W faces; from which A cottonwood, 12 in. diam., bears S 18¼° E, 16 li. dist., marked T 15 N, R 20 E, S 25, M C B T. A sycamore, 31 in. diam., bears S 74½° W, 25 li. dist., marked T 15 N, R 20 E, S 26, M C B T.
	Enter shallow channel, 1 to 2 ft. deep.
26.00	Across shallow channel, 64 li. wide, to sand bar parallel to river bank; thence on sand bar.
32.12	To right bank of main channel, course E; point for triangulation.
40.00	Point for ¼ sec. cor. falls in river.
	To determine the dist. across, I set a flag on line, on left bank; then measure a base, N 89° 59′ E, 20.00 ch. to a point, from which the flag bears N 49° 06′ W; from the flag the E end of base bears S 49° 6′ E; therefore, the dist. is tan. 40° 55′ × base, or .867 × 20 = 17.34 ch.; making the whole distance from meander cor., .64 + 6.12 + 17.34 = 24.1 ch., which added to 25.36, makes
49.46	To left bank of Yellowstone River; bank, 12 ft. high.
	Deposit a marked stone, 12 in. in the ground for meander cor. of frac. secs. 25 and 26, dig a pit, 36 × 36 × 12 in., 5 ft. N of cor. and raise a mound of earth, 4 ft. base, 2 ft. high, over deposit.
	In the pit drive a cedar stake, 2 ft. long, 2 in. sq., 12 in. in the ground, marked M C on S, T 15 N on N, R 20 E, S 26 on W, and S 25 on E faces.

SPECIMEN OF FIELD NOTES—(Continued)

Subdivision of T. 15 N, R. 20 E

Chains	
	Thence over level bottom land. Some small cottonwoods, none within limits suitable for bearing trees.
52.60	Leave bottom, begin ascent, bears E and W.
53.60	Top of ascent and edge of sandy plain, 40 ft. above river, bears E and W.
55.70	Wire fence, bears E and W.
62.80	Telegraph line, bears E and W.
80.00	Set a cedar post, 3 ft. long, 4 in. sq., with marked stone, 24 in. in the ground, for cor. of secs. 23, 24, 25, and 26, marked T 15 N, S 24 on N E, R 20 E, S 25 on S E, S 26 on S W, and S 23 on N W faces; with 2 notches on S and 1 notch on E edges; dig pits, 18 × 18 × 12 in. in each sec. 5½ ft. dist.; and raise a mound of earth, 4 ft. base, 2 ft. high, W of cor.
	Land, level.

39. Modifications and Changes.—There have been various modifications and changes in the methods of conducting the United States land surveys. In the surveys made previous to 1846, and in many of the surveys made since that time, closing corners were established on the north and west boundaries of every township, thus making a double set of corners on every township boundary. In these surveys, only two bearing trees were marked for any corner. Other trees were lettered for the township, range, and section, but their bearings and distances from the corners were not taken. These were spoken of as **witness trees** to distinguish them from bearing trees, but they do not appear in the field notes.

There has been much difference in the manner of marking bearing trees in the different surveying districts. One custom, quite prevalent in the earlier surveys, was to cut a blaze about a foot from the ground on the side of the tree facing the corner and then cut a notch in the middle of the blaze with two blows of the axe. The distance to the corner

was measured from this notch or witness mark. The tree was also blazed at the height of a man's head, and the required letters were cut in the tree within this blaze.

The early requirements for the surveys were not rigid with regard to either the directions or the lengths of the lines run, and such as they were, they were not enforced. Both base lines and meridians have in some instances been run by the magnetic needle in connection with, and as a part of, the survey of the township boundaries. The alinement was often bad and the chaining was no better. The surveyor who now retraces the lines of the United States land surveys will find in them every stage and condition of work from careful, well-executed surveys to works of the imagination made mainly on paper in the deputy surveyor's camp. Errors in direction, not of minutes merely, but of degrees, and errors of distance not of links merely, but of chains and tallies,* are not uncommon.

In the earlier surveys, the areas of fractional lots were computed by the deputy in the field, with the result that in many cases the areas thus given on the official plats cannot be verified from the field notes by any known method of computation. Indeed, they appear to have been guessed at without any computation whatever.

*Tally is the name applied to each distance of 10 chains, at the extremity of which the chainmen exchange pins and call *tally*.

UNITED STATES LAND SURVEYS
(PART 2)

SUBDIVISION AND RESURVEYS

SUBDIVISION OF SECTIONS

1. The general principles governing the subdivision of sections are to be found in the United States statute of February 11, 1805, of which the following are the leading provisions:

SECTION 100. The boundaries and contents of the several sections, half sections, and quarter sections of the public lands shall be ascertained in conformity with the following principles:

First.—All the corners marked in the surveys returned by the surveyor general shall be established as the proper corners of sections, or subdivisions of sections, which they were intended to designate, and the corners of half and quarter sections, not marked on the surveys, shall be placed as nearly as possible equidistant from two corners which stand on the same line.

Second.—The boundary lines, actually run and marked in the surveys returned by the surveyor general, shall be established as the proper boundary lines of the sections or subdivisions for which they were intended, and the length of such lines as returned shall be held and considered as the true length thereof. And the boundary lines which have not been actually run and marked shall be ascertained by running straight lines from the established corners to the opposite corresponding corners; but in those portions of the fractional townships, where no such opposite corresponding corners have been or can be fixed, the boundary lines shall be ascertained by running from the established corners due north-and-south or east-and-west lines, as the case may be, to the water course, Indian boundary line, or other external boundary of such fractional township.

Third.—Each section or subdivision of section the contents whereof have been returned by the surveyor general shall be held and considered

as containing the exact quantity expressed in such return; and the half sections and quarter sections the contents whereof shall not have been thus returned shall be held and considered as containing the one-half or the one-fourth part, respectively, of the returned contents of the section of which they may make part.

SECTION 101. In every case of the division of a quarter section the line for the division thereof shall run north and south, and the corners and contents of half-quarter sections which may thereafter be sold shall be ascertained in the manner and on the principles directed and prescribed by the section preceding, and fractional sections containing one hundred and sixty acres or upwards shall in like manner, as nearly as practicable, be subdivided into half-quarter sections, under such rules and regulations as may be prescribed by the Secretary of the Interior, and in every case of a division of a half-quarter section, the line for the division thereof shall run east and west, and the corners and contents of quarter-quarter sections, which may thereafter be sold, shall be ascertained, as nearly as may be, in the manner and on the principles directed and prescribed by the section preceding; and fractional sections containing fewer or more than one hundred and sixty acres shall in like manner, as nearly as may be practicable, be subdivided into quarter-quarter sections, under such rules and regulations as may be prescribed by the Secretary of the Interior.

It will be seen that the statute makes the corners and boundary lines of the United States land surveys unalterable, no matter what errors may have been committed in locating them. Although errors in the surveys have been numerous and some of them great, the surveys cannot be changed, and, however erroneous they may be, the boundaries and corners established by them are the true boundaries and corners. The statute also makes the length of the lines as returned by the surveyor general (which is the length given in the field notes) their true legal length. This defines and establishes a standard of measure for the line between every two corners of the government survey, which is the only legal standard for that line. All subsequent surveys under that system must make their measures conform to the standard of the measurement recorded in the field notes and marked on the ground by the corner monuments.

In subdividing a section, it may be necessary to first make a resurvey or retracement of the section in order to find or relocate any missing corner of the original survey. Supposing

all those corners to be known, there are four cases in subdividing sections not made fractional by waters or reservations. There is one general principle, however, that applies to all these cases, namely: *The section is divided into quarters by straight lines extending from the established quarter-section corners to the opposite corresponding corners, that is, from quarter post to opposite quarter post.*

The four cases in which the subdivision of a section, though the same with respect to the principle just stated, differs with respect to other conditions, are as follows:

2. Case I.—*Regular Sections.*—In the regular sections, such as Sections 8, 16, etc., all the corners on the exterior boundaries of the section are established by the original survey. The north-and-south lines or meridional boundaries, are 80 chains in length. The east-and-west lines, or latitudinal boundaries, though intended to be 80 chains in length, may or may not be that length, and usually are not. All the quarter-section corners are midway between the two section corners standing on the same section line. The section is divided into quarter sections by straight lines extending from each quarter-section corner to its opposite corresponding corner. The intersection of these lines establishes the legal center of the section; that is to say, the common corner of the four quarter sections.

To divide the quarter section, corners are first established on its north and south boundaries at points midway between the corners of the quarter section; that is, the corner on the section line is established midway between the section corner and the quarter-section corner, and the corner on the quarter-section line is established midway between the quarter-section corner on the section line and the corner at the center of the section. A north-and-south line running straight between the corners thus established divides the quarter into half-quarters. In a similar manner, the half-quarter is divided into quarter-quarters by an east-and-west line. This east-and-west dividing line may extend entirely across the quarter section, from a corner on the section line

midway between the section corner and quarter-section corner to a corner on the quarter-section line midway between the quarter-section corner on the section line and the corner at the center of the section, substantially the same as the north-and-south half-quarter line; or, in case it is desired to divide only one of the half-quarters, the line may extend from either of the above described corners to a corresponding corner established midway between the extremities of the north-and-south half-quarter line. It practically makes no difference whether the center of the quarter section is fixed at the intersection of the lines extending entirely through the quarter section, or at the center of the half-quarter line. The same result will be obtained in either case, but the latter method is usually the more expeditious when only one half-quarter is to be divided.

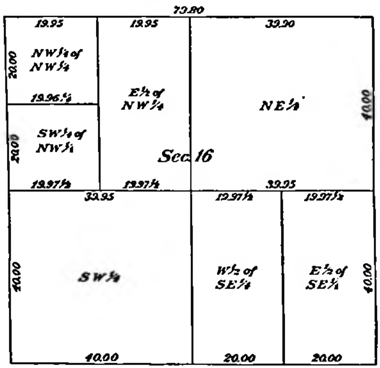


Fig. 1

Fig. 1 illustrates the manner of subdividing a regular nonfractional section, and the method of designating of the various subdivisions. The lengths of the latitudinal lines are merely assumed. In selling the public lands, the statute requires that these regular sections be "held and considered" as containing the exact 640 acres, and they are so sold, but as a matter of fact they seldom do contain exactly 640 acres, as is shown by the field notes of the United States survey. A complete subdivision would divide the section into quarter-quarters, or *forties*, as these lots are familiarly called. In the figure, the west half of the northwest quarter section is shown divided into quarter-quarters.

3. Case II.—*Fractional Sections Adjoining the North Boundary.*—This case relates to the subdivision of Sections 1, 2, 3, 4, and 5. These comprise all the sections adjoining the north boundary of the township except Section 6. The south half of each of these sections is regular and is subdivided in the manner described in Case I. Each quarter-section corner on a north-and-south section line through this tier of sections is at the regular distance of 40 chains north of the section corner on the section line forming the south boundary of the section. If the section adjoins a base line or standard parallel, such base line or standard parallel forms its north boundary, and its corners on such a line are not standard corners, but are merely closing corners, and do not affect the positions of any lines except those that close on them. In many of the older surveys, the same is true of these sections adjoining township lines that are not standard lines. These closing corners are the section corners for the sections adjoining the standard or township line on the south; that is, the sections now under consideration. *No corresponding quarter-section corners for these sections are marked in the original survey.* Quarter-section corners are established on the latitudinal township and standard lines when these lines are surveyed, but these quarter-section corners relate to the sections north of the lines; they do not in any way relate to the sections south of the lines or to the closing corners.

In subdividing the north half of the section under such conditions, the first thing necessary is to establish on the township line the quarter-section corner that relates to the section south of it. This is done by placing the quarter-section corner midway between the two corners of the section that stand on that line; that is, midway between the two closing corners.

The quarter sections adjoining the township line are divided into half-quarters by an east-and-west line that is 20 chains, *original measure*,* north of the corresponding

* By *original measure* is meant the measure actually laid down in the ground by the United States deputy surveyor when he ran the line and set the corners. A remeasure of the line is not to be expected to agree with this measure, but must be made to conform with it, as described elsewhere. See Art. 9.

quarter line, thereby throwing all of the fractions into the north tier of lots. The south half-quarters are thus of regular size. If either of the north or fractional half-quarters is further subdivided, this is done by a north-and-south line running midway between its end lines.

Fig. 2 illustrates the method of subdividing Sections 1, 2, 3, 4, and 5, and the method of describing the various lots. The fractional distances are of course merely assumed. The fractional lots are all in the north tier. The official plats give the area of these fractional lots for which they are sold.

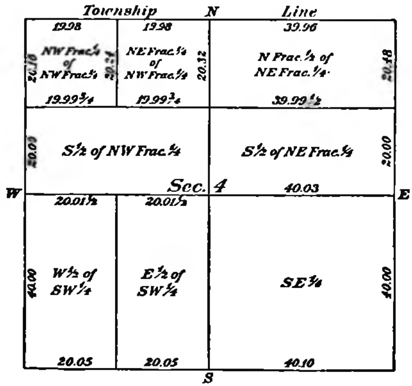


Fig. 2

The remaining lots are all sold as containing the exact 40, 80, or 160 acres, as the case may be. The fractional lots are also sold by number. The numbers commence with No. 1 for the northeast fractional quarter of the northeast fractional quarter, and run west to No. 4 for the northwest fractional quarter of the northwest fractional quarter. For the method of numbering all fractional lots, see the plat of a township in *United States Land Surveys*, Part 1.

4. Case III.—*Fractional Sections Adjoining the West Boundary.*—This case relates to the subdivision of Sections 7,

18, 19, 30, and 31. These comprise all the sections adjoining the west boundary of the township except Section 6. The east half of each section is regular and is subdivided as described in Case I. The quarter-section corner on each east-and-west section line through this range of sections is established at a distance of 40 chains west of the section corner on the section line forming the east boundary of the section, and consequently, the north-and-south quarter-section line through each section is parallel with and 40 chains distant from the east line of the section. In case the corners of the

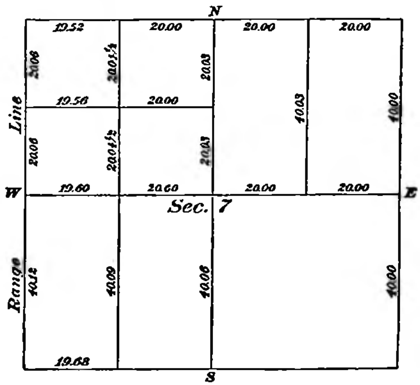


FIG. 8

section on the range line forming its west boundary are not standard corners but are merely closing corners, no quarter-section corner for the fractional section was planted by the original survey, and, consequently, the quarter-section corner for this section must be established on the range line midway between the two corners of the section that stand on that line. The fractional quarter sections adjoining the range line are divided into half-quarters by a north-and-south line 20 chains, by original measure, west from the corresponding quarter line, thus throwing the fractions all into

the west range of lots. The half-quarters are divided into quarter-quarters, the same as in Case I, by east and west lines. For method of numbering the fractional lots, see plat of township previously referred to.

Fig. 3 illustrates the method of subdividing Sections 7, 18, 19, 30, and 31. The lots adjoining the range line are sold as fractional, all others as containing the regular amount.

5. Case IV.—*Fractional Section in Northwest Corner.* Section 6, which is the fractional section in the northwest corner of the township, lying next to both the north and the west boundaries, is subject to the rules of subdividing applicable to both Case II and Case III. If the section corners at the northeast and southwest corners of the section are merely the closing corners for the section lines closing on the township and range lines, the quarter-section corners on the township and range lines forming the north and west boundaries of the section must first be established before the section can be divided into quarter sections. The quarter-section corner on the north boundary on the section should be placed in the township line at a distance of 40 chains, original measure, west from the closing section corner at the northeast corner of the section. The quarter-section corner on the west boundary of the section should be placed in the range line 40 chains, original measure, north from the section corner at the southwest corner of the section. The section is then divided into quarters by lines joining the opposite quarter-section corners, the same as in the other cases. The southeast quarter is regular and is subdivided as described in Case I. The northeast quarter is fractional and is subdivided in the same manner as described in Case II. The southwest quarter is fractional and is subdivided in the same manner as described in Case III. The northwest quarter is fractional in both its north-and-south and its east-and-west dimensions. It is divided into half-quarters by an east-and-west line parallel with and 20 chains, original measure, north from the east-and-west quarter line. The half-quarters are divided into quarter-quarters by a north-and-south line

parallel with and 20 chains, original measure, west from the north-and-south quarter line.

Fig. 4 illustrates the method of subdividing Section 6 and of numbering the fractional lots.

6. Quarter-Section Corners on Township Boundaries.—Under the present system of United States land surveys, no closing corners are allowed except on base lines and correction parallels. In all other cases, the section lines closing on township or range lines are run as random lines and corrected back so as to close on the true corners. As a

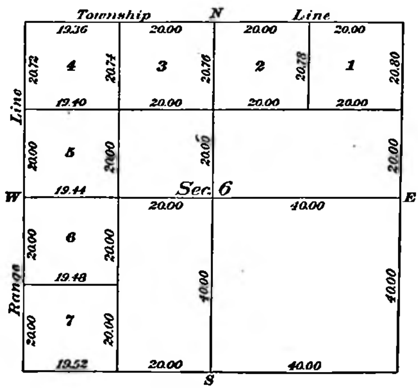


FIG. 4

consequence, the section and quarter-section corners on the township and range lines are common to the sections on both sides of the line, and form the basis of their subdivision. But in all the earlier surveys, closing corners were planted in the north and west boundaries of the township and no quarter-section corners were set on those boundaries for the fractional sections adjoining them on the south and east. These missing quarter-section corners are to be established by placing them midway between the two corners of the section; that is, the two closing corners that stand on

the same line. Section 6 is an exception to this rule. The quarter-section corners on the township and range lines are to be located at a distance of 40 chains, original measure, from the section corner at the northeast and southwest corners of the section, respectively, as described in the preceding article. This may readily be done by measuring along the township or range line from the standard quarter-section corner a distance equal to that between the standard and closing section corner at the northeast or southwest corner of the section, as the case may be, as found by actual measurement, and in the same direction that the closing section corner is from the standard section corner.

For example, suppose that it is stated in the field notes that the closing corner on the township line for the section line between Sections 5 and 6 is 135 links east of the standard corner for Sections 31 and 32 of the township north

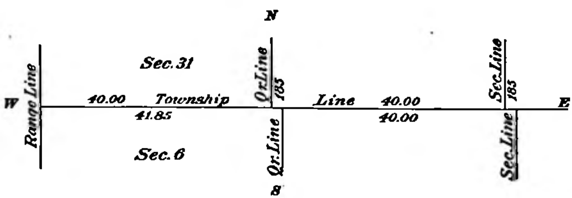


FIG. 5

of the township line, but by actual measurement the distance is found to be 185 links. In this case the quarter-section corner for Section 6 is placed on the township line at the same distance; namely, 185 links, actual measure, east of the standard quarter-section corner for Section 31 of the township north of the line. This is illustrated in Fig. 5, which shows the method of locating the north quarter post of Section 6.

7. Sections Made Fractional by Water, Etc.—The statute provides that "in those portions of fractional townships where no opposite or corresponding corners have been or can be fixed, the boundary lines not actually run and

marked shall be ascertained by running from the established corners due north-and-south or east-and-west lines, as the case may be, to the . . . external boundary of such fractional township." The section of the statute of 1805 from which the above is quoted evidently refers to surveys made previous to that time in which only the township boundaries were run and marked on the ground, and the subdivision into sections and the lesser tracts was made only on paper in the official plats. The general principle, however, has been applied to the location of the quarter-section lines in the subdivision of fractional sections.

The law presumes the section lines surveyed and marked in the field by the United States deputy surveyors to be due north-and-south or east-and-west lines, as they were intended to be, although as a matter of fact they are not actually so in all cases. The United States Land Office accordingly interprets the phrase "due north-and-south or east-and-west lines," as it occurs in the statute, to mean the lines surveyed and marked as such by the United States deputy surveyor, and holds that, in running the subdivision lines, mean courses must be adopted, or the subdivision lines must be run parallel to the existing section line when there is no section line on the opposite side of the subdivision line; that is, in case one of the quarter-section corners of a section was never set because of being inaccessible, so that the opposite quarter-section corner is the only corner that defines the position of the corresponding quarter-section line, this line must be run from the latter corner on a course that is a mean of the courses of the two section lines between which it is situated, and in case there is only one such section line, the course of the quarter-section line is made the same as

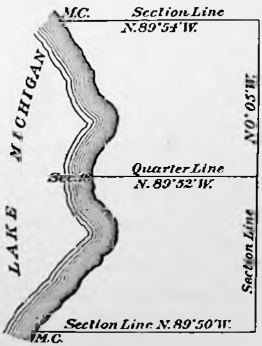


FIG. 6

that of this section line. Thus, suppose that in the fractional section represented in Fig. 6, it is found by trial that the course of the south line of the section is N 89° 50′ W, and that of the north line is N 89° 54′ W. Then the quarter line should be run westwards from the quarter-section corner on the mean course, which in this case is N 89° 52′ W. But if, for example, the lake extended eastwards entirely across the north portion of the section, so that there was no north section line for this section, the quarter-section line would be

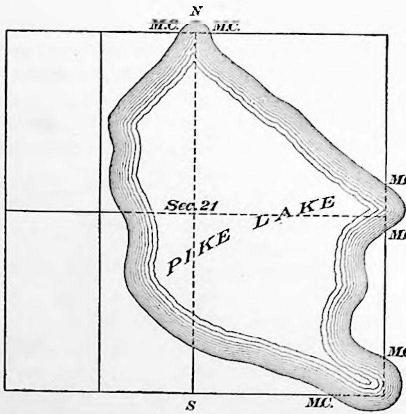


FIG. 7

run westwards parallel to the south section line, or on a course N 89° 50′ W in this case.

In case the missing corner can be established, as, for instance, by locating it on the ice in winter or by any other practical method, it should be located and the subdivision made in the manner described in Cases I, II, III, and IV. This is illustrated in Fig. 7, which represents a fractional section from which the north and the east quarter-section corners and the southeast section corner were omitted in the original survey by reason of being in a lake. When the

lake is frozen over, the positions of these corners can easily be located from the meander corners and the section can be subdivided in the regular way.

8. **Exceptions.**—There are many exceptions to the preceding methods for subdividing the fractional sections adjoining the township boundaries. The official plats in accordance with which the land was sold have been made up in various ways. In many cases, where the intersections of the section lines with the township boundaries were marked by closing corners, the north-and-south quarter-section lines of Sections 1, 2, 3, 4, 5, and 6 have been marked on the plats, and the areas of the fractional subdivisions computed, on the assumption that those lines were parallel with the east lines of the respective sections. In other cases, these quarter-section lines have been marked and the fractional areas computed on the assumption that the standard quarter-section corners on the township boundary were common to the sections on both sides of the line. This frequently made a wide discrepancy between the section lines and the intermediate quarter line in the sections bordering the township lines on the south. In Sections 6, 7, 18, 19, 30, and 31 the east-and-west quarter lines have sometimes been shown on the plats as being parallel with the south lines of the section.

In some cases, the areas of the fractions as laid down on the official plats can be accounted for only on the assumption that they were guessed at, as they undoubtedly were, without any computation whatever. As all sales of land are made by the United States Land Office in accordance with the official plats, the rule that has been adopted to meet all such cases is "to subdivide in such a way as to suit the calculation of the areas on the official plat." This cannot always be done.

Many of the surveys themselves have been made in a manner different from the present requirements. In some cases, the fractional lots have been thrown into the south and east tiers of sections in the township; and in other cases, where there were large streams or lakes, the surveys have

been made so as to throw all the fractions into the lots bordering on the water. In all exceptional cases, the official plat must be followed in making the subdivisions; and in cases where it is impossible to follow it literally and exactly, its intention must be interpreted as closely as possible.

9. Proportional Measures.—For each line of the public land survey, the length returned in the field notes is, by statute, made the true legal length of the line, as stated in Art. **1**. But it is rare that, in remeasuring a line of the original survey, its length, as remeasured, is found to be the same as the length given in the field notes. The length of such a line, as returned in the field notes, is here called its length by **original measure**, and its length as actually determined by measurement is called its length by *actual measure* or its *remeasured* length. Thus, if according to the field notes the distance between a section corner and the adjacent quarter-section corner is 40 chains, a distance of 20 chains, original measure, would be just one-half the distance between the corners, although this distance may be 21 chains by actual measure.

In subdividing sections, the surveyor must use the original measure; that is, he must make his measures agree with the recorded lengths of the lines. This is practically done in the field by determining the difference between the recorded and remeasured lengths of any given line and distributing this difference throughout the several portions of the line in proportion to their respective lengths. By this means the subdivision corners on the line are placed in such positions that the distance between two corners is to the corresponding recorded distance as the remeasured length of the line is to its recorded length. This will be easily understood by an example.

EXAMPLE.—Suppose that the returned length of the line running north from the quarter-section corner between Sections 5 and 6 to the section corner on the township line is 42 chains, and that this distance, as remeasured, is found to be 42.63 chains. (*a*) What should be the remeasured length of the south part of the line or distance from the quarter-section corner to the half-quarter corner? (*b*) What should

be the remeasured length of the north part of the line or distance of the latter corner from the section corner?

First Solution.—(*a*) The excess of the remeasured length of the line over its recorded length is equal to $42.63 - 42 = .63$ ch. This excess is divided between the two parts of the line in proportion to their respective lengths. The south part of the line should have a length of 20 ch., original measure, and if x denotes the amount of excess to be given to this part of the line, its value may be expressed by the proportion

$$42 : 20 = .63 : x$$

from which $$x = \frac{20}{42} \times .63 = .3 \text{ ch.}$$

Hence, the length of this portion of the line by the standard of the remeasure should be equal to $20 + .3 = 20.3$ ch. Ans.

(*b*) By a similar process, the amount of excess that must be given to the north part of the line is found to be equal to $\frac{22}{42} \times .63 = .33$ ch., and the length of this part of the line according to the remeasure should be equal to $22 + .33 = 22.33$ ch. Ans.

Second Solution.—(*a*) A rather more direct, though no more accurate, solution is obtained by letting x denote the required length of the part of the line under consideration. Then, for the length of the south part of the line, the proportion is

$$42 : 20 = 42.63 : x$$

from which $$x = \frac{20}{42} \times 42.63 = 20.3 \text{ ch. Ans.}$$

(*b*) Similarly, for the north part of the line, the proportion is

$$42 : 22 = 42.63 : x$$

from which $$x = \frac{22}{42} \times 42.63 = 22.33 \text{ ch. Ans.}$$

EXAMPLES FOR PRACTICE

1. If the returned length of the northern boundary of Section 6 was 77.75 chains, and this distance, as remeasured, is found to be 81 chains, what should be the remeasured length of that part of the line extending from the section corner on the township boundary between Sections 5 and 6 to the north quarter-section corner of Section 6? Ans. 41.67 ch.

2. Suppose that the distance from the section corner for Sections 9, 10, 15, and 16 to the section corner for Sections 3, 4, 9, and 10, as remeasured, is found to be 82.64 chains; at what distance from the nearer section corner should each quarter-quarter corner on this line be set in subdividing the section? Ans. 20.66 ch.

3. If the returned length of the line from the quarter-section corner between Sections 5 and 6 to the section corner on the township

boundary is 38.67 chains, and the length of the line as remeasured is found to be 39.72 chains: (*a*) what should be the remeasured length of the south part of this line or the distance from the quarter-section corner to the half-quarter corner? (*b*) what should be the remeasured length of the north part of the line or distance of the half-quarter corner from the section corner?

Ans. { (*a*) 20.54 ch.; (*b*) 19.18 ch. }

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10. Accretions or Alluvium.—Some of the most difficult problems encountered by the land surveyor are those that result from the changing of the courses of streams or the shores of lakes. Owners of land bordering on navigable waters hold to high-water mark. On our non-navigable waters they usually hold to the center of the stream or lake, as the case may be, subject to the public rights of fishing, etc. When the shore line is changed by the recession of the waters or by additions gradually made by deposit from the water, it frequently becomes necessary to extend boundary lines over the land thus formed. Such newly formed land is called **alluvium,** or **accretions.** It is also necessary at times to extend these lines under the water for various purposes. The principle by which the lines are extended both over accretions and under water is based on the common law. **Common law** is law that is the outgrowth of customs, usages, and judicial decisions, in distinction from **statute law,** which consists of laws enacted by the law-making power of the country, The common law is recognized as binding wherever there is no statute law to the contrary.

11. Under the common law, it is taken for granted that the water front is an important part of the value of a lot. Hence, in extending the boundary lines to or under the water, they are so extended that each shore owner will retain his proportional share of the water frontage. This will frequently result in very complicated problems for the surveyor who is called on to locate the boundaries across the newly formed land. The lines will not be straight extensions or continuations of the established boundaries, but will run at

some angle therewith and with the shore or thread of the stream. The **thread of the stream** is defined as its center line, as measured between the shores, regardless of the current. The rule for extending these boundary lines has been laid down by the courts as follows:

"Measure the whole extent of the ancient line on the river and ascertain how many feet each proprietor owned on this line. Divide the newly formed river line into an equal number of parts, and appropriate to each owner as many of these parts as he owned feet on the old line. Then draw lines from the point at which the proprietors, respectively, bounded on the old to the points thus determined as points of division on the newly formed shore." "This rule is to be modified under some circumstances, as for instance, if the ancient margin has deep indentations or sharp projections, the general available line of the river ought to be taken and not the actual length of the margin as changed by the indentations or projections."

This rule was adopted for land bordering on rivers, but the same principle has been applied by the courts to land bounding on other waters. The surveyor will find many cases, especially on lands bordering on small lakes of irregular outline, where he cannot apply any mathematical rule to their solution, but will have to use his common sense and good judgment to determine what line will best preserve the equities of the different proprietors. In general, however, the boundary of lots fronting on lakes is extended under the water on a line running at right angles with the general course of the shore. Lots fronting on non-navigable streams have their boundaries extended at right angles with the thread of the stream.

An exception to this is found in the state of Indiana, in which state this common-law rule is not applied. The courts of that state hold that the boundary lines of subdivisions of land of the United States survey bordering on waters shall be extended the same as they would have been if no water had been there. As a result of this ruling of the courts in the state of Indiana, the owner of land fronting on water in

that state may be deprived of his entire water frontage by the shifting of a stream or the recession of a lake, and, at the same time, the water frontage of the adjacent land owner may be greatly increased.

12. The manner in which boundary lines are extended over accretions is illustrated in Fig. 8, which represents a fractional section containing a lake that lies chiefly in the northeast quarter of the section, though it extends into each of the other three quarter sections, and also into the adjacent section. The line *a b c d e* represents the original shore of the lake, *a* and *e* being regular meander corners. After a number of years, it is found that the surface of the water in

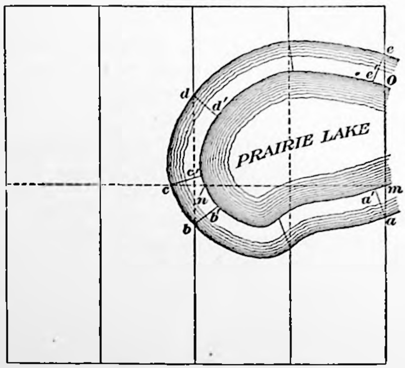

FIG. 8

the lake has become lowered, which has changed the shore line to the position represented by the line *a′ b′ c′ d′ e′*, and it is necessary to extend the boundary lines of the adjoining properties over the newly formed land to the present shore of the lake. According to the common-law rule for extending such boundary lines, they would be extended on the lines *a a′*, *b b′*, etc. in directions substantially at right angles to the shore line, thus giving each property owner his proportionate amount of the newly formed land and proportionate

length of the new shore line. If this fractional section were in the state of Indiana, however, the boundary lines would be extended the same as though no water had been there, that is, they would be continuations of the regular subdivision lines of the section, and would be in the same positions that these portions of the subdivision lines would have been in had the present shore $a'\,b'\,c'\,d'\,e'$ been the shore of the lake at the time of the original survey. As the effect of this, the shore line of the southeast fractional quarter section, which formerly extended from *a* to *b*, would now extend from *m* to *n*, and the shore of the northeast fractional quarter section, which formerly extended from *d* to *e*, would now extend from *n* to *o*, which would give this quarter section a much greater water frontage than it had originally. At the same time, the southwest and northwest quarter sections, which originally had the water frontages *b c* and *c d*, respectively, would be entirely deprived of their water frontage. If each fractional quarter section is divided into fractional half-quarters, the injustice in this distribution of the water frontage is even more apparent.

RESURVEYS

FINDING CORNERS

13. General Principles.—In an old settled country, much of the work that the land surveyor has to do consists in finding old corners, restoring them when obliterated or destroyed, and retracing old boundary lines, either as a basis from which to locate other lines or for the purpose of settling disputed questions of boundary. In doing this, as well as in some original surveys, the surveyor is controlled by certain legal requirements that do not at all times harmonize with accurate instrumental work or correct mathematical computations. In an original survey, the problem is to locate the corner or line where it ought to be. In a resurvey, it is to find where the corner or line actually was located, and not where it ought to have been located.

It is a leading provision of the United States law relative to the surveys of the public lands that the corners and lines actually marked on the ground and returned in the field notes of those surveys must be established as the proper corners or lines that they were intended to designate. After the land has been sold in accordance with that survey and record, no one—not even the government itself—has any right or authority to correct any errors that may have been made in the original survey, or to change it in any way. By common law, the same is true of the lot corners and lines of village or city lots where they are sold by plat and the corners and lines have been marked on the ground previous to the execution of the plat. It is also true of all other boundaries of land that are marked on the ground at the time of, or previous to, the sale and transfer made in accordance therewith. Though neither the directions nor distances of boundaries, nor the areas enclosed, correspond with the description or plat, the original stakes, monuments, and marked lines are conclusive as to the boundaries.

In the United States land surveys, no special weight or preponderance is given to any corner above that of any other corner of the same survey. A quarter-section corner is entitled to just as much respect or authority as a section corner or a township corner. So long as it can be found and identified, there it must stay and control the location of the subdivision lines of the section. If lost or destroyed, the corner must be found or relocated. By a lost corner is here meant a corner that is still in existence and not destroyed, but whose location is lost and cannot be determined without a resort to surveys from distant corners.

14. The Search for a Corner.—In the directions heretofore given for subdividing sections, it was presupposed that the section and quarter-section corners of the original survey were known. When a number of years have passed since the original survey was made, however, it is usually the case that some of these corners have been lost or totally destroyed. In such cases, it is the surveyor's first work to

find the corners if lost, or to relocate them if destroyed or obliterated.

To find a lost corner is largely a matter of skill and evidence. The evidence may be of any kind that would tend to prove the location of the corner. The best evidence is the monument itself as planted by the original survey. When looking for the monument, the surveyor should know, to begin with, just what he is looking for—whether a stake, a stone, a hole in a rock, or whatever else it may be. This he will learn from the field notes, if it is the original monument; from any other available testimony, in case a new monument has been planted in place of the original monument. The field notes will also describe the witness or bearing trees or objects, if there are any. If more than one bearing tree or object can be found, the point where the corner post or monument should be found will be indicated by the intersection of arcs drawn from these objects with radii, equal, respectively, to the distances of the objects from the corner, as given in the field notes. If only one bearing tree or object remains, it is necessary to measure from it the recorded distance in a direction just opposite to its recorded bearing, and at the extremity of the distance, search for the monument.

15. Decayed Wooden Post.—If a wooden post is the object sought, it may have decayed so that nothing but the rotten wood is left, and in that case careful work is required. The surface dirt should be removed to the depth of a few inches or to a point where the subsoil has not been disturbed. It is very essential not to drive the spade down into the earth and throw it out by spadefuls, as one may thus destroy what one is looking for without knowing it. The surface should be pared, shaved, or scraped down a little at a time, and very carefully, by clean cuts of the spade. If the search is made properly and in the right place, and the earth has not been previously removed or disturbed, the remains of the post are very likely to be found, even though a great many years have passed since it was planted. If the soil is a stiff clay, packed hard or covered with a sward, a hole will

be found of the size and shape of the post that made it, and it will contain the decayed remains of the post. By carefully cutting down at the side of the hole, its size, shape, and direction can be seen. Fig. 9 represents the appearance of a decayed wooden post when the earth has been carefully removed from the surface, then an excavation made at one side of the post and the side of the excavation toward the post shaved away carefully by vertical cuts of the spade until the decayed remains of the post are seen in the position in which it was driven.

The position of a corner is often as well marked by the decayed remains of the post as it was by the sound post. Such corner monuments often outlast those of iron or stone, being less subject to removal by vandal hands. They are best seen in light-colored clay subsoils. In sand, the cavity made by the decaying post is gradually filled by the falling sand, but the decayed wood mingles with this loose sand and discolors it to such an extent that where it has not been disturbed, the position of the post can be easily traced. In black muck, it is not so easy to distinguish the remains of the post near the surface, but such soils are soft, so that the posts, being driven easily, are usually driven deep, and where it is also wet, the bottom of the post will be preserved in a sound condition for a great length of time.

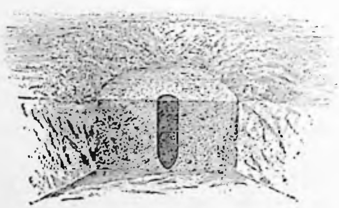

FIG. 9

16. Bearing Trees.—Both bearing trees and witness trees may usually be known by their kind and size, and by the scars on them. In some cases, the trees have grown to such an extent that it is necessary to cut away the wood that has grown over the marks, in order to be certain that they are the right trees, especially if there are other similar trees near by. As cutting away the wood injures the tree, it

should not be done unless absolutely necessary in order to establish the corner.

When the bearing trees themselves have entirely disappeared and no trace of them remains above the surface of the ground, unmistakable traces of the roots, sufficient to fully identify the trees, can often be found by digging into the undisturbed soil below the surface soil. Conditions much the same as those just stated regarding the traces and remains of decayed stakes also obtain with respect to the decayed roots of a tree. By carefully removing the surface soil to the depth to which it has been disturbed, which quite commonly is only to the depth to which the plow has penetrated, then shaving away the undisturbed soil by clean horizontal cuts of the spade, the exact form of the roots of the tree can usually be discerned unmistakably, especially in light-colored soil. In this way and by such means, the approximate size and position of the tree, and in many cases the variety of the tree, can be determined, if the surveyor is sufficiently skilled in work of this character. The roots of some varieties of trees penetrate much more deeply than those of other varieties. For example, a white oak tree or a hickory tree will have, besides the usual branching roots, what is called a top root, that is, a large root that extends nearly straight downwards from beneath the center of the tree and usually penetrates to a great depth. Most other kinds of oak have only branching roots that are usually of considerable size and extend outwards at very great depths below the surface of the ground. Some varieties of trees, such as the elm, have large branching roots that extend outwards at slight depths below the surface. Other varieties, such as the tamarack, usually have a large number of small branching roots that extend outwards very near the surface. From such conditions, the surveyor, who is familiar with the characteristics of the roots of different varieties of trees, can often determine the variety of a tree with sufficient certainty to identify it as a bearing tree. But this is a matter that requires personal observation and experience, and cannot be learned from books.

17. Errors in Field Notes.—It should be remembered that field notes are not infallible, and errors in them are not rare. A direction may be given as north instead of south, or as east instead of west, and vice versa; the bearing may have been wrongly read on the graduated circle of the instrument, as 64° instead of 56°; the figures denoting the bearing may have been transposed, as 53 for 35; the chain may have been wrongly read by counting the links from the wrong end, as 48 instead of 52; or they may have been counted from the wrong tally mark, as 42 instead of 32. In working from a bearing tree or object to find a corner, and finding no indications of a post at the place indicated in the notes, all these sources of error should be tested. For the post planted at the time of the original survey is the best evidence of the position of the corner it was intended to mark, provided that it has not been moved.

18. Evidences of Location of Corner.—An old fence will indicate in a rough way where to look for a corner, but the history of the fence with respect to whether it was originally built to the corner, should be carefully inquired into before accepting it as evidence of the position of the corner. Such a fence may be the best possible evidence of the location of a line, or it may be worse than worthless for such purpose. The evidence of persons who have been familiar with the location of the corner sought may be of great service in finding it, but there is a great difference in people in this regard. Some have an accurate sense of locality and can tell very closely where an object is located; others cannot tell anywhere near it. Their ability in this respect, as well as their means of knowledge, should be inquired into before placing implicit confidence in their testimony.

Measures and lines run from the nearest known corners of the same survey in each direction will assist in locating the point at which to look for a corner. But the measurements of the United States land survey have seldom been uniform in different sections and not always uniform throughout the same section, and, consequently, this method cannot be

depended on to relocate a corner in its original position. The surveyor should remember that the problem is to find where the corner of the original survey actually was, and not where it should have been. A pick and shovel, when used intelligently, are of great assistance in finding lost corners, especially where wooden posts were used to mark the corners. In searching for a lost corner, the surveyor should not give it up as destroyed until he has made every possible effort to find it and has failed.

TO REPLACE LOST AND OBLITERATED CORNERS

19. Rules for Restoring Obliterated Corners.—Some of the most troublesome matters encountered by the land surveyor are those relating to the restoration of corners that have become destroyed or obliterated. In many cases, it is impossible to decide whether a corner has become totally obliterated or is merely lost, until this has been determined by a careful and thorough search. Hence, the preliminary portion of the process for restoring an obliterated corner consists in the search for the corner, and is the same as the process of trying to find a lost corner. There is but one rule for restoring corners that have become destroyed or totally obliterated, and this rule has no exception.

The corners must be replaced in their original locations if possible, regardless of errors in the original survey.

Failing of nearer and better evidence by which to locate the missing points, resort is had to the nearest other corners and monuments in each direction; these may be monuments of the same survey, or of other surveys when their relative location is known. The following rules apply to the United States surveys:

(*a*) *On base lines, correction parallels, township and range lines*, the lost corners should be restored by placing them in line between the nearest known corners on the same line and at distances from the known corners proportional to those laid down in the field notes of the original survey.

This rule supposes the original line to have been a straight line, though it is frequently not so. If there is reason to

believe the line has angles in its course, measurements from known corners to the right and left of it will aid in determining its true position. It is sometimes found that the position of a closing corner cannot be located from the nearest standard corner, because in some cases the distance of the closing corner from the standard corner is not given in the field notes, and in other cases it is erroneously stated. In such cases, if a portion of the closing line is known, the rule is to prolong it to its intersection with the township boundary and there look for or locate the closing corner.

(*b*) *Lost interior section corners* should be restored at distances from the nearest known corners, north, south, east, and west, proportional to those distances as given in the field notes of the original survey.

This rule supposes that the measurements of the original survey were uniform on adjacent sections. But they are not generally so, and, consequently, before measuring any line, it is well for the surveyor to compare his chaining with that of the original survey by measuring between the known corners nearest to the missing one that were set by that survey.

(*c*) *Lost township corners*, when common to four townships, are restored in the same manner as interior section corners. When common to only two townships they are restored as in (*a*).

(*d*) *Lost quarter-section corners* are to be restored in line between the section corners and at distances from them proportional to those returned in the field notes of the original survey; that is, midway between the section corners except in north and west tiers of sections. The same rule applies to meander corners wherever it can be applied.

20. Other Cases.—There are many cases in which other methods for restoring corners will be more satisfactory than the rules given. For instance, a half-quarter corner that was established when the adjacent section and quarter-section corners were known, may be used to restore either of them, when lost or destroyed, by prolonging the line over the known corners and doubling the distance, that is, by prolonging the

line of the two known corners to a point at a distance beyond the half-quarter corner equal to its distance from the section or quarter-section corner. If the half-quarter corner is the corner of a fractional half-quarter in the north or west tier of sections, the line is prolonged a distance that, instead of being equal to the distance between the corners, is in the same proportion to it that the corresponding returned distances are to each other. Any other intermediate corner whose location is definitely known may be used in a similar manner. Most of the difficulties encountered in finding lost or restoring obliterated corners arise from imperfections or errors in the original survey, or in the field notes. Hence, it is difficult, and sometimes impossible, to restore a corner to its original location by surveys from distant corners. After making such surveys, the ground should again be examined most carefully for the nearer and better evidence of the monument or post itself, as described in the directions for finding corners.

RESURVEY AND SUBDIVISION OF A SECTION

21. The example below, which is from actual practice, is given to illustrate the ordinary methods of finding corners, restoring those that are destroyed, and subdividing the section. (See Fig. 10.) The notes are kept on the general plan of those of the United States land survey. The following is a copy of such parts of the original field notes as were needed in the survey:

FIELD NOTES OF THE UNITED STATES SURVEY
Section 5, T. 3 S, R. 9 W

Chains		
80.00	North between secs. 8 and 9. Set post cor. to secs. 4, 5, 8, and 9,	Y. oak 22 in. S 87 E 171 li. W. oak 36 in. N 1 E 140 li.
	North between secs. 4 and 5.	
6.00	Enter wet prairie.	
33.00	Left wet prairie.	
33.80	W. oak 8 in.	
40.00	Set post qr. sec. cor. secs. 4 and 5,	W. oak 18 in. S $60\frac{1}{4}$ E $47\frac{1}{2}$ li. W. oak 18 in. N 22 E 71 li.

FIELD NOTES OF THE UNITED STATES SURVEY—(Continued)
Section 5, T. 3 S, R. 9 W

Chains	
55.50	Enter wet prairie.
74.00	Brook 40 li. wide C. SW.*
79.80	Intersected N boundary 139 li. E of post. Set post at intersection cor. of secs. 4 and 5, W. oak 14 in. S 83¼ E 451 li. W. oak 14 in. S 86¾ W 720 li.
	North between secs. 7 and 8.
80.00	Set post cor. to secs. 5, 6, 7, and 8, W. oak 8 in. N 8 E 29 li. W. oak 12 in. S 44 E 43 li.
	West corrected line between secs. 5 and 8.
40.20½	Set post qr. sec. cor. W. oak 22 in. S 22 W 50 li. Y. oak 20 in. N 49 E 11 li.
80.41	Section corner.
	West between secs. 6 and 7.
17.71	W. oak 18 in.
40.00	Set. post qr. sec. cor. W. oak 10 in. S 87 E 48½ li. R. oak 13 in. S 87 W 15½ li.
	North between secs. 5 and 6.
25.17	W. oak 18 in.
40.00	Set post qr. sec. cor. Aspen 14 in. S 41½ E 80 li. W. oak 14 in. N 57 E 41 li.
66.47	Stream 40 li. wide, C. SW.
68.44	Same stream, C. SE.
69.40	Same stream, C. SW.
81.41	Intersected N boundary. (Post removed.) Set post at intersection cor. to secs. 5 and 6, W. oak 10 in. S 53 E 268 li. W. oak 14 in. S 81 W 166 li.
	North boundary of sec. 5.
	West on S boundary to sec. 33, T. 2 S, R. 9 W.
80.00	Set post cor. to secs. 32 and 33, W. oak 12 in. N 74 E 299 li. W. oak 10 in. N 88 W.

*Abbreviation for *course southwest.*

FIELD NOTES OF THE UNITED STATES SURVEY—(Continued)
Section 5, T. 3 S, R. 9 W

Chains	West on S boundary of sec. 32, T. 2 S, R. 9 W.
14.69	W. oak .9 in.
40.00	Set post qr. sec. cor. { W. oak 36 in. N 75 E 42 li. W. oak 6 in. N 45¼ W. 33 li.
53.85	W. oak 12 in.
80.00	Set post cor. to secs. 31 and 32, { W. oak 15 in. N 43 W 91 li. W. oak 16 in. N 30¼ E 255 li.

The following are the complete field notes of the resurvey of the section, made for the purpose of finding the corners, restoring those that were destroyed, and subdividing the section. These notes give a large amount of valuable and practical information, and should be studied carefully.

Resurvey and Subdivision of Section 5, T. 3 S, R. 9 W, May 23–26, 1883

Chains	
	Begin at the corner of secs. 4, 5, 8, and 9. Found bearing tree north standing dead and decayed. Ran thence S 1° W 140 li. where I found one foot beneath the surface of the road a stake hole one foot deep, with the decayed wood plain and distinct. Set a stone 18 × 10 × 6 in. in its place marked + on top and put broken brick around for cor. and mark { Maple 16 in. N 46 E 72 li. Maple 12 in. N 61 W 89 li.
	Set up a flag 20 li. east of cor. From cor. measured north alongside of line fence.
33.90	W. oak line tree now 24 in. dia.
40.12	Qr. sec. cor. dug out in highway. Stumps of two bearing trees are standing, showing marks plainly. From these stumps I strike arcs of 47½ li. and 71 li., respectively, and at their intersection plant an earthenware post 2 ft. long, 3 in. dia. and mark { W. oak 12 in. S 62 W 112 li. W. oak 10 in. S 3 W 46 li.
	I then set up the transit 20 li. east of the qr. sec. cor., backsight on flag 20 li. east of sec. cor. and prolong line north.
74.20	Brook.
80.00	Set temporary stake 20 li. west of random line and look for cor. in wet prairie or marsh. Found stump of bearing tree, showing marks, on high ground SE. Run thence

Resurvey and Subdivision of Sections 5, T. 3 S, R. 9 W, May 23–26, 1883
(Continued)

Chains	
	N 83¼ W 451 li.; set temporary stake and run thence S 86¾ W 720 li.; set temporary stake and look for evidence of bearing tree.
	By digging away the surface earth I find, where a tree has stood, the decayed roots being still plainly visible. This is S 88 W 6 li. from temporary stake. I locate the center

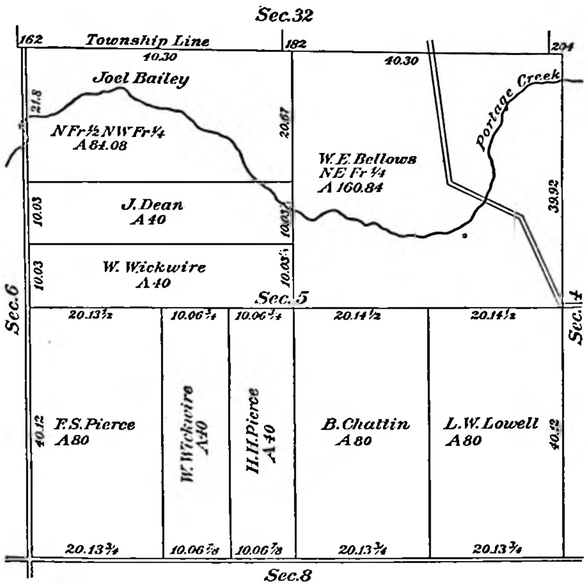

Fig. 10

of the tree as nearly as possible and run thence N 86¾ E 720 li. Renew search for cor. and find the sound remains of a stake 2 ft. below surface of marsh. This stake is 80.04 ch. from cor. of secs. 4, 5, 8, and 9, and 2 li. west of line prolonged over sec. cor. and qr. post. I then go to

Resurvey and Subdivision of Sections 5, T. 3 S, R. 9 W, May, 23–26, 1883
(Continued)

Chains	
	the cor. of secs. 32 and 33 and find post standing in correct position as shown by stump of both bearing trees. I also find that the jog between the two corners is 204 li. instead of 139 li., as given in the field notes. Set new post with brick around for cor. of secs. 4 and 5. No tree or bearing object near. Left flag on the cor.
	I then return to qr. sec. cor. of secs. 4 and 5, set up transit 10 li. north of it, sight to flag on the sec. cor. north, which line I assume as a meridian, and run thence on random S 88° 50′ W along N side of line fence, setting temporary stakes every 10 ch. at an offset distance of 10 li. south of the random line.
80.00	Set temporary stake on east side of road. Cor. post and bearing trees both dug out in the highway and no trace of cor. left. I then go to the cor. of secs. 5, 6, 7, and 8. This is also dug out in the highway and no evidence of it to be found. I next go to the qr. sec. cor. of secs. 7 and 8, which I find and identify, and run thence north on a random line along the center of the road without noting the course.
40.00	Set temporary stake.
80.00	Set temporary stake.
121.78	Intersect north boundary 12 li. west of post, which is decayed and in its correct position, with both bearing trees standing. Set stone 18 × 10 × 5 in., marked +, for new corner post of secs. 5 and 6, and put broken glass and crockery around it. I then go to the cor. of secs. 31 and 32, T. 2 S, R. 9 W, and find a stone planted for cor., and stumps of both bearing trees standing. The jog between the cors. is 162 li. I then go to the qr. sec. cor. of secs. 6 and 7, which I find and identify, and measure east along the center of the road.
40.00	Set temporary stake 40.12 ch. north from the qr. sec. cor. of secs. 7 and 8 and set up flag.
	Thence measure east along center of road, setting temporary stakes every 10 chains.
40.00	All traces of qr. sec. cor. destroyed.
80.62	Cor. of secs. 4, 5, 8, and 9.
	West corrected line by single sight to flag near cor. of secs. 5, 6, 7, and 8.
20.13¾	Set for ¼ qr. sec. cor. a stone 20 × 12 × 8 in. and put brick around it and mark { W. oak 16 in. S 26 E 46 li. W. oak 12 in. N 14 E 42 li.

Resurvey and Subdivision of Section 5, T. 3 S, R. 9 W, May 23–26, 1883
(Continued)

Chains	
$40.27\frac{1}{2}$	Set iron plow coulter 18 in. long with broken brick and crockery around for qr. sec. cor. of secs. 5 and 8, and mark $\begin{cases}\text{Cherry 12 in. S 47 W 143 li.}\\ \text{Blk. walnut 16 in. N 23 W 76 li.}\end{cases}$
$50.34\frac{3}{8}$	Set plow point with brick around for cor.
$60.41\frac{1}{4}$	Set black walnut post with broken glass around for $\frac{1}{4}$ qr. sec. cor. and mark $\begin{cases}\text{Blk. walnut 20 in. N 20 W 51 li.}\\ \text{Blk. walnut 18 in. N 45 E 76 li.}\end{cases}$
80.55	Cor. of secs. 5, 6, 7, and 8. By my measure the distance from the qr. cor. of secs. 7 and 8 north to the township boundary is 121.78 ch. as compared with the returned distance, which is $40 + 81.41 = 121.41$ ch., a surplus of 37 li. Hence, I place the corner at a distance of $\frac{40}{121.41} \times 121.78 = 40.12$ ch. from the qr. sec. cor. of secs. 7 and 8 and at a distance of $\frac{81.41}{121.41} \times 121.78 = 81.66$ ch. from the cor. of secs. 5 and 6 on the township boundary.
	I also find that the distance from the qr. sec. cor. of secs. 6 and 7 east to the corner of secs. 4, 5, 8, and 9 is 120.62 ch., a surplus of 21 li. as compared with the original measure of $40 + 80.41 = 120.41$ ch. Hence, I place the corner at 40.07 ch. from the qr. sec. cor. of secs. 6 and 7 and 80.55 ch. from the cor. of secs. 4, 5, 8, and 9. Put in a 2-ft. length of 3-in. sewer pipe for cor. of secs. 5, 6, 7, and 8 and mark $\begin{cases}\text{Hickory 12 in. S 17 W 193 li.}\\ \text{W. oak 18 in. N 42 W 73 li.}\end{cases}$
	I leave a flag on the cor. and go to the qr. sec. cor. of secs. 5 and 6 and put my transit in line between the flags at the sec. cors. and on the true line plant an iron plow beam $26 \times 4 \times 1$ in. for qr. sec. cor. of secs. 5 and 6, and mark $\begin{cases}\text{Hickory 10 E 42 li.}\\ \text{Cherry 16 S 26 E 104 li.}\end{cases}$
	This cor. is 40.12 li. from the sec. cor. south and 41.54 li. from the sec. cor. north. Thence
	North on true line from qr. sec. cor. of secs. 5 and 6.
10.03	Set stake with brick around for cor.
20.06	Punch hole with iron bar 3 ft. deep and 2 in. diam. in the ground and fill it with Portland cement mortar for $\frac{1}{4}$ qr. sec. cor. and mark $\begin{cases}\text{Maple 6 in. N 24 E 165 li.}\\ \text{Maple 8 in. N 72 W 213 li.}\end{cases}$

Resurvey and Subdivision or Section 5, T. 3 S, R. 9 W, May 23–26, 1883
(Continued)

Chains	
	I then return to temporary stake set near qr. sec. cor. of secs. 5 and 6 and continue the random line from the east on a course S 88° 50′ W.
80.56	Intersect the section line 14 li. north of the qr. sec. cor. Deducting the 10-link offset made to the north on this line leaves 4 li. correction to be prorated on the stakes of the random. Thence east corrected.
20.14	Set temporary stake 3 li. south of random to true line.
30.21	Set temporary stake 2½ li. south of random to true line.
40.28	Set temporary stake 2 ii. south of random to true line.
	I set up transit over this last stake, backsight to flag on qr. sec. cor. of secs. 5 and 8 and prolong the line north.
10.00	Set temporary stake.
20.00	Set temporary stake.
40.73	Intersected township boundary 184 li. east of the qr. sec. cor. of sec. 32. I found the closing distance between cors. at NW cor. of sec. 5 to be 162 li. and at the NE cor. of sec. 5 to be 204 li., the half sum of which is 183 li. Hence, I plant a temporary stake in the township boundary 183 li. east of the qr. sec. cor. of sec. 32, and remeasure the north line of sec. 5 and find the middle point to be 182 li. east of the qr. sec. cor. of sec. 32, at which point I plant for qr. sec. cor. of sec. 5 a piece of 1½-in. gas pipe 3 ft. long and mark { W. oak 16 in. S 23 E 26 li. Hickory 12 in. S 44 W 53 li.
	By ruling of the general land office the distance from this point to the center of sec. 5 is $\frac{39.80 + 41.41}{2} = 40.60\frac{1}{2}$ ch.; hence, I have $40.73 - 40.60\frac{1}{2} = 12\frac{1}{2}$ li. surplus to be prorated. Therefore, I go south on corrected line.
20.66¾	Set stone 16 × 9 × 6 in. for ½ qr. sec. cor. Put broken crockery around it and mark { Apple 10 in. N 46 W 176 li. Maple 16 in. N 17 E 227 li.
30.69¾	Set marked stone for cor.
40.73	At intersection of quarter lines set stone 20 × 12 × 4 in. with cross-mark on it for center of sec. 5.
	Thence west on true line from center of sec. 5.
10.06¾	Set marked stone 12 × 8 × 4 in. with brick around.
20.13½	For ½ qr. sec. cor. set marked stone 20 × 9 × 5 in. with brick around.
	East on trne line from center of sec. 5.

Resurvey and Subdivision of Section 5, T. 3 S, R. 9 W, May 23–26, 1883
(Continued)

Chains	
20.14½	For ½ qr. sec. cor. set stake with broken glass around and mark $\begin{cases}\text{W. oak 6 in. S 87 E 46 li.}\\ \text{W. oak 18 in. S 5 W 23 li.}\end{cases}$

NOTE.—It is not expected that the surveyor will follow any special order in subdividing a section. The conditions are not likely to be the same in any two sections, and the subdivision of each section will be controlled by local circumstances, and should be made in such order as may be most expedient for that section.

LEGAL PRINCIPLES CONTROLLING SURVEYS

22. General Statement.—There are various statutes and common-law principles that control surveys. If in the endeavor to secure mathematical accuracy of his work, the surveyor fails to observe these principles, he is quite certain to be discredited if the case comes into court. For instance, the United States statute requires that quarter-section lines be straight lines across the section from corner to corner, although such lines will seldom divide the section into four equal parts. Most of the common-law principles that follow are universally accepted by the courts. On some points, the decisions vary in different states. Every surveyor should inform himself on the decisions of the supreme court of the state in which he surveys. They are to be found in the state reports, which are usually kept by the proper officials at the county seat, and by leading lawyers. The surveyor has to properly construe descriptions in deeds, and in resurveys he has to consider and weigh evidence of every kind that tends to show the location of corners and boundary lines.

The word *call* is commonly used in land law with reference to the descriptions in surveys or grants of land. When used in this sense, it means any fact, condition, or requirement called for or specified in the description, and that serves to define or identify the survey. For example, the description,

"Thence N 40° E 24 chains to a granite boulder at an angle in the south boundary of the Wellstown turnpike," contains six distinct calls, namely: (1) a call for the course, as *N 40° E;* (2) a call for the distance, as *24 chains;* (3) a call for the object at the extremity of the course, on which the course closes, as *a granite boulder;* (4) a call describing the position of this object, as *at an angle;* (5) a call further describing and identifying the object, as *in the south boundary;* (6) a call identifying the boundary and thus further identifying the object, as *of the Wellstown turnpike.*

23. Interpretation of Descriptions in Deeds.—In construing descriptions in deeds, the courts have laid down the following principles:

(*a*) The description is to be taken most strongly against the grantor. If it is capable of more than one meaning, that one should be adopted which will give the grantee the greatest amount of land.

(*b*) A deed must be construed according to the condition of things at the time it was made. The written descriptions are to be interpreted in the light of the facts known to, and in the minds of, the parties at the time, and with reference to any plats, facts, and monuments on the ground, which are referred to in the deed. If the descriptions are uncertain, the construction given by the parties, as manifested by their acts on the ground, is deemed the true one unless the contrary is clearly shown.

(*c*) Every call in the description must be answered if it can be done, and none is to be rejected if they can all stand consistently together. If one part of the description is false and impossible, but by rejecting that part a perfect description remains, the false part should be rejected and the deed held good. Those boundaries are to be retained which best subserve the prevailing intention manifested on the face of the deed. The certain description must prevail over the uncertain in the absence of controlling circumstances.

(*d*) Where the description calls for land "owned and occupied," the actual line of occupation is a material call to

be considered, but where land is conveyed "beginning at" and "bounding on" certain land, the point of beginning and boundary is on the true line of that land.

(*e*) Where land is described as running a certain distance by measure to a known line, the land will extend to that line whether the measure is correct or not. Not so if the line is not definitely marked, fixed, or known.

(*f*) Where land is conveyed as "beginning at and bounding land of B," the point of beginning and boundary is the true line of B's land.

(*g*) A conveyance by metes and bounds will carry all the land included in them, although it be more or less than is stated in the deed. The mention of quantity after a definite description, being the least certain, does not control, but if the boundaries are in doubt, quantity may become a controlling consideration.

(*h*) "Northward" or 'northerly" means due north when nothing is mentioned to show deflection to the east or west. A course from corner to corner means a straight line, but may be explained by other matters to be a marked line, as following a hedge or stream. The use of the word "about" indicates that exactness is not intended, but where nothing more certain can be found, the grant is limited to the exact courses* and distances given.

(*i*) Where lines are laid down on a map or plan, and are referred to in a conveyance of the land, all the particulars shown on the map or plan are considered as much the true description as though they were expressly recited in the deed. A reference in a description to the government patent makes that patent and the government survey a part of the deed. So where for greater certainty any survey is referred to in a deed, the survey referred to legally forms a part of the deed.

(*j*) A grant of land bounded by the highway takes to the center of the highway, unless the highway is excluded

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* The use of the word *course* in the sense of *bearing* is universal with the courts and is common everywhere. Hence, in quoting the principles laid down by the courts, this usage has been retained.

in explicit terms. The monuments referred to in a deed, whether they are natural or artificial, control the courses and distances.

24. Descriptions of Land Bordering on Water.—In construing descriptions of land bordering on waters, the surveyor will need to inform himself regarding the laws of the locality, as they vary in different states. The following principles are stated as a general guide:

(*a*) It is a universal rule that grants of land bordering on navigable waters take only to high-water mark, while grants on non-navigable streams take to the center of the stream. The same rule applies to non-navigable lakes in some states. But what is a navigable stream? According to the common law, a navigable stream is one emptying into the ocean, and into which the tide ebbs and flows. By the federal law, it is one capable of being used as a highway of commerce. In some states one law prevails, and in some the other. Under the Michigan law, the Detroit river is not a navigable stream, and adjacent proprietors own to the center of the river, although it is a great highway of commerce. A similar contradiction exists in the case of the upper Mississippi; a state on one side holds it to be a navigable river, while a state on the opposite side holds the reverse. The center or thread of a stream is measured between the low-water marks, regardless of the main channel.

(*b*) A boundary on, or by, or to a stream includes flats at least to low-water mark and in some cases to the thread of the stream. A boundary on the bank referring to fixed monuments limits the grant to the bank. The words "along the bank" exclude the river and its bed. A bank is the continuous margin where vegetation ceases. The shore is the space between the bank and low-water mark. That only is the river bed which the river occupies long enough to wrest it from vegetation. When by action of the water the river bed changes, the ownership of adjoining land changes with it. Meander lines are not boundary lines. A patent from the United States, of land bounded by a lake or stream, conveys

land to the water edge, although the meander line does not coincide with the shore line.

25. Locating Corners and Boundaries.—The following principles govern and should be fully recognized in locating the corners and boundaries of surveys:

(*a*) In locating a deed on the ground, the surveyor is to rely: (1) on the actual lines originally surveyed; (2) on lines run from acknowledged calls and corners; (3) on lines run according to the courses and distances.

(*b*) When the boundaries are fixed by known monuments, though neither courses, distances, nor contents correspond, the monuments govern. Surplus lands do not vitiate a survey, nor does a deficiency of the area called for operate against it.

(*c*) Course and distance yield to monuments, but where the monuments are wanting and course and distance cannot be reconciled, there is no rule that compels any preference of one over the other. Local circumstances will usually determine which should be preferred. Where no bounds were established, the line must be run by aid of the measurements in the deeds, the oldest title receiving its full measure first.

(*d*) Boundary may be proved by any evidence that is admissible to establish any other fact. A long-established fence is better evidence of actual boundary settled by practical location than any survey made after the monuments of the original survey have disappeared.

(*e*) Where a survey is made previous to the plat and there is a difference between the survey and the plat regarding the location of the lines and monuments, the lines and monuments originally marked as such are to govern. Purchasers of town lots have a right to locate them according to the stakes that they find planted and recognized. If the stakes were planted by authority and the lots were purchased and taken possession of in reliance on them, no subsequent survey can be allowed to unsettle them. When surveys made many years apart disagree, and the original corners

and witness trees are gone at the time of the later survey, the probabilities favor the earlier survey, especially if the line has remained unquestioned for many years.

(*f*) Streets that are well defined and marked by natural or artificial monuments govern course and distance in fixing boundaries of lands. Ancient reputation and possession in regard to streets in a town are entitled to more respect in deciding on the boundaries of lots than any experimental survey afterwards made.

(*g*) Where lots are sold by numbers and a plat, any surplus or shortage in a block is to be divided pro rata between the lots.

(*h*) The original surveys by which the government sold and conveyed its land establish the rights of the parties as to boundaries. Land sold under the United States surveys pass according to the descriptions of the legal subdivisions, whether those subdivisions contain the legal quantity of land or not.

(*i*) Each section or subdivision of a section is independent of any other section or subdivision in the township and is governed by its own boundaries. The corners established by the original survey of the United States public lands are conclusive, and no error in placing them can be corrected by an individual or state surveyor. Quarter posts, however distant from the direct line, are to be as much respected as section or township corners.

(*j*) Field notes must yield to actual monuments erected by the original surveyor. They are only to be relied on as evidence to assist in finding the exact situation of the monuments. When there are conflicting monuments, that is to be considered the true one which most nearly conforms to the field notes.

(*k*) Any difference in the length of a line by actual measure, as compared with that indicated by the government survey, should be divided between the parts in proportion to their respective lengths, as shown by the survey. In the absence of evidence to the contrary, it is not permitted to presume that the variation arose from the defective survey of any part.

(*l*) A fence line between two properties that is agreed on by the owners of the properties to be the division line between them, or is openly recognized by them as such, and undisputed, for a term of years, becomes the legal division line between those properties, regardless of whether it is or is not in the true position of the original line. But such fence line does not in any way affect the position of any other boundary line, and in case it is supposed, or was intended, to be on a line of the public-land survey, as, for example, a section or a quarter-section line, but really is not on such line, the fact that the fence is the legal division line between the adjacent properties does not in any way affect the position of the survey line if the latter can be determined.

THE EARLIER SURVEYS

26. Surveys by Metes and Bounds.—As the present rectangular system of land surveys was not inaugurated until 1784, the land surveys made previously were not in conformity with this system. In the older states of the American Union, the original surveys of the public lands were made by what is commonly known as **metes and bounds.** Each tract, whatever its shape, was enclosed by a traverse line that started from some given point and, extending entirely around the tract, closed on the starting point. In many cases, and probably in most cases, these surveys were imperfectly made and were full of errors. This was due to two principal causes, namely, the cheapness of the lands and the lack of skill in the surveyors. The traverse was always a compass traverse, and usually the magnetic bearings only were taken, without noting the magnetic declination. In some cases, the starting point and angular points of the traverse were marked more or less permanently and described definitely, but in very many cases they were marked merely with a wooden stake driven into the ground, without anything else whatever to identify them, and sometimes they were not marked at all. It is often found that the traverse, when run accurately according to

the description, cannot be made to close on the starting point. Boundary lines described in deeds and shown on maps as straight are found to be crooked on the ground. Tracts commonly contain less or more land than called for in descriptions. Records of adjoining tracts often make one tract overlap another or leave an unclaimed gore between them. These discrepancies and blunders often render the work of the surveyor exceedingly difficult when retracing such old boundaries or locating their corners, and great tact and judgment are often necessary in making amicable and satisfactory adjustments of contending claims. In general, old boundaries, such as line trees, stone monuments, and fences, are sustained by the courts as holding; but, before retracing the lines, the surveyor should, if possible, secure the consent of adjacent owners to abide by such monuments and boundaries, irrespective of the lines or quantities called for in contracts or deeds.

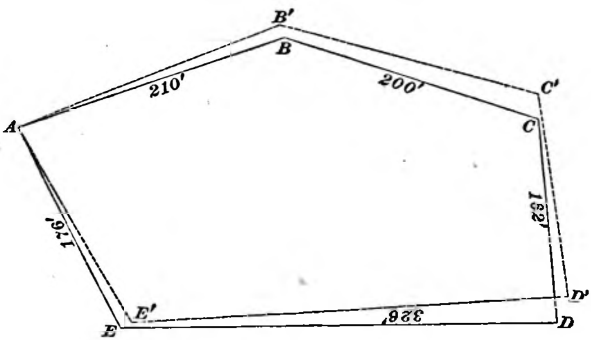

FIG. 11

27. Effect of Magnetic Variation.—It must be borne in mind that the bearings of lines are each year undergoing a slight change which, in a long period, amounts to several degrees, and if the lines were rerun according to original bearings as given in descriptions, they would enclose a tract quite different from that included in the original survey.

The surveyor must accordingly determine the amount of magnetic variation or change that has taken place between the date of the original survey and that of the survey about to be made, and having determined such change or variation, he must make his bearings conform to those of the original survey when making the resurvey. Before commencing the resurvey, the surveyor should correct all the bearings and write them out in their proper order together with the original bearings.

Fig. 11 illustrates the effect of magnetic variation in altering the direction of lines. The polygon *A B C D E* shows the outline of a tract according to the original survey, and *A′ B′ C′ D′ E′* the relative directions of the boundaries when resurveyed with the original bearings, there having been during the intervening time a west magnetic variation, or a westerly change in the magnetic declination. When the old bearings and the variation are known, the new bearings are determined in the same general way used for changing magnetic to true bearings, or vice versa, when the declination is known. (See *Compass Surveying.*)

Below are given the original and the corrected bearings of a survey, it having been ascertained that during the time elapsed between the original survey and the resurvey of the tract shown, the variation of the needle was 3° 00′ west. Let the student verify the values of the corrected bearings. In case of doubt, draw a diagram showing the new magnetic meridian 3° west of the original.

1 Courses	2 Bearings	3 Distances	4 Corrected Bearings
A B	N 68° 00′ E	210	N 71° 00′ E
B C	S 73° 00′ E	200	S 70° 00′ E
C D	S 8° 00′ E	162	S 5° 00′ E
D E	S 87° 00′ W	326	S 90° 00′ W
E A	N2 8° 00′ W	176	N 25° 00′ W

EXAMPLES FOR PRACTICE

1. If the variation in the magnetic declination is 3° 00′ east, what is the corrected bearing of a line whose original bearing was N 88° 30′ W? Ans. S 88° 30′ W

2. If during the period that has elapsed since a survey was made the variation in the magnetic declination is 4° 25′ west, what are the present bearings of the following lines, whose original bearings were: (*a*) S 82° 36′ E, (*b*) N 87° 35′ E, (*c*) S 42° 48′ W, and (*d*) N 32° 06′ W?

Ans. (*a*) S 78° 11′ E; (*b*) S 88° 00′ E; (*c*) S 47° 13′ W; (*d*) N 27° 41′ W

3. The original bearing of a certain line of a survey was S 25° 28′ E, and its present bearing is S 30° 40′ E; what should be the present bearings of certain other lines of the same survey whose original bearings were: (*a*) N 5° 30′ E, (*b*) S 89° 10′ E, and (*c*) S 75° 20′ W?

Ans. (*a*) N 0° 18′ E; (*b*) N 85° 38′ E; (*c*) S 70° 08′ W

4. A certain line of a survey is found to have a bearing N 27° 55′ E, and its bearing as recorded in a former survey was N 25° 00′ E; what should be the present bearings of certain other lines of the same survey, whose original bearings, as recorded in the notes of the former survey, were: (*a*) N 89° 00′ E, (*b*) S 1° 30′ E, (*c*) S 88° 45′ W, and (*d*) N 45° 00′ W?

Ans. (*a*) S 88° 05′ E; (*b*) S 1° 25′ W; (*c*) N 88° 20′ W; (*d*) N 42° 05′ W

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28. How to Determine Magnetic Variation.—If the date of the original survey is known, the amount of variation may be determined approximately from a chart and tables published by the United States Coast and Geodetic Survey, which give the approximate yearly variation for different sections of the country. But no method that depends on computation for determining the amount of change in the declination can be anything better than a rough approximation. Moreover, the date of the survey is often omitted; the date of the deed must not be taken as the date of the survey.

If one of the original boundaries remains unobliterated and can still be traced, the magnetic variation can be determined

at once by taking the present bearing of the line. The difference between the present bearing and that of the original survey is the required correction. The corrections are then to be applied to the original bearings of the other lines and the resulting courses run out.

If any two corners of the original boundary can be identified, the true bearings and distances between them can be found. A traverse can be run as a random line between the two known corners, according to the notes of the original survey, the direction and distance found between the corresponding points of the original survey and the random traverse, and from them the corrected bearings and distances computed, as explained in *Transit Surveying*, Part 2.

29. Straightening Boundaries.—Where the description and map show a boundary to be a straight line and the actual boundary is found to be crooked, it is sometimes good policy to establish a new and straight boundary by the principle of "give and take," provided that the owners of the adjoining lands will agree to the adjustment.

Fig. 12 illustrates the principle that is frequently employed in correcting such boundaries. Let A and E be two corners of a survey, and suppose that the boundary line joining them is described and shown on the map as a straight line, but that the irregular line $A\,B\,C\,D\,E$ represents the actual

FIG. 12

boundary. It is evident that the dotted straight line $A\,E$ may be substituted for the irregular line $A\,B\,C\,D\,E$, and equitably divide the adjoining properties. The principle of give and take is applied, the adjoining owners making exchanges of equal areas.

The location of the new boundary is determined by making a careful survey of the old boundary and platting it to a large scale; a fine thread is then stretched on the plat and a line of division made as closely as can be estimated by the eye.

The areas of the equalizing triangles are then calculated by scaling their dimensions, and if they do not balance, the dividing line can readily be shifted until the desired result is obtained. The line is then laid out and marked on the ground, and permanent corners established. Where the boundary is in woodland, careful search must be made for line and bearing trees. Blaze marks are very enduring, being easily recognized on most varieties of trees after a lapse of many years.

MAPPING

(PART 1)

PLATTING ANGLES AND LINES

INTRODUCTION

1. **A map** is a drawing representing the outlines, dimensions, and natural and artificial divisions and features of a part of the earth's surface by means of lines whose forms, lengths, and relative positions are determined by a survey.

2. **Scale of Map.**—Nearly all the straight lines shown on a map represent either boundaries or divisions of the given surface; they have only the properties of direction and length. Since the map is a representation of the surface of a given locality, the distance between any two points on the map bears a certain definite ratio to the distance between the two points represented by them. The ratio of a distance on the map to the corresponding actual distance is called the **scale** of the map. Thus, if 1 inch on the map represents 200 feet on the ground, the scale of the map is $\frac{1}{12} \div 200$, or $\frac{1}{2400}$. The scale is uniform for the entire map; consequently, all distances and areas shown on the map are proportional to their actual values. Hence, the distance between any two points shown on a map, or the area of any portion of the surface represented by it, can be measured directly on the map by means of the scale used in drawing it. The scale of a map depends on the size of the map and the purposes for which the map is to be used.

3. Measuring Scales.—A **measuring scale**, or simply a **scale**, is a graduated ruler used for making measurements on a drawing. Of the many kinds used by draftsmen, the one best adapted to the needs of the surveyor is the triangular scale divided decimally. A perspective view of it is shown in Fig. 1. The scale is 1 foot long, and contains six systems of graduations, one on each side of each edge; practically, therefore, it is a combination of six scales. Each scale is divided into inches and fractional parts of an inch, these parts being submultiples of 10—tenths, twentieths, fiftieths, etc. The number of parts into which each inch is divided is indicated by a large number in the center of the scale. Thus, the numbers *10* and *50* in Fig. 1 indicate that the upper scale there shown is divided into inches and tenths, and the lower into inches and fiftieths. Except in the 10-scale, the numbers opposite the graduations do not indicate inches; but, when multiplied by 10, they indicate the number of divisions from the zero of the scale. Thus, the number 12, opposite a division line on the 50-scale, denotes $\frac{12 \times 10}{50} = \frac{120}{50} = 2\frac{20}{50}$ inches measured from the zero of the scale. So, too, the number 22 on the 60-scale denotes $\frac{22 \times 10}{60} = \frac{220}{60} = 3\frac{40}{60}$ inches measured from the zero of the scale. In practice, it is not necessary to perform these operations. To determine, for instance, the value of the division marked 12 on the 50-scale (that is, the distance of that division from the zero mark), take on the scale the first number preceding 12 that is divisible by 5: this number is 10, which divided by 5 gives 2; to this add the number of divisions between 10 and 12, which is 20, thus obtaining $2\frac{20}{50}$ inches. Likewise, in using the 60-scale, the distance

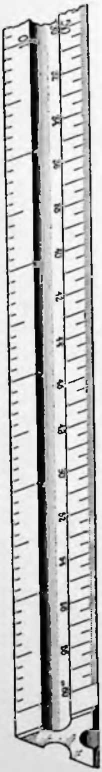


FIG. 1

that the division marked 22 is from the zero of the scale is obtained by dividing 18, which is the next lower multiple of 6, by 6, and adding the number of divisions between the 18-mark and the 22-mark, which in this case is 40; the result is, then, $\frac{18}{6} + \frac{40}{60} = 3\frac{40}{60}$ inches. Similarly for other divisions.

DRAWING THE PLATES

4. Drawing Plates.—The principles and methods of mapping described in *Mapping*, Parts 1 and 2, are fully illustrated by drawings. These comprise, besides the illustrations that appear in various parts of the text, seven drawing plates that are sent to the student, three with *Mapping*, Part 1, and four with *Mapping*, Part 2. The examples given in the plates are similar to those met in practical field and office work, and in each case the field notes of the survey are given in the text, the parts not required for the platting being omitted for brevity. From these field notes the student is required to draw the plates to the scale stated in each case.

5. Size of Plates.—As explained in *Geometrical Drawing*, the size of each finished plate is to be 14 by 18 inches. A border line is to be drawn $\frac{1}{2}$ inch from each edge of the plate, thus making the size of the plate within the border line 13 by 17 inches. The sheet itself, when first placed on the board, must be somewhat larger than the finished plate, so that the holes made by the thumbtacks will not appear on the finished drawing. The extra margin is very convenient for testing the pen in order to see whether the ink flows freely and the lines have the proper width.

6. Titles of Mapping Plates.—The titles of all maps and plates drawn in connection with this part of the Course are to be in capital letters of the style shown in Fig. 2 (*b*). Such letters are commonly known as **Italic caps.**, and are used extensively for the titles of maps. The size of letters must be governed by the purpose for which they are to be used, but they should be of uniform size throughout each title, name, or statement. The letters in the title

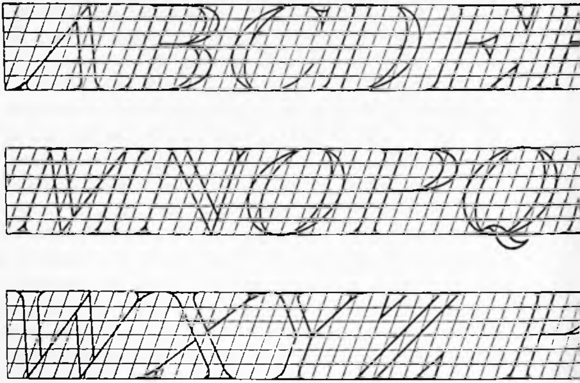
should be the largest letters on the drawing, but should harmonize with the rest of the lettering. Their size should depend somewhat on the size of the drawing. The letters in the titles to the mapping plates should be capitals and should have a uniform height of $\frac{3}{10}$ inch.

The student who has had no practice in making this style of letters will find it advantageous to construct the letters of the titles in the manner illustrated in Fig. 2 (*a*). The construction may be described as follows: The height of the letters, $\frac{3}{10}$ inch, is divided into six equal parts, and horizontal lines are drawn through the points of division. These horizontal lines are thus spaced $\frac{1}{20}$ inch apart. The lower horizontal line is then laid off in spaces of $\frac{1}{20}$ inch, and from the points of division parallel slanting lines are drawn to the upper horizontal line. The slanting lines are thus at the same distance apart horizontally that the horizontal lines are vertically. These parallel horizontal and slanting lines, which are to be drawn lightly with a fine-pointed pencil, form a series of construction lines on which to outline the letters. It is very important that all letters in the title have the same slant, though just what the slant is, within reasonable limits, is not important. In Fig. 2, each slanting line has a horizontal projection equal to two spaces, and this slant will be used for the letters of the titles to the mapping plates.

There is no rigid rule for the ratio of the width to the height in letters of this class, though, in general, this ratio should be less for large than for small letters. The horizontal widths of the letters shown in Fig. 2, expressed in terms of the spaces, are as follows:

Width of 1 space, letter I
Width of 4 spaces, letter J
Width of $4\frac{1}{2}$ spaces, letters L, N, U
Width of $4\frac{3}{4}$ spaces, letter F
Width of 5 spaces, letters A, B, D, E, H, K, P, S, V, Y, Z
Width of $5\frac{1}{2}$ spaces, letters C, G, R, T, X
Width of 6 spaces, letters M, O, Q
Width of 8 spaces, letter W

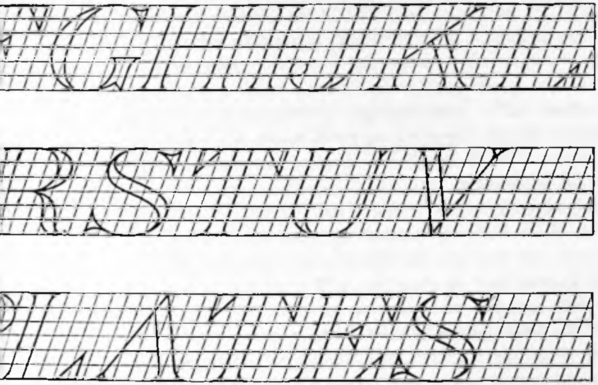
These widths are measured from outside to outside of the



(a)

(b)

FIG.



FGHIJKL

RSTUV

PLATES

slant lines, and do not include the projections of the short horizontal lines at the tops and bottoms of the letters. These horizontal lines should project one-half of one space at the top of the letter and three-fourths of one space at the bottom.

The letters should be spaced at regular intervals. For the letters here described, the spacing shown in Fig. 2 (*a*) gives a satisfactory appearance. The horizontal distance between the slant lines in adjacent letters should usually be uniform, except where *A* follows *P* or is adjacent to *T*, *V*, or *W*. In any of these cases, the extremity of the projecting horizontal line at the bottom of *A* should be in the same slant line with the top extremity of *P*, *T*, *V*, or *W*.

The spacing should be laid out and the construction lines drawn in pencil. The outlines of the letters should then be sketched carefully in pencil. In inking, the straight lines that compose a part of nearly every letter should usually be drawn with the ruling pen, giving all such lines the same slant and width, though after a draftsman becomes expert in lettering he may do this freehand. Each letter should then be finished carefully freehand, the shade parts inked black, and the construction lines erased. The letters should then have the same general appearance as the letters shown in Fig. 2 (*b*).

Expertness in lettering can be obtained only by constant practice. In practicing, the student should try to give each mark its proper position and form by a single stroke of the pen; then, if the result is reasonably satisfactory, it should not be changed, since trying to change it is not likely to improve it but is more likely to make it worse. Bold, sharp lines give the best appearance in lettering, and too fine lines detract from, rather than add to, the appearance of the letters. Attempting to make very fine lines is a common fault with beginners, and should be avoided.

Although the lettering of the title is the last work to be done on each mapping plate, the construction of the letters in the title is described here, since it is a matter that relates to all the mapping plates. Before the student begins to draw the title to the first mapping plate, he should study this article carefully and practice the construction of the letters.

DRAWING PLATE, TITLE: PLATTING ANGLES—I

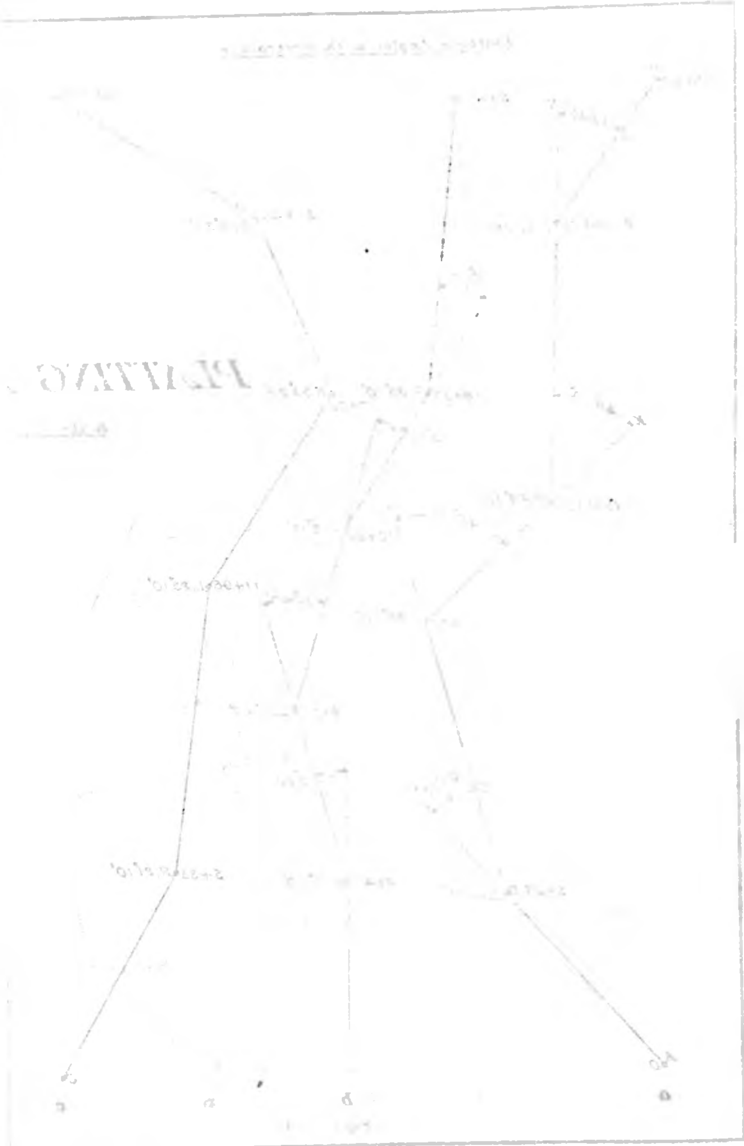
NOTE.—In order to avoid confusion, a star (*) will be used to distinguish figures in the plates from those in the text. Thus, Fig.* 1 means Fig. 1 in the plate, while the usual notation Fig. 1 means Fig. 1 in the text.

7. Preliminary Explanations.—This plate shows six angle lines, three of which are grouped under Fig.* 1 and three under Fig.* 2. The three lines *a*, *b*, and *c*, under Fig.* 1, should be drawn to a scale of 200 feet to the inch, platting the angles by means of a protractor, the use of which is fully explained in *Geometrical Drawing*.

These lines are to be platted according to the directions given below. Care should be taken to locate each line approximately in the same relative position that it occupies on the plate. This statement also applies to all the plates to be drawn from the data given in this text. In some of these plates, distances are expressed in stations of 100 feet, as explained in *Compass Surveying*, Part 1. The direction of each line is referred to that of the immediately preceding line prolonged, and the angle is recorded as being to the right or left, as explained in *Transit Surveying*, Part 1, in connection with traversing. In practical office work, the lines prolonged are drawn lightly in pencil, and erased as soon as the angles are laid off. In the lines *a* and *b*, Fig.* 1, the lines prolonged are dotted, and the angles marked by dotted arcs, in order that the method may be clearly understood. The dimensions of the plates and the directions for drawing the border lines are the same as for the plates in *Geometrical Drawing*. The notes for line *a* in Fig.* 1 are given in the accompanying table.

NOTES FOR LINE *a*

Station	Angle
25 + 84	End of line
21 + 94	L. 32° 35′
15 + 53	R. 44° 10′
11 + 72	L. 60° 30′
5 + 25	L. 25° 15′
0	



PLATTING AN

Scale 200 ft = 1 i

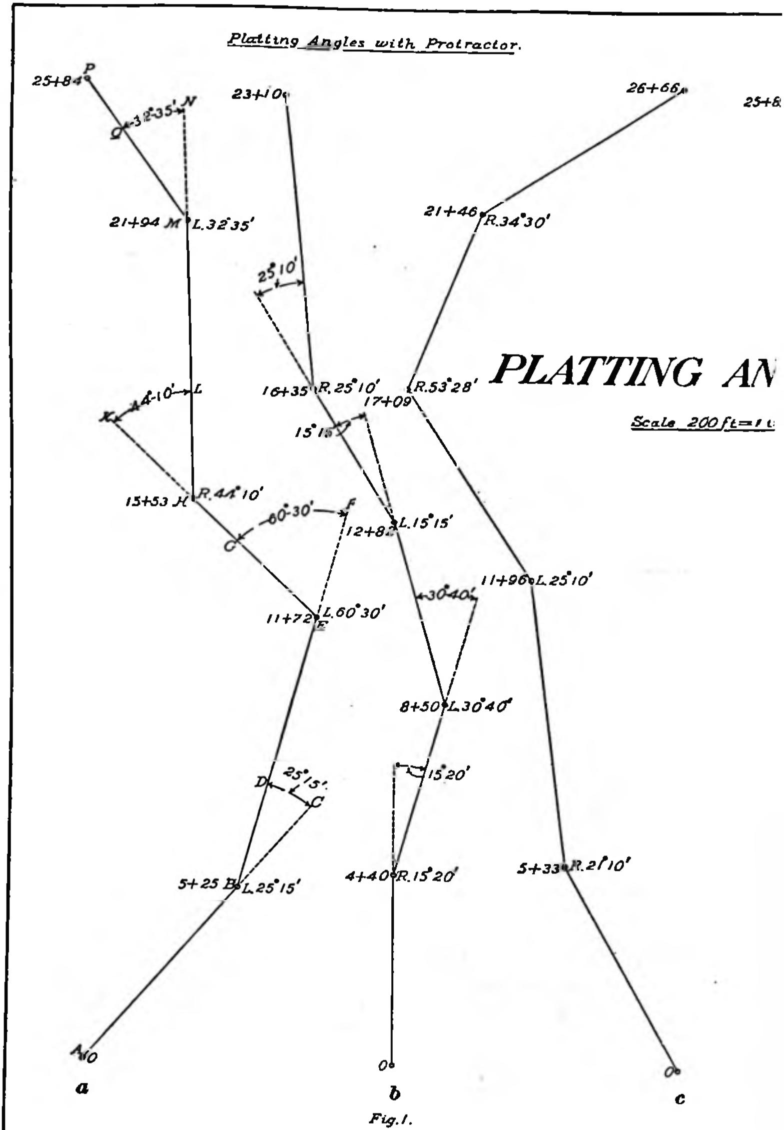
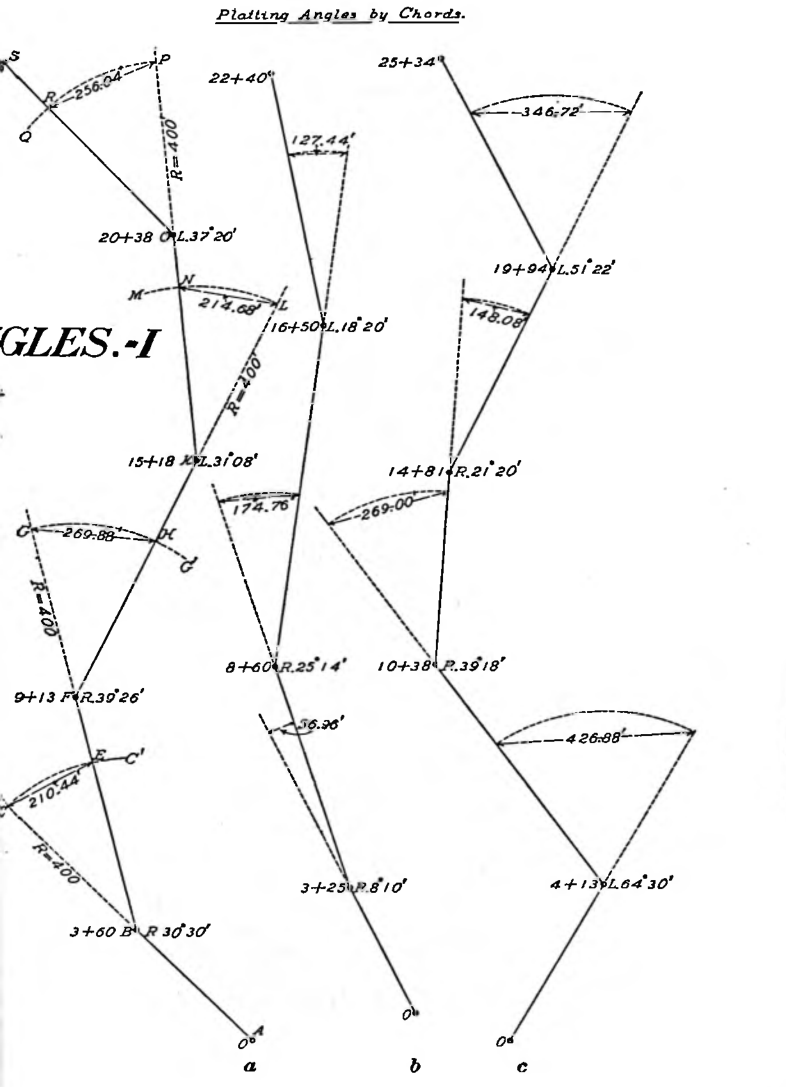


Fig. 1.

GLES.-I



Platting Angles by Chords.
S
P
R
256.04'
Q
R=400
20+38 O L.37°20'
M
N
214.68'
L
22+40'
127.44'
16+50 L.18°20'
R=400
15+18 K L.31°08'
174.76
G
269.88'
H
G'
R=400
9+13 F R.39°26'
E
C'
210.44'
R=400
3+60 B R 30°30'
0° A
8+60 R.25°14'
56.96'
3+25 R.8°10'
0°
25+34'
346.72'
19+94 L.51°22'
148.08'
14+81 R.21°20'
269.00'
10+38 R.39°18'
426.88'
4+13 L.64°30'
0°
a
b
c

Fig. 2.


б
в

8. Platting Angle Lines by Protractor.—The starting point *A* of line *a*, Fig.* 1, is Station 0. This point should be located in the same relative position that it occupies on the plate, namely, at a distance of ¾ inch from the lower and left-hand border lines. Starting from the point *A* as thus located, draw the line *A C* of indefinite length and in the same direction that it has in the plate. In order to determine this direction, draw a light pencil line, through *A* parallel to the vertical border line, on both the plate and the drawing, measure with the protractor the angle between this line and the line *A C* in the plate, and lay off the same angle between the lines in the drawing. From the point *A*, scale off on the line *A C* the length of the first course, 525 feet, thus locating the point *B*, which is Station 5 + 25. At this point, an angle of 25° 15′ is turned to the left. In order to lay off this angle, prolong *A B* to *C*, making *B C* a little longer than the radius of the protractor; then place the center of the protractor on the point *B*, with the zero point on the line *B C*, and on its graduated edge lay off the angle 25° 15′ to the left of *B C*, marking the point *D* of angle measurement with a needle point. Through the points *B* and *D* draw a straight line. The angle *C B D* is 25° 15′, and the line *B D* is the direction of the next course. Prolong the line *B D* to *F*, and on the line *B F* scale off from *B* the length of the second course *B E*, which is found by subtracting 525 from 1,172, giving a difference of 647 feet. The second angle point, Station 11 + 72, is thus located at *E*. Any number of minutes smaller than the number contained in the smallest subdivision of the protractor should be estimated and laid off by the eye.

The remaining angles and courses are platted in a similar manner. At *E*, lay off to the left of the line *E F* the second angle *F E G*, equal to 60° 30′. Prolong *E G* to *K*, and from *E* scale off on the line *E K* the length of the third course *E H*, which is equal to 1,553 − 1,172 = 381 feet. This determines the position of the third angle point, Station 15 + 53. At *H* lay off to the right of *H K* the third angle *K H L* equal to 44° 10′, and through the point *L* thus

determined draw the line HLN; the length of the fourth course is 2,194 − 1,553 = 641 feet, and this distance, scaled off from H along that line, fixes the position of Station 21 + 94 at M, the fourth angle point. Finally, lay off to the left from MN the angle NMO equal to 32° 35′, and from M, along the line MOP, scale off the length of the last course, which is equal to 2,584 − 2,194 = 390 feet. The fifth and last point P, which is Station 25 + 84, is thus established.

When commencing to plat the lines b and c, the notes for which are given in the accompanying table, care should be

NOTES FOR LINE *b*

Station	Angle
23 + 10	End of line
16 + 35	R. 25° 10′
12 + 82	L. 15° 15′
8 + 50	L. 30° 40′
4 + 40	R. 15° 20′
0	

NOTES FOR LINE *c*

Station	Angle
26 + 66	End of line
21 + 46	R. 34° 30′
17 + 09	R. 53° 28′
11 + 96	L. 25° 10′
5 + 33	R. 21° 10′
0	

exercised to locate the starting point of the line b at a distance of $\frac{3}{4}$ inch from the lower border line, and $4\frac{1}{4}$ inches from the left-hand border line, and Station 0 of line c at $\frac{3}{4}$ inch from the lower border line, and $7\frac{1}{4}$ inches from the left-hand border line. The first part of each line should be drawn in the same direction with reference to the border lines as on the plate, so as not to have the lines run too near together in platting. The rest of the work is similar to that explained for the line a.

9. To Lay Off an Angle by Its Chord.—In platting an angle by means of its chord, the length of the chord is obtained from a table of chords. Such a table gives the lengths of chords for all angles from 0° to 90° in a circle whose radius is 1. A radius of any convenient length may

be assumed, and the chord length corresponding to that radius obtained by multiplying the length of the chord given in terms of radius 1 by the length of the assumed radius. Thus, let it be required to lay off an angle of 40° 10′ to the left from the line *A B*, Fig. 3. The line *A B* is prolonged to *C*, making *B C* = 400 feet, the length assumed for the radius.

A table of chords gives .6868 as the chord of an angle of 40° 10′ in terms of a radius 1. Multiplying this chord by 400, the length of the assumed radius, gives 274.72 feet as the length of the required chord. From *B* as a center, and with a radius *B C* = 400 feet (to scale), describe to the left of *B C* the indefinite arc *C D*, making the length of the arc slightly greater than the length of the required chord. Then from *C* as a center, with a radius of 274.72 feet, describe an arc intersecting the arc *C D* at *C′*, and connect *B* and *C′* by a straight line. The angle *C B C′* thus formed is the required angle measuring 40° 10′. This method of platting angles is more accurate, though less rapid, than platting with a protractor.

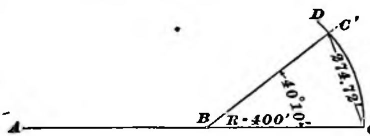


FIG. 3

The table of chords used for these calculations may be found in almost any engineers' pocketbook. If the student does not possess a pocketbook that contains this table, he can easily find the required chord from his table of natural sines; it is equal to twice the sine of half the given angle. Thus, the chord of $40° \; 10' = 2 \sin \frac{40° \; 10'}{2} = 2 \sin 20° \; 05'$ $= 2 \times .34339 = .68678$, which, written to only four decimal places, is .6868, the same as given in a table of chords.

10. Platting a Survey Line by Chords.—The method of platting a survey line by chords is illustrated in Fig.* 2 of this plate, in which the angles for the lines *a*, *b*, and *c* are laid off by this method. The notes for the line *a* are

given in the accompanying table, and from these notes the line is platted to a scale of 200 feet to the inch in the following manner:

The starting point or initial point A of this line, also numbered 0, should be located on the drawing $\frac{3}{4}$ inch from the lower border line, and 11 inches from the left-hand border line, and the line AB should be drawn in the same direction, with respect to the border line, as is shown on the plate. The direction of this line once established, the point B, which is Station 3 + 60, is located by scaling off from A on the line AB, to the adopted scale, the length of the first course, which is 360 feet. At B, the line deflects 30° 30′ to the right from the line AB. Prolong AB 400 feet, which is the length of radius assumed in calculating the chord length for laying off the angles in these examples. Then from B as a center, with a radius of 400 feet, describe to the right of AB prolonged the indefinite arc CC', containing at least 30° 30′. The chord of an angle of 30° 30′ corresponding to a radius 1 is .5261, which, when multiplied by 400, the length of the assumed radius, gives 210.44 feet as the length of the required chord. From C as a center, with a radius of 210.44 feet, describe an arc intersecting the arc CC' at the point E. The line joining B and E forms with BC an angle CBE = 30° 30′, the required angle. Prolong BE and scale off along this line the length of the second course, which is equal to 913 − 360 = 553 feet, thus locating the point F, which is Station 9 + 13. Prolong BF 400 feet to G. With F as a center, and a radius FG of 400 feet, describe to the right of FG the indefinite arc GG', containing at least 39° 26′. The chord of 39° 26′ corresponding to a radius 1 is .6747, which, multiplied by 400, gives 269.88 feet as

NOTES FOR LINE *a*

Station	Angle
25 + 80	End of line
20 + 38	L. 37° 20′
15 + 18	L. 31° 08′
9 + 13	R. 39° 26′
3 + 60	R. 30° 30′
0	

the length of the required chord. With G as a center, and this chord as a radius, describe an arc intersecting the arc $G G'$ at H. The line joining F and H forms with the radius FG the required angle $GFH = 39° 26.'$ Prolong FH and lay off along this line the length of the third course, equal to 1,518 − 913 = 605 feet, thus locating the point K, which is Station 15 + 18. Prolong FK 400 feet to L. With K as a center and KL as a radius, describe to the left of KL the indefinite arc LM. The chord corresponding to 31° 08′ is .5367, which multiplied by 400 gives 214.68 feet as the length of the required chord. With this chord as a radius and L as a center, describe an arc intersecting the arc LM at the point N. Join K and N, forming with KL the angle LKN = 31° 08′, and along the continuation of KN scale off the fourth course equal to 2,038 − 1,518 = 520 feet. Station 20 + 38 is thus located at O. Produce KO 400 feet to P. Then with O as a center and OP as radius, describe the indefinite arc PQ. The chord of 37° 20′, the last angle, is .6401, which multiplied by 400 gives the length of the required chord as 256.04 feet. With P as center and this chord as radius, describe an arc intersecting PQ at R. The line joining O and R forms with OP the required angle POR = 37° 20′. Along the continuation of OR, scale off the length of the last course equal to 2,580 − 2,038 = 542 feet, thus locating the final Station 25 + 80 at the extremity S of this course.

NOTES FOR LINE *b*

Station	Angle
22 + 40	End of line
16 + 50	L. 18° 20′
8 + 60	R. 25° 14′
3 + 25	R. 8° 10′
0	

NOTES FOR LINE *c*

Station	Angle
25 + 34	End of line
19 + 94	L. 51° 22′
14 + 81	R. 21° 20′
10 + 38	R. 39° 18′
4 + 13	L. 64° 30′
0	

In a similar manner, plat the lines *b* and *c* according to the accompanying notes, locating the starting point of line *b* 1 inch from the lower border line and 13 inches from the left-hand border line, and the starting point of line *c* $\frac{3}{4}$ inch from the lower border line and 14 inches from the left-hand border line.

DRAWING PLATE, TITLE: PLATTING ANGLES—II

11. To Lay Off an Angle by Its Tangent.—In laying off an angle by its tangent, the line from which the angle is turned is prolonged a certain distance, and the tangent of the given angle, as obtained from a table of natural tangents, is multiplied by the distance that the line is prolonged; this result, commonly called the **calculated tangent**, is platted on a line drawn perpendicular to the line prolonged at its extremity. A line joining the angular point with the extremity of the calculated tangent gives the direction of the required line, on which the given distance is then laid off to scale.

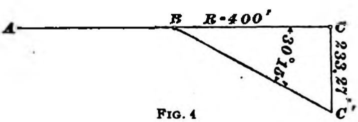


FIG. 4

Let *A B*, Fig. 4, be a given line from which an angle of 30° 15′ is to be laid off to the right at the point *B*. Prolong *A B* to *C*, making *B C* = 400 feet, the distance chosen

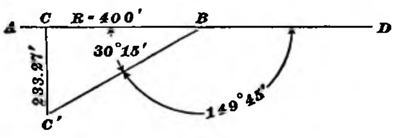


FIG. 5

in this case. Any other distance would do just as well. The tangent of 30° 15′, as given in a table of natural tangents, is .58318, which, when multiplied by 400 feet, gives 233.27 feet as the length of the calculated tangent. At *C* draw the perpendicular *C C′* to the right of *A C*, making its length equal to that of the calculated tangent, and join *B*

PLATTING ANGLES.-II

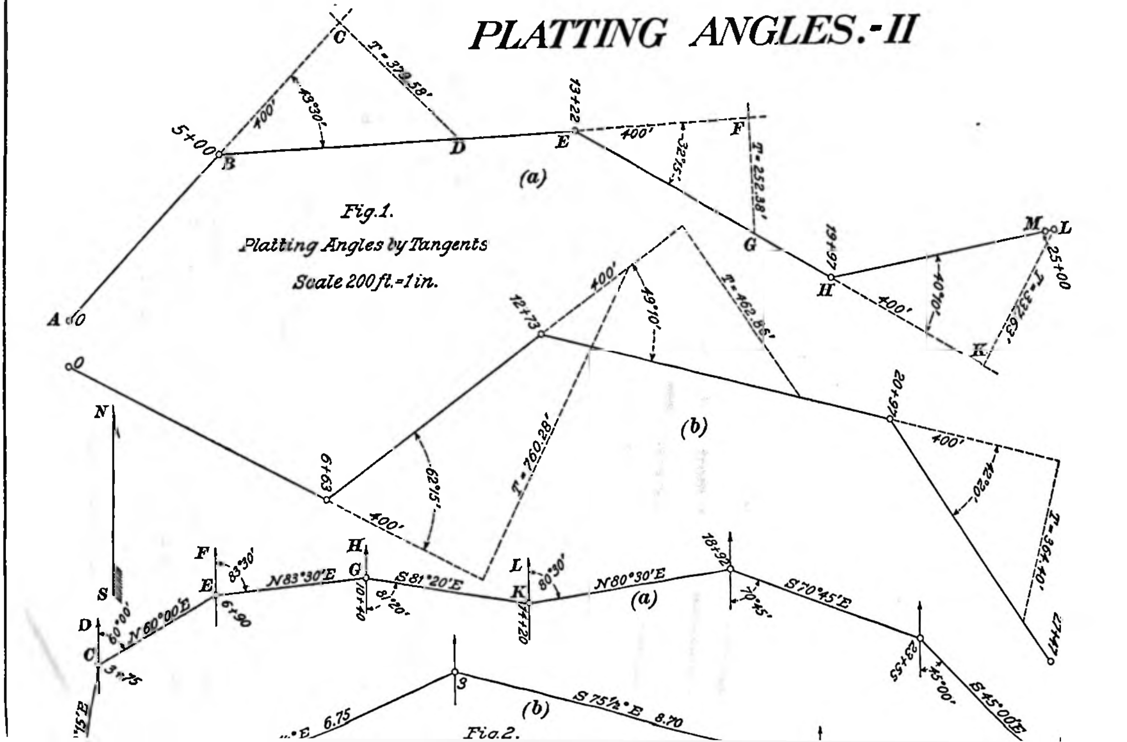
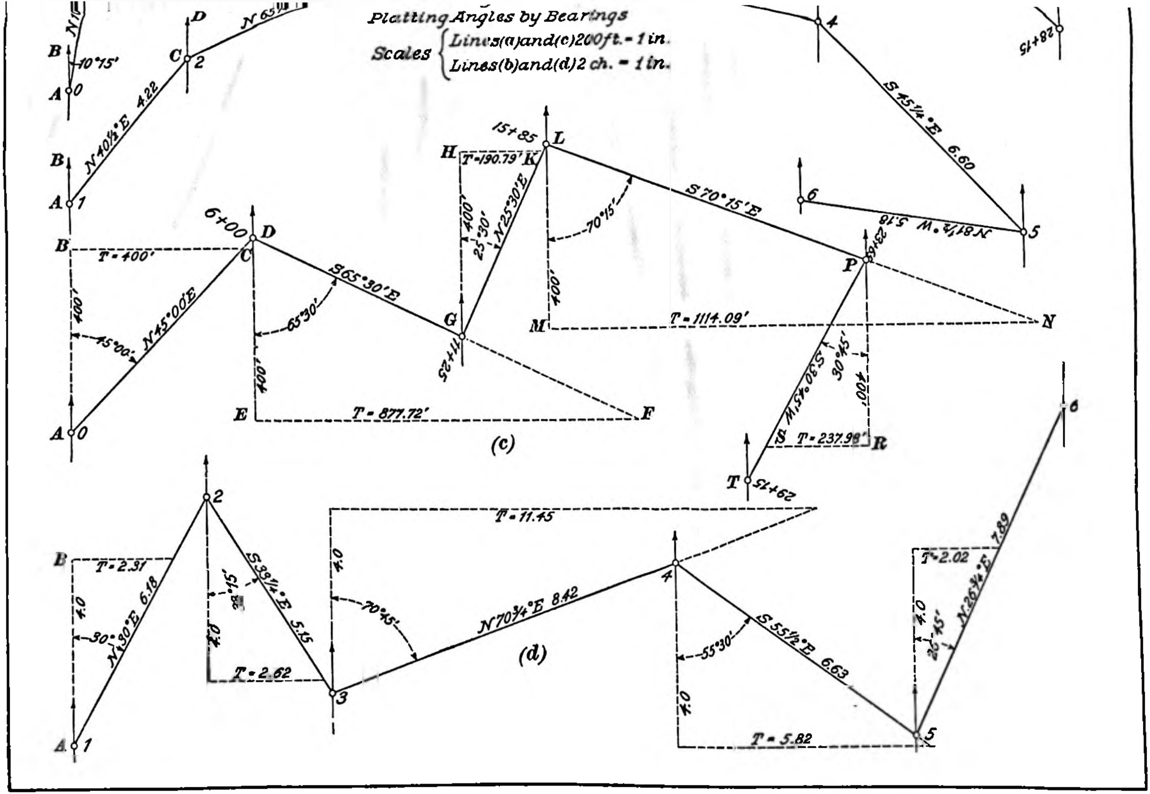


Fig. 1.
Platting Angles by Tangents
Scale 200 ft. = 1 in.

Fig. 2.

Platting Angles by Bearings

Scales $\begin{cases}\text{Lines (a) and (c) 200 ft.} = 1\text{ in.}\\ \text{Lines (b) and (d) 2 ch.} = 1\text{ in.}\end{cases}$



and C'. The angle $C B C'$ thus formed is the required angle of 30° 15′, and the line $B C'$ gives the direction of the required line.

To plat an obtuse angle $D B C'$, Fig. 5, turned off to the right of $B D$, produce $D B$, and construct, by the method explained above, the angle $C B C'$ equal to the supplement of $D B C'$, that is, equal to 180° minus the required obtuse angle.

12. Platting a Survey Line by Tangents.—This method is illustrated by the platted traverse lines *a* and *b* in Fig.* 1 of this plate. The notes for the line *a* are given in the accompanying table and are platted as follows: The starting point A, which is Station 0, should be located in the same relative position that it occupies on the plate, namely, $\frac{3}{4}$ inch from the left-hand border line and $3\frac{3}{4}$ inches from the upper border line. The direction of the line $A B$ can be determined in a manner similar to that described in Art. 8. After drawing the line $A B$ in the required direction, lay off on it from A, to a scale of 200 feet to the inch, the length of the first course, 500 feet, thus locating the point B, which is Station 5. At this station, the line deflects 43° 30′ to the right. Prolong $A B$ 400 feet. The natural tangent of 43° 30′ is .94896, which, when multiplied by 400, gives 379.58 feet as the length of the calculated tangent. At C, the extremity of the line prolonged, draw the perpendicular $C D$ to the right, and on this perpendicular scale off from C the calculated tangent distance, 379.58 feet, to the point D. The line joining B and D forms an angle of 43° 30′ with the line $A B$ prolonged and gives the required direction of the second course of the traverse. On the line $B D$ prolonged, scale off the distance 1,322 − 500 = 822 feet, thus locating the point E, which is Station 13 + 22. At this point the line

NOTES FOR LINE *a*

Station	Angle
25 + 00	End of line
19 + 97	L. 40° 10′
13 + 22	R. 32° 15′
5 + 00	R. 43° 30′
0	

deflects 32° 15′ to the right. Prolong BE 400 feet to the point F. The natural tangent of 32° 15′ is .63095, which, when multiplied by 400, gives 252.38 feet as the length of the calculated tangent. At F, the extremity of the prolongation of BE, draw the perpendicular FG to the right, and on this perpendicular scale off the calculated tangent distance, 252.38 feet, to the point G, and join E and G. The angle FEG thus formed is 32° 15′, and the line EG is the required direction of the third course of the traverse. Scale off on the line EG prolonged the distance 1,997 − 1,322 = 675 feet to H, thus locating Station 19 + 97, at which point the line deflects 40° 10′ to the left. Prolong EH 400 feet to K and at this point draw the perpendicular KL to the left. The natural tangent of 40° 10′ is .84407, which, when multiplied by 400, gives 337.63 feet as the calculated tangent. Scale off on the line KL the calculated tangent distance of 337.63 feet to L, and join H and L. The angle KHL thus formed is 40° 10′. Scale off on the line HL the distance 2,500 − 1,997 = 503 feet, thus locating the point M at the end of the line, which is Station 25 + 00. In a similar manner plat the line b according to the notes here given, locating the starting point, Station 0, ½ inch below the point A of the preceding line, and ¾ inch from the left border line.

NOTES FOR LINE *b*

Station	Angle
27 + 47	End of line
20 + 97	R. 42° 20′
12 + 73	R. 49° 10′
6 + 63	L. 62° 15′
0	

13. To Lay Off Angles by Bearings.—By this method of laying off angles, the direction of each line is referred to a meridian line. In platting the traverse of a survey, a pencil line giving the direction to the meridian is drawn through each station at which the bearing of a line is taken. Usually, it is convenient to make the meridian parallel to the sides of the drawing sheet. The lines showing the direction of the meridian at each station can then be drawn by

means of the ordinary **T** square and triangles, and the angles can be laid off by means of a protractor, by chords, or by tangents, as may be desired.

Let it be required to plat a line having a bearing of N 60° E followed by a line having a bearing of N 30° E. The point *A*, Fig. 6, is assumed as the position of the station at which the first bearing is taken. Through this point the line *A B* is drawn along the edge of the triangle, perpendicular to the straight edge of the **T** square, to represent the direction of the meridian. Since the bearing is east, the angle *C A B*, equal to 60°, is laid off to the right of *A B*. From

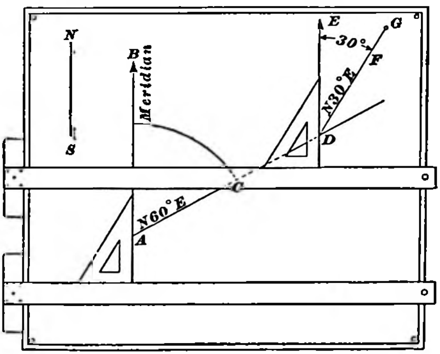


FIG. 6

the notes, find the length of the first course, and lay it off along *A C* to the adopted scale, thus locating at its extremity the point *D*, where the next bearing is taken.

The bearing of the next line, as taken at *D*, is N 30° E. Slide the **T** square upwards, and also slide the triangle across the board until the edge that is perpendicular to the **T** square passes through *D*, and then draw the meridian *D E* through *D*. From *D*, lay off an angle *E D G* equal to 30°, to the right of *D E*.

14. Platting Bearings by Protractor.—This method of platting bearings is illustrated in Fig.* 2. The four angle

lines or traverse lines shown in this figure, designated as *a*, *b*, *c*, and *d*, respectively, are all platted by bearings. The bearings of the lines *a* and *b* are platted by means of the protractor, and those of the lines *c* and *d* are platted by the tangent method; the latter method is the more accurate, though it is much the slower. For the lines *a* and *c*, the distances are given in stations of 100 feet, and should be platted to a scale of 200 feet to the inch; for the lines *b* and *d*, the distances are given in chains, and should be platted to a scale of 2 chains to the inch. The notes for line *a* are here given in tabular form.

NOTES FOR LINE *a*

Station	Bearing
28 + 15	End of line
23 + 55	S 45° 00′ E
18 + 92	S 70° 45′ E
14 + 20	N 80° 30′ E
10 + 40	S 81° 20′ E
6 + 90	N 83° 30′ E
3 + 75	N 60° 00′ E
0	N 10° 15′ E

The point *A*, which is Station 0 of the line *a*, should be located in about the same relative position as it has on the plate, that is, $\frac{3}{4}$ inch from the left-hand border line and $7\frac{1}{2}$ inches from the lower border line. The arrow *N S* represents the direction of the meridian. The bearing of the first course is N 10° 15′ E. Through *A*, draw a line *A B* parallel to the meridian line *N S*. The line so drawn represents the direction of the meridian at the point *A*, and the direction of the first course is then at an angle of 10° 15′ to the right of the line *A B*. At the point *A*, lay off an angle of 10° 15′ to the right of *A B* by means of the protractor, and then draw the line *A C*, which thus has a bearing N 10° 15′ E and represents the direction of the first course. Scale off the length of the first course, 375 feet, from *A* along the line *A C*, thus locating the point *C*, which is Station 3 + 75. Draw the line *C D* through *C* parallel to the meridian line *N S*; the line *C D* then represents the direction of the meridian at the point *C*. Since the bearing of the second course is

N 60° 00′ E, lay off at C an angle of 60° 00′ to the right of CD, and draw the line CE, which thus represents the direction of this course. Scale off from the point C along the line thus drawn the length of the second course, which is equal to $690 - 375 = 315$ feet, thus locating the point E, which is Station $6 + 90$. Through E draw the line EF parallel to the meridian line NS; and at the point E lay off to the right of the line EF the bearing of the third course, which is N 83° 30′ E, and draw the line EG representing the direction of this course. Scale off on this line the distance $1{,}040 - 690 = 350$ feet, thus locating the point G, which is Station $10 + 40$. Draw the meridian line GH, and lay off from the line GH the bearing of the next course, S 81° 20′ E. Since this course is south and east, the angle is laid off to the right of GH from the south end of the protractor, in a direction contrary to the movement of the hands of a watch. After laying off this angle, draw the line GK representing the direction of this course, and scale off on it the distance $1{,}420 - 1{,}040 = 380$ feet, thus establishing the point K, which is Station $14 + 20$. In a similar manner locate the remaining stations on this line.

The notes for line b are given in tabular form below, and should be platted according to the method just described. In the notes of this line, the stations at the angles of the line are numbered consecutively, beginning with Station 1 at the starting point, which is located $1\frac{1}{4}$ inches below the point A of the preceding line and $\frac{3}{4}$ inch from the left-hand border line. This station is written in the first or top line of the notes, which read from the top downwards, as is not unusual in the notes of ordinary land surveys with the compass, instead of from the bottom upwards, as in surveys where more or less sketching

NOTES FOR LINE b

Station	Bearing	Distance Chains
1	N $40\frac{1}{2}$° E	4.22
2	N $65\frac{1}{4}$° E	6.75
3	S $75\frac{1}{2}$° E	8.70
4	S $45\frac{1}{4}$° E	6.60
5	N $81\frac{1}{2}$° W	5.18
6	End of line	

is necessary. In these notes, the numbers of the stations do not indicate the lengths of the various courses; these lengths are given in the third column of the notes.

15. Platting Bearings by Tangents.—Lines *c* and *d* of Fig.* 2 are platted by means of the tangents of the bearings of the various courses. The direction of each line is laid off from the meridian by means of the tangent of its bearing in substantially the same manner that an angle is laid off by its tangent, as explained in Art. **11.** This is one of the best methods of platting bearings, and when a **T** square and triangles are used, the work can be performed expeditiously. The notes given here for line *c* are platted in the following manner:

NOTES FOR LINE *c*

Station	Bearing
29 + 15	End of line
23 + 65	S 30° 45′ W
15 + 85	S 70° 15′ E
11 + 25	N 25° 30′ E
6 + 00	S 65° 30′ E
0	N 45° 00′ E

The point *A*, which is the initial point of this line, should be located in the same relative position that it occupies on the plate, that is, $\frac{3}{4}$ inch from the left-hand border line and $2\frac{1}{2}$ inches below the point *A* of the line immediately preceding. The line *N S* is assumed to have the direction of the meridian. The bearing of the first course is N 45° 00′ E. Through *A* draw the meridian line *A B* parallel to the meridian *N S*, and on this line scale off from *A* the distance by which the tangent of the bearing is multiplied, in this case 400 feet, to the point *B*. At *B*, the extremity of this distance, draw *B C* to the right perpendicular to the meridian line *A B*. The tangent of 45° 00′ is 1.0000, which when multiplied by 400 gives 400 feet as the length of the calculated tangent for this bearing and distance. Scale off the distance of 400 feet on the perpendicular *B C*, thus fixing the point *C*, and draw the line *A C*. The angle *B A C* that the line *A C* forms with the meridian is 45° 00′, and the bearing of the line *A C* is N 45° 00′ E. On the line *A C*

prolonged, scale off the length of the first course, 600 feet, thus locating the point D, which is Station 6. The bearing of the second course is S 65° 30′ E. Through D draw the meridian line DE. Since the bearing is south and east, it should be laid off below the point D and to the right of DE. On the line DE scale off the distance, 400 feet, to the point E. At E draw the perpendicular EF to the right of DE. The tangent of 65° 30′ is 2.19430, which, when multiplied by 400, gives 877.72 feet for the calculated tangent. On the perpendicular EF, scale off the distance of 877.72 feet to the point F, and draw the line DF; the angle EDF thus formed is 65° 30′, and the bearing of the line DF is S 65° 30′ E. On the line DF scale off the length of the second course, which is equal to 1,125 − 600 = 525 feet, thus locating the point G, which is Station 11 + 25. In a similar manner plat the directions of the remaining courses, and locate the stations at their extremities.

NOTES FOR LINE *d*

Station	Bearing	Distance Chains
1	N 30° E	6.18
2	S $33\frac{1}{4}$° E	5.15
3	N $70\frac{3}{4}$° E	8.42
4	S $55\frac{1}{2}$° E	6.63
5	N $26\frac{3}{4}$° E	7.89
6	End of line	

The notes for line *d* are given here in tabular form; they are platted in substantially the same manner as those for line *c*, except that, as the distances are in chains, a distance of 4 chains, equal to 400 links, is laid off on the meridian line through each angular point, and is used as the multiplier of each tangent. The point A of line *d* is located $\frac{3}{4}$ inch from the left-hand border line and $\frac{1}{2}$ inch above the lower border line, and is marked as Station 1. Through this point, draw the meridian line AB parallel to NS, and on this line scale off from A the distance of 4 chains to the point B. At B erect a perpendicular to the right of AB. The bearing of the first course is N 30° 00′ E, and the tangent of 30° 00′ is

.57735, which, when multiplied by 4, gives 2.3094 chains as the length of the calculated tangent for this bearing and distance. Scale off this calculated tangent distance on the perpendicular and draw a line joining the point thus determined with the point *A*; the line thus drawn forms with the line *A B* an angle of 30° 00′, and has a bearing of N 30° 00′ E. On this line, scale off the length of the first course, 6.18 chains, thus locating Station 2. In a similar manner plat the remaining courses and locate Stations 3, 4, 5, and 6, marking them as shown in the plate.

The bearing of each course should be written above it plainly and distinctly, the letters reading in the same direction in which the course is measured. In all railroad surveys and in the more recent highway surveys, the distances are reckoned in station intervals of 100 feet, and each station is numbered according to its distance in station intervals from the initial point of the survey. When this system is used, the length of each course is given by the difference between the numbers of the stations at its two extremities. But in land surveys, and in the older highway surveys, the lengths of the courses are recorded in surveyors' chains, and the stations at the angular points or extremities of the courses are numbered consecutively, beginning with Station 1 at the initial point of the survey. In this system, the lengths of the courses are not indicated by the station numbers, but must be recorded separately.

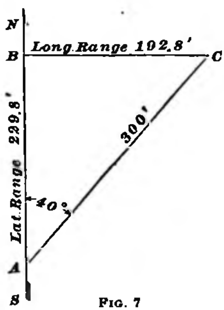


FIG. 7

16. To Lay Off an Angle by Latitude and Longitude Ranges. Suppose that the bearing of a course is N 40° 00′ E, and its length is 300 feet. The ranges of the course, calculated as explained in *Compass Surveying*, Part 2, are: latitude range, +229.8 feet; longitude range, +192.8 feet.

Let *A*, Fig. 7, be the station at which the bearing is taken.

Through A draw the meridian $N S$. From A lay off along $N S$ the distance $A B$ equal to the calculated latitude range, or 229.8 feet, and at the extremity of this distance draw the perpendicular $B C$ to the right, or toward the east. On this perpendicular, scale off the calculated longitude range, 192.8 feet, to the point C, and draw the line $A C$. The angle $B A C$ thus formed is an angle of 40°, and the length of $A C$ is 300 feet.

17. Platting a Survey by Ranges.—In this method, each course is platted by means of its latitude and its longitude range, the operation described in the last article being repeated at every station. As will be observed, it is not necessary to scale off the distances, since the extremities of each course are determined by its ranges.

18. Platting a Survey by Latitudes and Longitudes.—This method was fully explained in *Compass Surveying*, Part 2. It is, by far, more accurate than any of the methods that have been described, because the position of each station is referred directly to the initial point or first station of the survey and is platted independently.

In surveys of such character as preliminary railroad surveys, ordinary land surveys, etc., angles and bearings are often platted by tangents; but on such work as difficult railroad location, where dependence must be placed on accurate platting for the purpose of obtaining a satisfactory paper location, the method by latitudes and longitudes should be used, and the line should be platted to a scale large enough to show complete topographical details.

19. Parallel Rulers.—There are two general kinds of parallel rulers. One kind, known as the **folding parallel**

FIG. 8

ruler, consists of two rulers connected to each other by two light bars of equal length and having jointed ends that

permit the rulers to be opened apart or folded together, but hold them constantly parallel to each other. This form of parallel ruler is shown in Fig. 8; it is not much used at the present time. The other kind, called the **rolling parallel ruler,** is very convenient for platting and is used very extensively. An illustration of it is given in Fig. 9. It consists of an ordinary ruler fitted with milled rollers of equal diameter attached to a common axis; it is usually made of metal, and is therefore of considerable weight. This instrument is used for the purpose of transferring the direction of a line from one part of the plat to another, such as drawing a line through any point on a map parallel to the meridian. For instance, if it is required to draw a meridian line through a certain point on a plat, the straight edge of the parallel ruler is made to coincide with the line representing the given meridian, and the ruler is then rolled across the paper until

FIG. 9

its straight edge passes through the desired point. A line drawn through the point along the straight edge of the ruler will then be the required meridian line.

20. Protractor Sheets.—In platting surveys by means of the ordinary movable protractor, more or less error is likely to occur in transferring the meridian lines from point to point on the plat and in adjusting the protractor to the meridian line for laying off the bearing of each course. This liability to error is reduced to a minimum by drawing the plat on what is commonly called a **protractor sheet.** This is a sheet of drawing paper, tracing paper, or bristol board having a protractor printed in the center of the sheet, and is sometimes called a **paper protractor.** The protractor is a full circle, from 8 to 14 inches in diameter, graduated to half or quarter degrees, according to size, and is printed from

accurately engraved plates. It is usually printed in red on Whatman's drawing paper. The numbers of the divisions of the protractor are not printed on the sheet, but are written

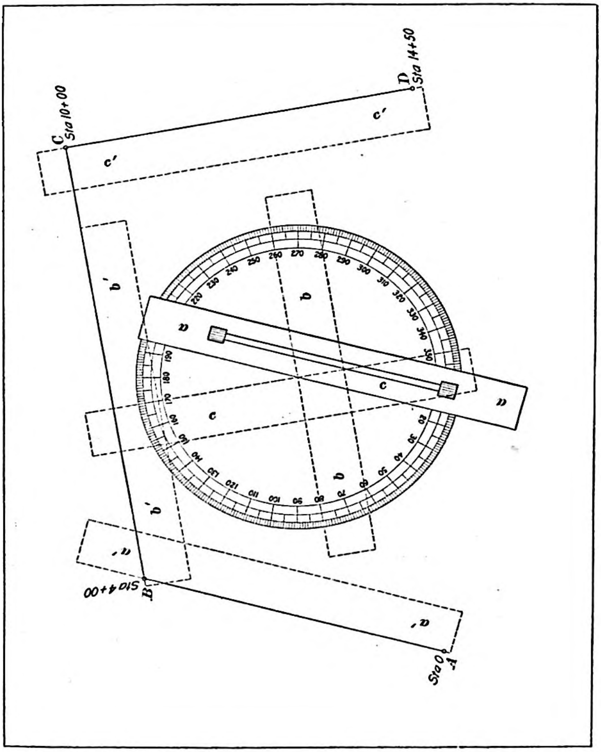
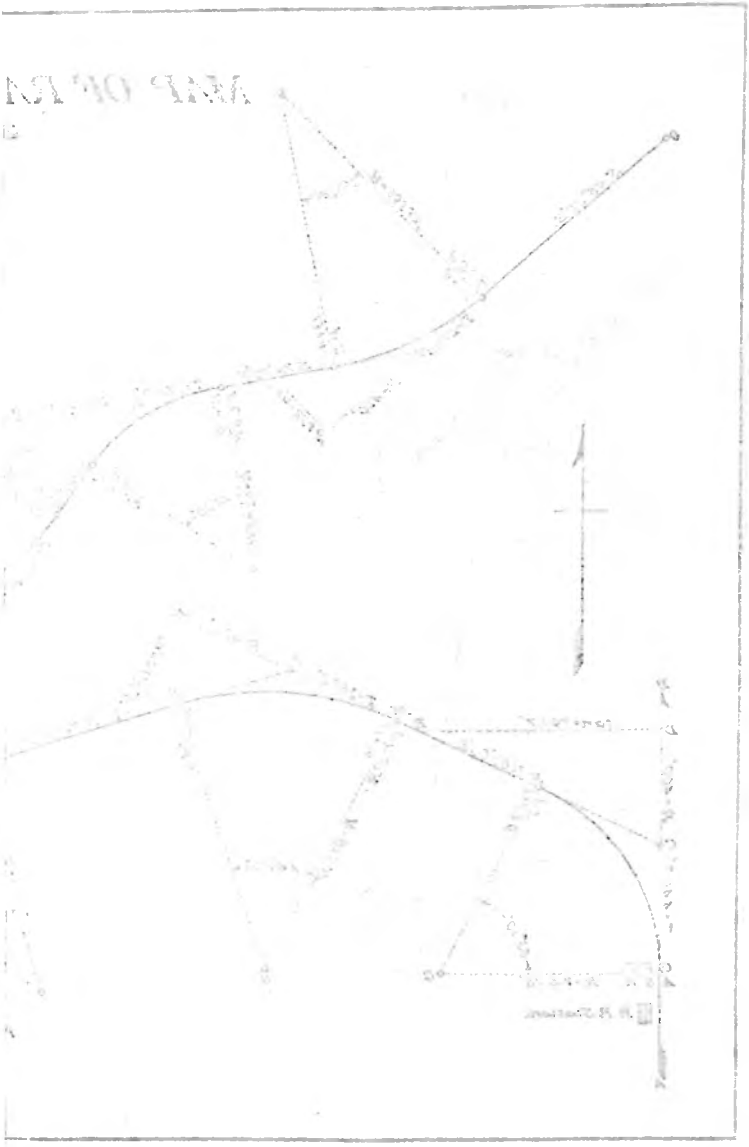


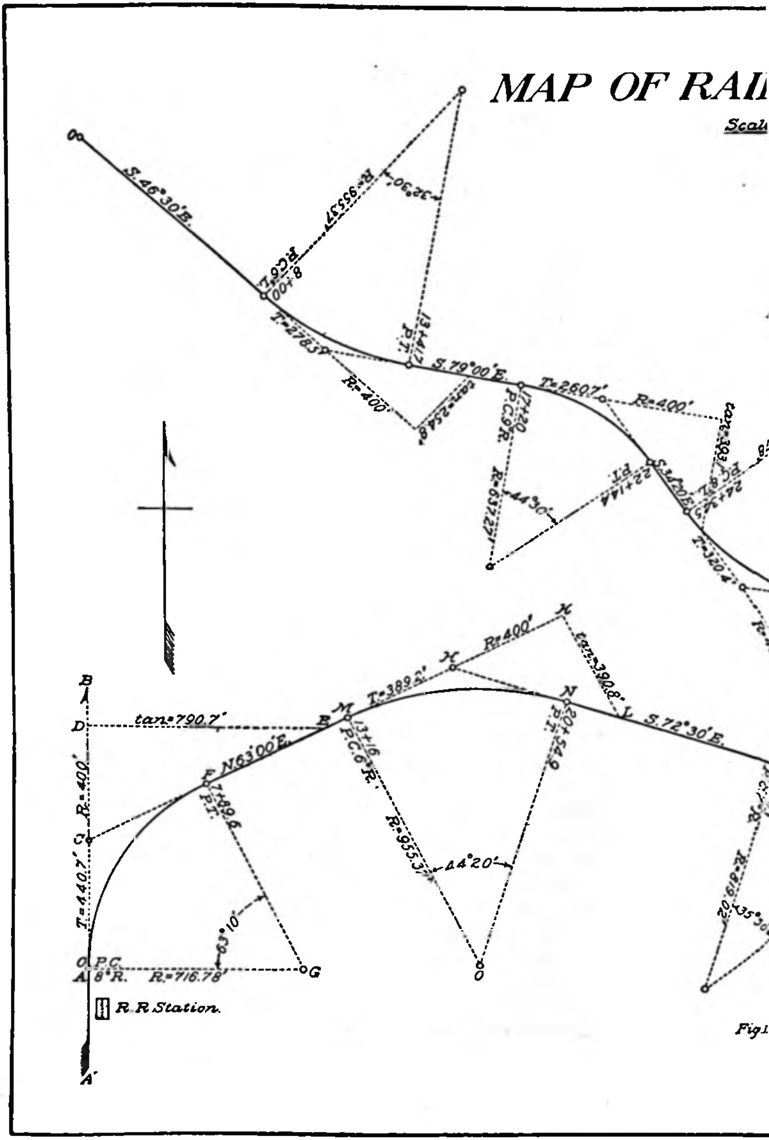
Fig. 10

on afterwards-by the draftsman in the manner most convenient for the plat for which the sheet is used. Fig. 10 represents a protractor sheet, and illustrates the manner in

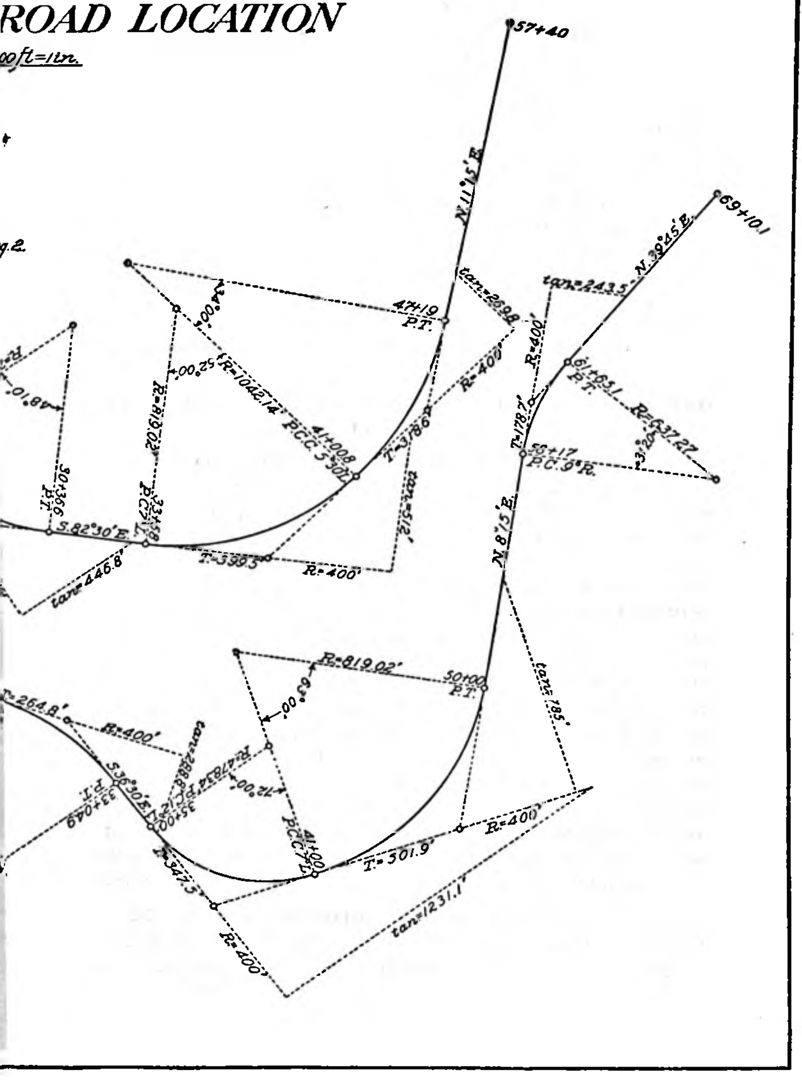
which an azimuth traverse is platted by means of the parallel ruler. The notes of the traverse are given in the accompanying table. In this case the meridian line is assumed to be parallel to the left edge of the drawing paper, and the divisions on the protractor are numbered accordingly. As this is for the platting of an azimuth traverse, the degrees are numbered from 0 to 360 around the complete circle, with the zero mark toward the north or the south edge of the paper, according as azimuths are reckoned from the north or from the south. The point A is chosen for the initial point, or Station 0 of the traverse. Since the forward azimuth of the first course from Station 0 is 195°, the parallel ruler is placed on the protractor in the position $a\,a$, so that the straight edge of the parallel ruler is on the division marked 195° and the division directly opposite, which is the division marked 15°. The number of the division on the protractor directly opposite the division representing the given azimuth will always be equal to the difference between 180° and the given azimuth. When the straight edge of the parallel ruler is placed on two such divisions, it will always pass through the center of the protractor. By using opposite divisions, instead of one division and the center of the protractor, greater accuracy is obtained. When the edge of the parallel ruler is on the 195° and the 15° division marks, it is in a position parallel to the direction of the line that is to be platted through the point A to represent the first course of the traverse, whose azimuth is 195°. The parallel ruler is then rolled to the point A, where it will have the position $a'\,a'$, and, with its straight edge passing through the point, the line $A\,B$ is drawn. On this line the length of the first course, 400 feet, is laid off to the scale of the plat, thus locating the point B, which is Station 4. The azimuth reading for the

Station	Azimuth	Magnetic Bearing
14 + 50		
10 + 00	350° 00′	N 10° W
4 + 00	260° 00′	S 80° W
0	195° 00′	S 15° W




MAP OF RAI
Scal
S.46°30'E.
R.=955.37
32°30'
P.C.6°L.
8+00
T.=278.57'
P.T.
13+41.7
R.=400'
tan.=254.8'
S.79°00'E.
T.=260.7'
R.=400'
P.C.9°R.
17+20
R.=637.27
44°30'
P.T.
22+14.4
24+24
T.=320.4'
R.R. Station.
B
D
tan.=790.7'
E
M
T.=389.2'
R.=400'
N
tan.=390.8'
L
S.72°30'E.
N63°00'E.
F
7+89.6
P.T.
13+16
P.C.6°R.
P.T.
20+54.9
R.=400'
C
T.=440.7'
R.=955.37
44°20'
63°10'
O P.C.
A 8°R.
R.=716.78'
G
O
A'
Fig.

ROAD LOCATION



course from Station 4 to Station 10 is 260°. The parallel ruler is therefore placed in the position *b b*, so that its edge is on the division of the protractor marked 260° and the opposite division, marked 80°, and is then rolled to the point *B*, in the position *b′ b′*, and the line *B C* is drawn. On the line thus drawn, the distance 1,000 − 400 = 600 feet is scaled off, thus locating the point *C*, which is Station 10. In a like manner the line *C D* is platted, locating the point *D*, which is Station 14 + 50. If a parallel ruler is not at hand, two triangles can be used for the same purpose, though not quite so expeditiously. The method of drawing parallel lines by means of two triangles is fully explained in *Geometrical Drawing*.

DRAWING PLATE, TITLE: MAP OF RAILROAD LOCATION

21. The Line of a Railroad Survey.—The line of a preliminary railroad survey is usually a succession of straight lines; such a line is not uncommonly called an *angle line*, but it is correctly designated as a *traverse line*. The **located line**, as the final line of a railroad survey is called, is a succession of straight lines and curves. In platting a located line, the straight lines, commonly called **tangents**, should be laid out and measured from one point of intersection to another, and the angles between the tangents, that is, the intersection angles, should be platted either by tangents or by ranges. The points of tangency, that is, the point of curve and point of tangent for any required curve joining two tangents, should then be located by calculating the tangent distances and scaling them off from the point of intersection backwards and forwards on the two connecting tangents. The center of each curve is best determined by describing intersecting arcs from the points of tangency as centers, with a radius equal to that of the given curve.

22. General Description of the Plate.—On this plate are shown two platted lines, each illustrating the operation of platting the line of a railroad location. This map is

called a map of railroad location, but the following description will apply as well to the platting of any similar alinement consisting of straight lines and curves, such as that for a canal or pipe line. In this plate, all the angles are laid off by the method of tangents described in Art. **11**, though any other satisfactory method may be used. The notes from which these two lines are to be platted are given in detail on pages 28 to 33; they should be gone over carefully and each calculation verified before beginning the platting. The magnetic meridian is assumed to be parallel to the right and left border lines of the plate, and the starting point, or Station 0, of each line should be platted in the same position with reference to the border lines as it occupies on the plate. The direction of the first tangent of each line should be determined from the direction of the magnetic meridian, and should be platted carefully with reference thereto. Without these precautions, the lines are liable to run off the paper, necessitating a repetition of the work, and involving the erasure of lines, which soils and injures the paper, and mars the appearance of the drawing. The notes should be platted to a scale of 300 feet to the inch.

23. Drawing the Plate.—As has been stated, the notes of two lines of railroad location are given in detail in the following pages. All calculations necessary for platting these notes are assumed to have been made, and written on the right-hand page of the notebook, as shown. All these calculations for each line should be verified by the student before beginning to plat the line. The lines are then platted from the notes as follows: First draw a meridian line parallel to the right-hand and left-hand border lines, then locate the starting point *A*, Fig.* 1, $\frac{3}{4}$ inch from the left-hand border line, and $1\frac{3}{4}$ inches from the lower border line. Through this point, which is Station 0 of this line, draw the line *A B* parallel with the meridian line. From the notes, it is found that Station 0 is the P. C. of an 8° curve to the right, and that the direction of the tangent *A' A*, preceding the curve, is due north and south; this tangent is assumed to be part of

a line already constructed. The intersection angle is 63° 10′, and the radius of an 8° curve is 716.78 feet; from these the tangent distance is found to be 440.7 feet, as recorded in the notes. Using a scale of 300 feet to the inch, this distance is scaled off upwards from the point *A*, on the tangent *A′ A* prolonged, thus locating the point of intersection *C* of the back and forward tangents, or tangents preceding and following the curve. In order to determine the direction of the forward tangent *CE*, it is necessary to lay off to the right, at the point of intersection *C*, the intersection angle of 63° 10′. On the prolongation of the back tangent, scale off, from the angle of intersection to the point *D*, the distance *CD*, equal to the multiplier of the tangent, which in this case is taken as 400 feet. The intersection angle of this curve is 63° 10′, and the tangent of this angle is 1.97681. The length of the calculated tangent is therefore $400 \times 1.97681 = 790.7$ feet. Since this curve is to the right, draw from *D* a line to the right perpendicular to *AB*, and on this perpendicular scale off the calculated tangent, 790.7 feet, locating the point *E*. A line drawn from *C* through the point *E* gives the direction of the forward tangent. On the line *CE*, scale off from *C* the tangent distance of the curve, 440.7 feet, thus locating the P.T. of the first curve at the point *F*, which is at Station 7 + 89.6, since the P. C. is at Station 0 and the length of the curve is 789.6 feet. From *A* and *F* as centers, with a radius of 716.78 feet, the radius of an 8° curve, describe arcs intersecting at *G*; then, with *G* as a center, and the same radius, describe a curve joining the points *A* and *F*. The curve *AF* is an 8° curve tangent to the lines *AA′* and *FE* at the points *A* and *F*.

From the notes it is found that the tangent *FE* extends to Station 13 + 16, which is the P. C. of a curve of 6° to the right, whose central angle is 44° 20′. The radius of this curve is 955.37 feet, and its tangent distance is 389.2 feet, as given in the notes. The distance from the point of intersection of the first curve to the point of intersection of the second curve is calculated next. This distance is composed of three parts; namely, the tangent of the preceding curve, which is 440.7 feet; the intermediate tangent, extending

NOTES FOR FIG.* 1

Station	Deflection	Total Angle	Magnetic Bearing	Calculated Bearing
40	12° 00′			
39	6° 00′			
38 + 00	18° 00′	36° 00′		
37	12° 00′			
36	6° 00′			
35 + 00	P. C. 12° L.			
34				
33 + 4.9	10° 40.3′ P. T.	35° 50′	S 36° 30′ E	S 36° 40′ E
33	10° 30′			
32	7° 00′			
31	3° 30′			
30	7° 14.7′	14° 29.4′		
29	3° 44.7′			
28	0° 14.7′			
27 + 93	P. C. 7° R.			
24				
21				
20 + 54.9	10° 38.8′ P. T.	44° 20′	S 72° 30′ E	S 72° 30′ E
20	9° 00′			
19	6° 00′			
18	3° 00′			
17	11° 31.2′	23° 2.4′		
16	8° 31.2′			
15	5° 31.2′			
14	2° 31.2′			
13 + 16	P. C. 6° R.			
12				
11				
10				
9				
8				
7 + 89.6	15° 35′ P. T.	63° 10′	N 63° 00′ E	N 63° 10′ E
7	12° 00′			
6	8° 00′			
5	4° 00′			
4	16° 00′	32° 00′		
3	12° 00′			
2	8° 00′			
1	4° 00′			
0	P. C. 8° R.		North	North

NOTES FOR FIG.* 1

	Remarks	June 28, 1894
Int. Ang. = 72° 00′ 12° curve, L. R. = 478.34 ft. T. = 347.5 ft. P. C. = 35 + 00 Length of curve = 600 ft. P. C. C. = 41 + 00 Def. 100 ft. = 6° 00′ Def. 1 ft. = 3.6′	From intersection to intersection. Tan preceding curve = 2 6 4.8 ft. Tan between curves = 1 9 5.1 ft. Tan 12° curve = 3 4 7.5 ft. Total, = 8 0 7.4 ft. tan 72° 00′ = 3.07768 400 ft. × 3.07768 = 1,231.1 ft.	
Int. Ang. = 35° 50′ 7° curve, R. R. = 819.02 ft. T. = 264.8 ft. P. C. = 27 + 93 Length of curve = 511.9 ft. P. T. = 33 + 4.9 Def. 100 ft. = 3° 30′ Def. 1 ft. = 2.1′	From intersection to intersection. Tan preceding curve = 3 8 9.2 ft. Tan between curves = 7 3 8.1 ft. Tan 7° curve = 2 6 4.8 ft. Total, = 1 3 9 2.1 ft. tan 35° 50′ = .72211 400 ft. × .72211 = 288.8 ft.	
Int. Ang. = 44° 20′ 6° curve, R. R. = 955.37 ft. T. = 389.2 ft. P. C. = 13 + 16 Length of curve = 738.9 ft. P. T. = 20 + 54.9 Def. 100 ft. = 3° 00′ Def. 1 ft. = 1.8′	From intersection to intersection. Tan preceding curve = 4 4 0.7 ft. Tan between curves = 5 2 6.4 ft. Tan 6° curve = 3 8 9.2 ft. Total, = 1 3 5 6.3 ft. tan 44° 20′ = .977 400 ft. × .977 = 390.8 ft.	
Int. Ang. = 63° 10′ 8° curve, R. R. = 716.78 ft. T. = 440.7 ft. P. C = 0 Length of curve = 789.6 ft. P. T. = 7 + 89.6 Def. 100 ft. = 4° 00′ Def. 1 ft. = 2.4′	Radius 1 = 400 ft. tan 63° 10′ = 1.97681 400 ft. × 1.97681 = 790.7 ft.	

NOTES FOR FIG.* 1—*Continued*

Station	Deflection	Total Angle	Magnetic Bearing	Calculated Bearing
69 + 10.1		End of line		
61 + 65.1	15° 40′ P. T.	31° 20′	N 39° 45′ E	N 39° 40′ E
61	12° 44.1′			
60	8° 14.1′			
59	3° 44.1′			
58 + 17	P. C. 9° R.			
55				
54				
53				
52				
51				
50 + 00	17° 30′ P. T.	63° 00′	N 8° 15′ E	N 8° 20′ E
49	14° 00′			
48	10° 30′			
47	7° 00′			
46	3° 30′			
45	14° 00′	28° 00′		
44	10° 30′			
43	7° 00′			
42	3° 30′			
41 + 00	18° 00′ P. C. C. 7° L.	72° 00′	N 71° 15′ E	N 71° 20′ E

NOTES FOR FIG.* 2

Station	Deflection	Total Angle	Magnetic Bearing	Calculated Bearing
13 + 41.7	10° 15′ P. T.	32° 30′	S 79° 00′ E	S 79° 00′ E
13	9° 00′			
12	6° 00′			
11	3° 00′			
10 + 00	6° 00′	12° 00′		
9	3° 00′			
8 + 00	P. C. 6° L.			
5				
3				
0			S 46° 30′ E	

NOTES FOR FIG.* 1—*Continued*

	Remark	June 28, 1894
Int. Ang. = 31° 20′ 9° curve, R. R. = 637.27 ft. T. = 178.7 ft. P. C. = 58 + 17 Length of curve = 348.1 ft. P. T. = 61 + 65.1 Def. 100 ft. = 4° 30′ Def. 1 ft. = 2.7′		From intersection to intersection. Tan preceding curve = 5 0 1.9 ft. Tan between curves = 8 1 7.0 ft. Tan 9° curve = 1 7 8.7 ft. Total, = 1 4 9 7.6 ft. tan 31° 20′ = .60881 400 ft. × .60881 = 243.5 ft.
Int. Ang. = 63° 00′ 7° curve, L. R. = 819.02 ft. T. = 501.9 ft. P. C. C. = 41 + 00 Length of curve = 900 ft. P. T. = 50 + 00		From intersection to intersection. Tan preceding curve = 3 4 7.5 ft. Tan between curves = 0.0 ft. Tan 7° curve = 5 0 1.9 ft. Total, = 8 4 9.4 ft. tan 63° = 1.96261 400 ft. × 1.96261 = 785 ft.

NOTES FOR FIG.* 2

	Remarks	June 28, 1894
Int. Ang. = 32° 30′ 6° curve, L. R. = 955.37 ft. T = 278.5 ft. P. C. = 8 + 00 Length of curve = 541.7 ft. P. T. = 13 + 41.7 Def. 100 ft. = 3° 00′ Def. 1 ft. = 1.8′		Radius 1 = 4 0 0.0 ft. From Sta. 0 to P. C. = 8 0 0.0 ft. Tan 6° curve = 2 7 8.5 ft. Total from P. C. to P. I. = 1 0 7 8.5 ft. tan 32° 30′ = .63707 400 ft. × .63707 = 254.8 ft.

NOTES FOR FIG.* 2—*Continued*

Station	Deflection	Total Angle	Magnetic Bearing	Calculated Bearing
57 + 40		'End	of line	
47 + 19	8° 46.3′ P. T.	34° 00′	N 11° 15′ E	N 11° 20′ E
47	8° 15′			
46	5° 30′			
45	2° 45′			
44 + 00	8° 13.7′	16° 27.4′		
43	5° 28.7′			
42	2° 43.7′			
41 + 00.8	14° 01.7′ P.C.C. 5° 30′ L.	52° 00′	N 45° 15′ E	N 45° 20′ E
41	14° 00′			
40	10° 30′			
39	7° 00′			
38	3° 30′			
37 + 00	11° 58.2′	23° 56.4′		
36	8° 28.2′			
35	4° 58.2′			
34	1° 28.2′			
33 + 58	P. C. 7° L.			
32				
30 + 36.6	13° 27.8′ P. T.	48° 10′	S 82° 30′ E	S 82° 40′ E
30	12° 00′			
29	8° 00′			
28	4° 00′			
27 + 00	10° 37.2′	21° 14.4′		
26	6° 37.2′			
25	2° 37.2′			
24 + 34.5	P. C. 8° L.			
24				
23				
22 + 14.4	9° 39′ P. T	44° 30′	S 34° 20′ E	S 34° 30′ E
22	9° 00′			
21	4° 30′			
20 + 00	12° 36′	25° 12′		
19	8° 06′			
18	3° 36′			
17 + 20	P. C. 9° R.			
17				
15				

NOTES FOR FIG.* 2—*Continued*

Remarks	June 28, 1894
Int. Ang. = 34° 00′ 5° 30′, L. R. = 1,042.14 ft. T. = 318.6 ft. P. C. C. = 41 + 00.8 Length of curve = 618.2 ft. P. T. = 47 + 19 Def. 100 ft. = 2° 45′ Def. 1 ft. = 1.65′	From intersection to intersection. Tan preceding curve = 3 9 9.5 ft. Tan between curves = 0.0 ft. Tan 5° 30′ curve = 3 1 8.6 ft. Total, = 7 1 8.1 ft. tan 34° 00′ = .67451 400 ft. × .67451 = 269.8 ft.
Int. Ang. = 52° 00′ 7° curve, L. R. = 819.02 ft. T. = 399.5 ft. P. C. = 33 + 58 Length of curve = 742.8 ft. P. C. C. = 41 + 00.8 Def. 100 ft. = 3° 30′ Def. 1 ft. = 2.1′	From intersection to intersection. Tan preceding curve = 3 2 0.4 ft. Tan between curves = 3 2 1.4 ft. Tan 7° curve = 3 9 9.5 ft. Total, = 1 0 4 1.3 ft. tan 52° 00′ = 1.27994 400 ft. × 1.27994 = 512 ft.
Int. Ang. = 48° 10′ 8° curve, L. R. = 716.78 ft. T. = 320.4 ft. P. C. = 24 + 34.5 Length of curve = 602.1 ft. P. T. = 30 + 36.6 Def. 100 ft. = 4° 00′ Def. 1 ft. = 2.4′	From intersection to intersection. Tan preceding curve = 2 6 0.7 ft. Tan between curves = 2 2 0.1 ft. Tan 8° curve = 3 2 0.4 ft. Total, = 8 0 1.2 ft. tan 48° 10′ = 1.11713 400 ft. × 1.11713 = 446.9 ft.
Int. Ang. = 44° 30′ 9° curve, R. R. = 637.27 ft. T. = 260.7 ft. P. C. = 17 + 20 Length of curve = 494.4 ft. P. T. = 22 + 14.4 Def. 100 ft. = 4° 30′ Def. 1 ft. = 2.7′	From intersection to intersection. Tan preceding curve = 2 7 8.5 ft. Tan between curves = 3 7 8.3 ft. Tan 9° curve = 2 6 0.7 ft. Total, = 9 1 7.5 ft. tan 44° 30′ = .98270 400 ft. × .98270 = 393.1 ft.

from the P. T. of the preceding curve at Station 7 + 89.6 to the P. C. of the second or 6° curve at Station 13 + 16, a distance of 526.4 feet; and the tangent of the 6° curve, which is 389.2 feet; making a total distance of 1,356.3 feet. On the line CE prolonged, scale off from C a total distance of 1,356.3 feet, thus locating the point of intersection H of the 6° curve, and on the prolongation of the same line lay off the additional distance HK, equal to 400 feet, in order to lay off the intersection angle of this curve. The tangent of 44° 20′ is .97700, which, multiplied by 400, gives 390.8 feet as the calculated tangent for laying off this angle. From K draw a line to the right perpendicular to HK, and on it scale off this calculated tangent, 390.8 feet, thus locating the point L. The line joining H and L determines the direction of the forward tangent of the second curve. Next, from the point of intersection H, scale off on both back and forward tangents the tangent distance of 389.2 feet, thus locating the P. C. of this curve at M, which is Station 13 + 16, and its P. T. at N, which is Station 20 + 54.9. Then, with M and N as centers, and a radius of 955.37 feet, the radius of a 6° curve, describe arcs intersecting at O, and with O as a center and the same radius, describe a curve between the points M and N. The curve MN thus drawn is a 6° curve and is tangent to the lines FH and HL at the points M and N.

In platting these lines of railroad location, the tangent distances, the radii of the curves, and the tangents for laying off the intersection angles, should be drawn in dotted lines, as they are merely construction lines. The line of survey should be drawn in a full, bold line, as shown in the plate. The points of intersection and the points of curve and tangent are to be marked by small circles, the latter points being also designated by their station numbers. Dotted radial lines should be drawn from the center of each curve to its P. C. and P. T. On one of these radial lines, the length of the radius of the curve should be written, and the amount of the central angle should be marked within the radial lines. No further directions are deemed necessary for

platting the remainder of this line or the notes for example 2, a plat of which is shown in Fig.* 2 of the same plate, except that the starting point, Station 0, of example 2 is to be located ¾ inch from the left-hand border line and 1½ inches from the upper border line.

24. Railroad Curves.—For fitting in and platting the curves of a railroad location, what are known as **railroad curves** are very convenient. These consist of thin curved strips of hard rubber, pearwood, metal, or cardboard, cut to different radii according to a uniform scale, which is usually 100 feet to the inch. The two edges of each curved strip are the arcs of two curves having different radii. The degree of curvature of each arc, and the scale to which it has the given degree of curvature, are stamped distinctly on each strip, as

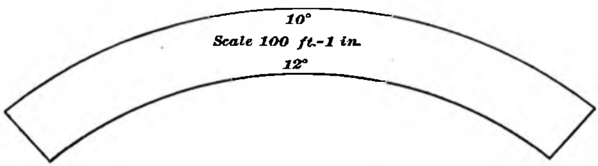


FIG. 11

shown in Fig. 11. A set of such railroad curves contains from 10 to 40 curves of different radii, according to the assortment. The curves can be adapted to any scale. For example, a 10° curve to a scale of 100 feet to the inch will serve for a 5° curve to a scale of 200 feet to the inch, or a 2° 30′ curve to a scale of 400 feet to the inch. In the same way, a 12° curve to a scale of 100 feet to the inch can be used for a 6° curve to a scale of 200 feet to the inch, or a 24° degree curve to a scale of 50 feet to the inch, etc. The principal object of these curves is to enable the engineer to readily select the curve that will best fit the ground lying between two tangents, as platted on the topographical map. The curves are applied directly to the topographical map on which the tangents of the line have been platted, and the curve that is best fitted to the ground and the tangents, and will give the best grades

and most economical construction, is selected. A satisfactory curve having been decided on, the tangent distances are then calculated and the curve is drawn in by means of a railroad curve or a pair of compasses, as may be most convenient.

25. Beam Compass.—When the radius of a curve is of considerable length, it is difficult, and sometimes impossible, to describe the arc of a true circle with ordinary compasses and lengthening bar. In such cases, an accurate and convenient substitute for the ordinary compasses and lengthening bar is found in the instrument known as a **beam compass.** Such an instrument is shown in Fig. 12. It consists of two

FIG. 12

upright legs *C* and *D* attached to metal pieces that clamp on the wooden beam *A B*. The leg *C* is attached rigidly to the metal clamp and carries a needle point, as shown. The leg *D* is attached to the metal clamp by means of a hinge joint that is adjustable by means of a lever and the milled-headed thumbscrew *E*. It carries either a pencil or a pen, by which the curve is described about the needle point as a center. The two legs *C* and *D* are clamped to the wooden beam *A B* by means of milled-headed screws, the heads of which are shown in the figure. By means of the milled-headed thumbscrew *E*, the pen or pencil can be accurately adjusted to the desired radius.

MAPPING

(PART 2)

TOPOGRAPHICAL DRAWING

REPRESENTATION OF TOPOGRAPHY

1. Topographical maps are graphical representations of the relative elevations, as well as of the dimensions and geographical positions, of the different natural and artificial features included in any given portion of the earth's surface. They show all inequalities of surface, such as hills, hollows, valleys, and plains, and the location of towns, highways, canals, railroads, streams, lakes, etc. Detailed topographical maps also show buildings and other structures, property lines, boundaries of fields, names of property owners, extent and varieties of timber, degree of curvature, character of soil and vegetation, etc. The contour map is valuable in the location of railroads, highways, canals, town sites, reservoirs, dams, parks, etc. In railroad, highway, and canal locations, it is used extensively for determining the best location with regard to alinement and grade.

2. Systems of Representing Topography.—The three most common systems of representing relative elevations on a topographical map are as follows: (1) by contour lines, (2) by hachures, (3) by shade from vertical light. A fourth system of representation is by means of what is called a **relief map,** which is constructed of papier mâché or other suitable material. On such a map is shown a series of

miniature hills and valleys, representing the exact form of the natural surface to a greatly reduced scale.

3. Contour Lines.—The method of representing topography by contour lines is the one generally adopted among engineers. As explained in *Topographic Surveying*, a **contour line** represents the intersection of a horizontal surface with the surface of the earth, and is therefore a line drawn through points of equal elevation. A map containing the outline of a given surface, together with the contour lines representing its form and inequalities, is called a **contour map** of the surface. Contour lines may conveniently be used to represent any form of surface.

Figs. 1 to 5, inclusive, illustrate the most common types of

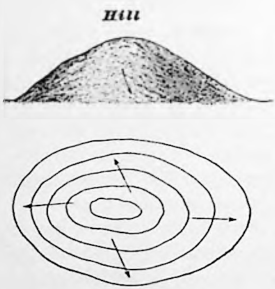


Fig. 1

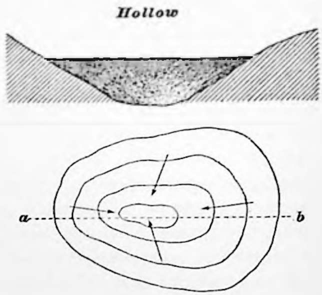


Fig. 2

the natural-surface formations of the ground. In the upper portion of each figure the natural formation is shown in elevation or section, and in the lower part the same formation is represented in plan by means of contour lines, as on a topographical map. In order that these may be more readily understood, arrows are marked on each figure to indicate the direction in which the water would flow. The vertical sections are in each case taken on the line *a b* in the contour plan. Fig. 1 represents a *hill;* Fig. 2, a *hollow;*

Fig. 3, the end of a *ridge*, commonly known as a *shoulder;* Fig. 4, the end of a *valley;* and Fig. 5, the end of two valleys

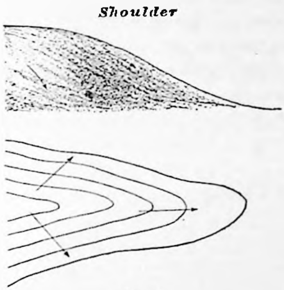


Fig. 3

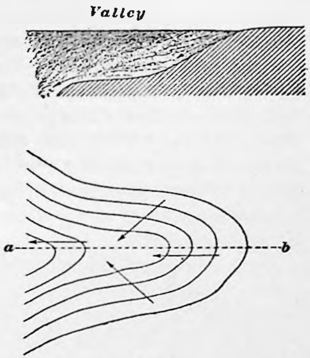


Fig. 4

meeting between two hills, forming what is commonly designated as a *saddle*. From the foregoing figures it is evident that where the contour lines run close together the slope of the natural surface is steeper, and where they are far apart the surface becomes more nearly level. Each contour line must be a distinct and continuous line, until it either runs off the map or closes on itself. In representing a cliff or a steep hillside, the contour lines run close together, and in case the cliff becomes

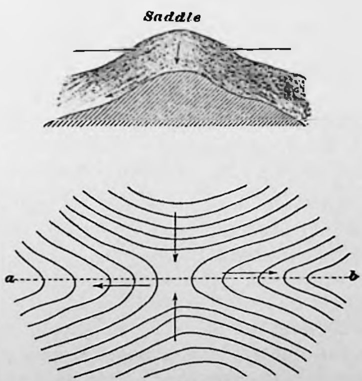


Fig. 5

absolutely vertical, two or more contour lines may run

together in a single line, or may even cross each other, as in the case of an overhanging cliff.

4. Hachures.—Slopes are sometimes represented by a system of short disconnected lines having the directions of lines of greatest slope; that is, the directions that water would take in running off the surface of the slope. Such lines are called **hachures, or hatchings.** Since contour lines are lines of constant elevation, hachures must always be drawn perpendicular to the contour lines. An example of this system of representing the form and inequalities of the earth's surface is shown in Fig. 6.

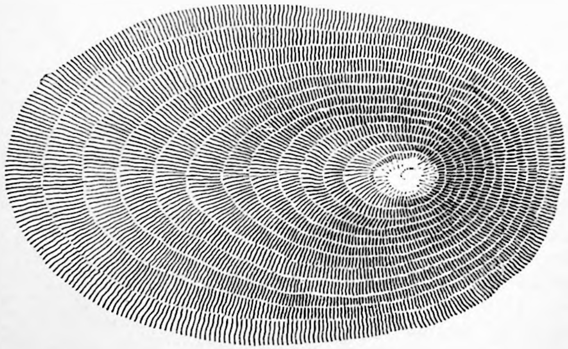


FIG. 6

In sketching topography by this system, the topographer should hold the book directly in front of him so that it will correspond with his position on the ground, and draw the lines toward him. If standing at the top of a slope, he should begin by drawing the lines from the bottom, and vice versa. To guide the hachures, he should first sketch in the contour lines lightly with a pencil, especially when the slopes are steep and irregular. The hachures must be drawn truly perpendicular to the contour lines, which are not drawn in ink, but are indicated by the spaces between the rows of

vertical hachures. Where the contour lines curve sharply, it is often well first to draw in the hachures at considerable intervals, as a guide to the direction of those drawn afterwards. Hachures in adjoining rows should not be continuous, but should be so drawn as to break joints. They must not overlap, and should be drawn in slightly wavy lines. The hachures should have their thickness and distance apart proportional to the steepness of the slope. The lines are made heavier as the slope is steeper, being fine for gentle slopes, while for very steep slopes the blank spaces are but half the breadth of the lines.

5. Shades From Vertical Light.—This system of representing the form and inequalities of the earth's surface depends on the principle that the steeper any slope is the less vertical light it receives, and consequently, the darker it appears. This difference in the degree of light is imitated and much exaggerated by the shading, when slopes are represented by this method. For work of this kind, the United States Coast and Geodetic Survey uses definite amounts of shade for slopes of different degrees. The shading is made dark for steep slopes and light for gentle slopes; it is deep black for slopes of 75° and steeper, and is graded to about midway between black and white for slopes of 30°, and so on, level surfaces being white or unshaded.

The shading is applied in various ways. A rapid method, and one sufficiently accurate for many kinds of work, is to sketch in the contours and then apply the shading in the form of India ink diluted with water. This shading, or India-ink tinting, is applied with a brush, and each varying tint is applied with its particular brush, care being taken not to allow any tint to dry before the succeeding tint is applied. By applying the tints in this way, they can be so blended as to give a smooth and finished effect to the work.

CONVENTIONAL SIGNS

6. Certain conventional signs are commonly used in topographical mapping to represent the natural and artificial features of the surface; the most common are shown in

Figs. 7 to 21. In making these conventional signs, great care should be exercised to draw them neatly and clearly, for if they are made in a slovenly manner they will greatly mar the appearance of the drawing. As a means of economy in making large maps, some offices have stamps for such conventional signs as those for grass, underbrush, woods, swamp, marsh, clearing, orchard, cultivated ground, etc., by means of which the signs can be quickly stamped on the portions of the map where required, thus saving considerable time and expense. It should be noted, however, that the practice regarding conventional signs is not entirely uniform; the same signs are not used in all offices to represent the same topographical features.

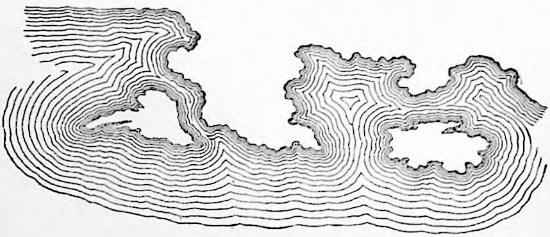


FIG. 7

7. Shore Line.—The shore line of a lake or other body of water is represented by a line that follows all the windings and indentations of the shore. Near the shore line, on the water side, is drawn a parallel line, and near this, but at a slightly greater distance from it, is drawn another parallel line, and so on, the distance between the lines gradually increasing and the lines becoming less irregular, as shown in Fig. 7.

8. Rocky Shore.—An abrupt and rocky shore is represented as shown in Fig. 8. The irregular dotted surfaces surrounded by shore lines represent sand bars, and the dotted outlines beyond the shore line represent shoals or submerged rocks.

9. Sand Shore and Sand Dunes.—A sand shore is represented by fine dots, as shown in Fig. 9. A gravel shore is represented in a similar manner, except that the dots are made larger or heavier. The black line represents the shore line proper, usually the high-

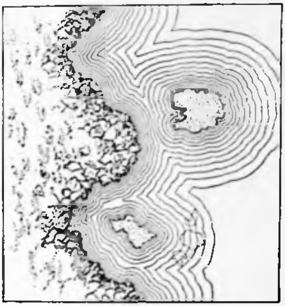


FIG. 8

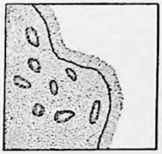


FIG. 9

water line. The irregular dotted surfaces inland from the shore line represent *sand dunes.*

10. Rivers.—The shore lines of large rivers are usually represented in a manner somewhat similar to the shore lines

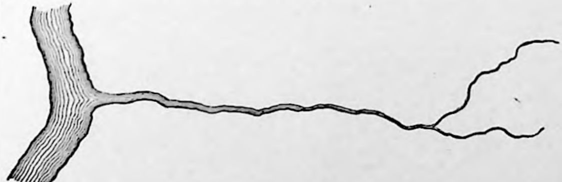


FIG. 10

of other bodies of water, as shown in Fig. 10. Large brooks or creeks are represented by two parallel lines, and small ones by a single line.

11. Grass and Cultivated Ground.—Grass is represented by irregular groups of short diverging lines, as shown in Fig. 11. Cultivated ground is represented by a

series of parallel continuous lines alternating with dotted lines, as shown in Fig. 12.

12. Orchards are represented by irregularly scalloped outlines of approximately circular form, arranged in regu-

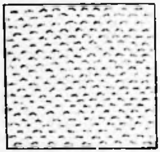


FIG. 11

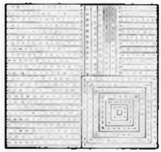


FIG. 12

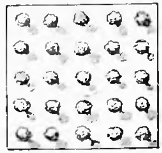


FIG. 13

lar rows to represent the trees, and shaded on the lower right-hand side, as shown in Fig. 13.

13. Woods.—The conventional signs used most commonly for representing woodlands consist of scalloped and shaded outlines similar to those used to represent orchards, except that they are arranged irregularly, being placed close together or far apart according as the forest is dense or open, and intermingled with stars in an irregular manner, as shown in Fig. 14. In this method of representing woods,

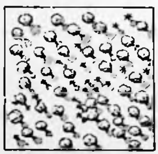


FIG. 14

FIG. 15

the scalloped outlines represent the deciduous trees, and the stars the evergreens. Woods are also sometimes represented by means of trees shown in elevation, as in Fig. 15. By reason of the trees being shown in elevation, this method is not so consistent for a map, and is not used so extensively, as the other, although it is very effective in

distinguishing the different varieties of trees, especially the deciduous from the evergreen trees.

14. Clearings are commonly represented by outlines resembling stumps placed irregularly and interspersed with the signs for grass and bushes, as shown in Fig. 16.

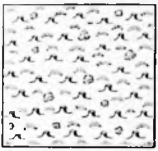


FIG. 16

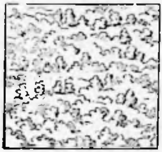


FIG. 17

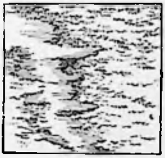


FIG. 18

15. Underbrush is usually represented by very irregular lines that are more or less scalloped and slightly shaded in places, the very irregular outlines somewhat resembling those of the underbrush, as shown in Fig. 17.

16. Swamp.—Swamp land is commonly represented by a combination of the conventional signs for grass, bushes, and water, giving the general effect shown in Fig. 18.

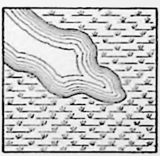


FIG. 19

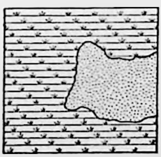


FIG. 20

17. Fresh-Water Pond and Marsh.—A fresh-water pond is represented in the same manner as an ordinary body of water. A fresh-water marsh is represented by irregular groups of short diverging lines and a series of parallel lines broken at irregular intervals, as shown in Fig. 19.

18. Salt-Water Pond and Marsh.—The surface of a salt-water pond is represented by small dots. A salt-water marsh

is represented by irregular groups of short diverging lines and a series of unbroken parallel lines, as shown in Fig. 20.

19. Rice Dikes and Ditches.—A rice dike is represented by two rows of short parallel lines; and a ditch, by two continuous parallel lines. The rice is represented by irregular groups of short diverging lines, having a number of light parallel shade lines drawn below each group, as shown in Fig. 21.

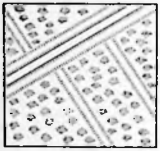


FIG. 21

20. Essentials to Mapping.—In the construction of maps, clearness, accuracy, legibility, and neatness are indispensable to good work. A neat explicit title, including the scale, date of survey, name of surveyor, direction of meridian, and key to any topographical symbols used, is very essential, and a neat border line adds much to the appearance of the map. Where time and cost are not to be considered, the lower sides of the contour lines may be shaded or hatched as though water were draining off them, and the valleys and low places tinted with a light shade of India ink. In sketching the contour lines on a map, gaps or spaces should be left in the lines at suitable intervals for writing the elevations of the contours.

PLATE, TITLE: CONTOUR MAP AND PROFILE

NOTE.—As stated in *Mapping*, Part 1, figures in the plates are referred to by placing a star after the abbreviation Fig. Thus, Fig.* 1 means Fig. 1 in the plate, while Fig. 1 means Fig. 1 in the text.

21. Preliminary Remarks and Field Notes.—Fig.* 1 is a contour map of a rectangular area, a portion of which, adjacent to the northeast corner, is occupied by a lake. Fig.* 2 represents profiles along the lines AA' and FF' of the contour map; the former line extends along the western edge of the field and the latter line extends over the summit of a hill and across an arm of the lake. The vertical scale of these profiles is five times the horizontal scale, thus exaggerating the slope of the surface. Before drawing this plate,

PROFILES

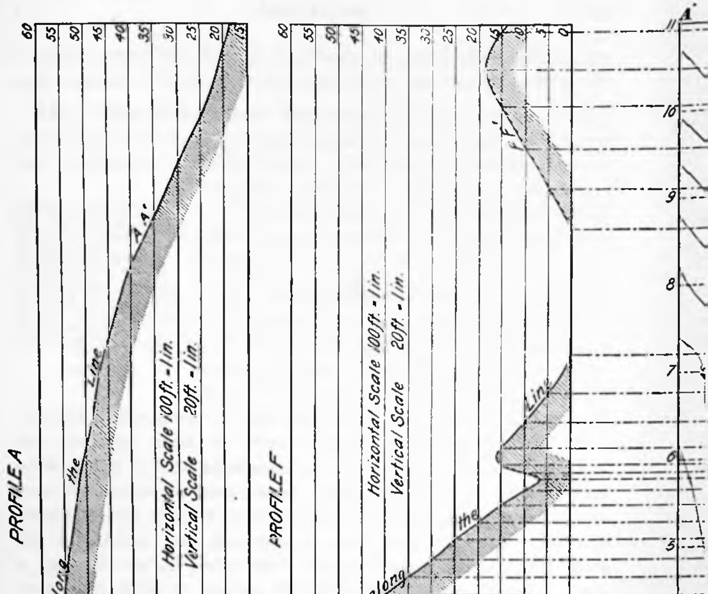


Fig.2

CONTOUR MAP

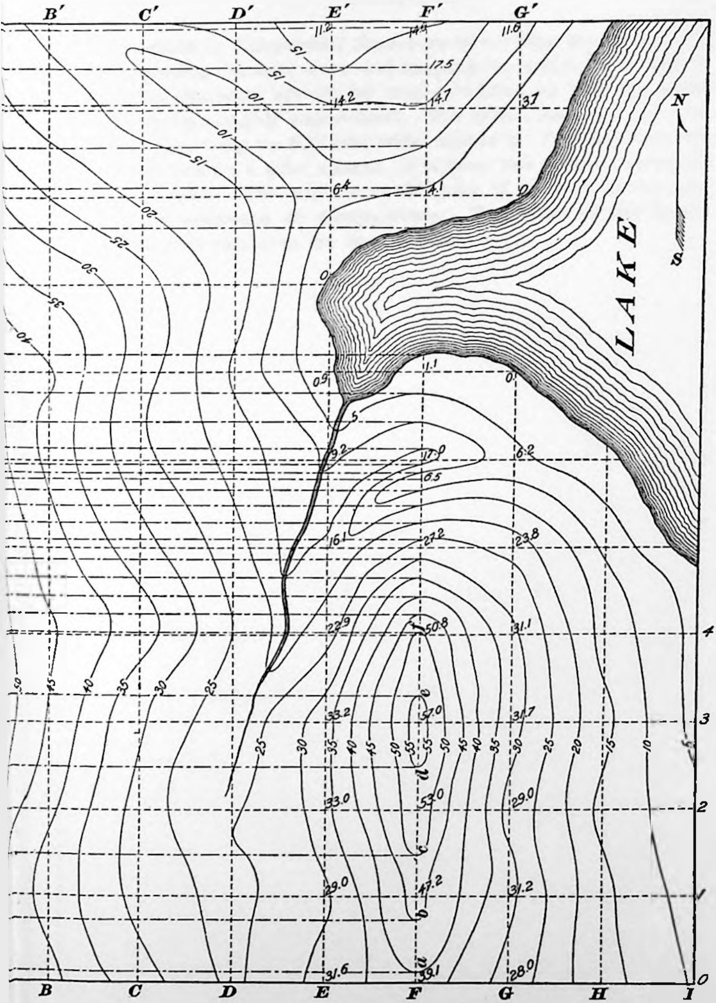
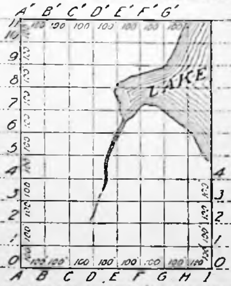


Fig. 1

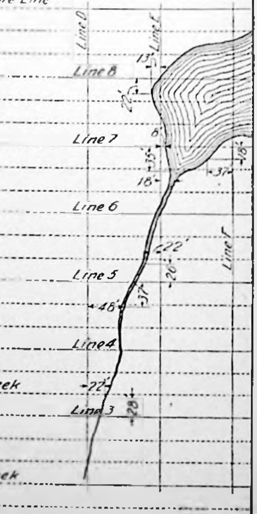
the article in *Topographic Surveying* describing the method of performing the field work and keeping the notes for a survey of this character should be read carefully, so that the work will be thoroughly understood. The area shown in Fig.* 1 is 1,100 feet long by 800 feet wide, and is divided into squares of 100 feet on a side, except, of course, the portion occupied by the lake. The surface of the lake is adopted as the surface of reference, or datum plane. The notes of the levels taken over this area are here given.

Station	Back-Sight	Height of Instrument	Foresight	Elevation of Surface			Remarks
Line I							
	11.2	11.2		0.0			Surface of Water
0			2.1	9.1			
1			3.8	7.4			
2			4.9	6.3			
3			7.6	3.6			
4			7.6	3.6			
4+80			11.2	0.0			Shore of Lake
Line H							
6+16			11.2	0.0			" " "
6			9.7	1.5			
5			1.9	9.3			
Line 6							
H+15			11.2	0.0			Shore of Lake
Line 5							
H+72			11.2	0.0			" " "
T.P.	10.8	209	1.1	10.1			
Line H							
4			7.3	13.6			
3			4.9	16.0			
2			6.8	14.1			
1			5.0	15.9			
0			4.4	16.5			
T.P.	11.7	32.0	06	203			
Line G							
0			4.0	28.0			
1			0.8	31.2			
2			3.0	29.0			
3			0.3	31.7			
4			0.9	31.1			
5			8.2	23.8			
T.P.	1.2	21.8	11.4	20.6			
T.P.	2.7	15.1	9.4	12.4			
6			8.9	6.2			
7			15.1	0.0			Shore of Lake



Station	Back Sight	Height of Instrument	Foresight	Elevation of Surface			Remarks
		15.1					
8+95			15.1	0.0			Shore of Lake
10			11.4	3.7			
11			3.5	11.6			
Line 11							
6+98			15.1	0.0			Shore of Lake
Line 10							
6+53			15.1	0.0			" " "
Line F							
11			0.5	14.6			
T.P.	3.8	17.9	1.0	14.1			
Line 11							
F+60			2.2	15.7			
Line F							
10+45			0.4	17.5			
10			3.2	14.7			
9			13.8	4.1			
8+60			17.9	0.0			Shore of Lake
7+16			17.9	0.0			" " "
7			16.8	1.1			
6			0.9	17.0			
5+80			11.4	6.5			
T.P.	10.2	27.9	0.2	17.7			
5			0.7	27.2			
T.P.	11.6	38.7	0.8	27.1			
T.P.	11.8	50.1	0.4	38.3			
T.P.	10.3	59.2	1.2	48.9			
4			8.4	50.8			
3			2.2	57.0			
2			6.2	53.0			
1			12.0	47.2			
T.P.	1.1	48.9	11.4	47.8			
0			9.8	39.1			
T.P.	2.0	41.3	9.6	39.3			

Station	Back Sight	Height of Instrument	Foresight	Elevation of Surface			Remarks
Line E		413					
0			9.7	316			
1			123	290			
2			8.3	330			
3			81	332			
T.P.	08	309	112	301			
4			80	229			
TP	12	212	109	200			
5			51	161			
6			12.0	9.2			
7			203	0.9			Shore Line 8 ft East of Line E E
7+46			212	00			Shore Line
8+10			212	00			
T.P.	3.1	17.9	64	14.8			
9			115	64			
10			3.7	14.2			
10+07			2.9	15.0			
10+44			2.9	15.0			
11			6.7	11.2			
Line D							
11			0.5	17.4			
10			10.9	7.0			
T.P.	11.7	268	2.8	15.1			
9			10.6	16.2			
8			9.3	17.5			
7			127	141			
6			8.6	18.2			
Line 5							
0+64			13.6	13.2			Creek
Line D							
5			4.6	222			
Line 4							
0+57			8.4	18.2			Creek
Line D							
4			18	250			



Station	Back Sight	Height of Instrument	Foresight	Elevation of Surface			Remarks
Line 3		26.8					
D+15			4.1	22.7			Creek
C+70			3.0	23.8			
Line D							
3			3.5	23.3			
2+40			2.6	24.2			Creek
Line 3							
D+50			0.3	26.5			
Line 2							
C+50			3.5	23.3			
Line D							
2			2.2	24.6			
1+72			1.8	25.0			
1			3.5	23.3			
0			2.7	24.1			
T.P.	11.9	37.8	0.9	25.9			
Line C							
0			5.7	32.1			
1			9.5	28.3			
2			10.3	27.5			
3			7.4	30.4			
4			0.7	37.1			
5			6.0	31.8			
6			11.1	26.7			
7			10.2	27.6			
8			10.7	27.1			
T.P.	0.4	26.3	11.9	25.9			
9			7.2	19.1			
10			13.2	13.1			
10+58			17.2	9.1			
11			12.0	14.3			
Line B							
11			12.5	13.8			
10			8.7	17.6			
9			1.3	25.0			

Station	Back Sight	Height of Instrument	Foresight	Elevation of Surface			Remarks
T.P.	11.4	36.9	0.8	25.5			
8			2.6	34.3			
T.P.	11.5	47.2	1.2	35.7			
7			6.7	40.5			
6			8.5	38.7			
5			4.6	42.6			
4			1.2	46.0			
3			2.6	44.6			
2			6.7	40.5			
1			8.8	38.4			
0			5.8	41.4			
Line A							
0			0.6	46.6			
T.P.	10.9	57.7	0.4	46.8			
1			9.5	48.2			
2			3.9	53.8			
3			0.0	57.7			
4			0.7	57.0			
5			4.5	53.2			
6			7.2	50.5			
7			10.6	47.1			
T.P.	0.7	47.2	11.2	46.5			
8			5.6	41.6			
T.P.	0.9	36.4	11.7	35.5			
9			2.6	33.8			
10			11.8	24.6			
T.P.	1.0	26.2	11.2	25.2			
11			8.2	18.0			
T.P.	0.5	15.3	11.4	14.8			
T.P.	1.6	9.3	7.6	7.7			
			9.3	0.0			Check–Surface of Water

LINE FF'
Contours 0 to 1
0 + 11 = contour 40
0 + 73 = contour 45
Contours 1 to 2
1 + 48 = contour 50
Contours 2 to 3
2 + 50 = contour 55
Contours 3 to 4
3 + 32 = contour 55
Contours 4 to 5
4 + 03 = contour 50
4 + 24 = contour 45
4 + 45 = contour 40
4 + 66 = contour 35
4 + 87 = contour 30
Contours 5 to 6
5 + 09 = contour 25
5 + 28 = contour 20
5 + 47 = contour 15
5 + 66 = contour 10
5 + 87 = contour 10
5 + 97 = contour 15
Contours 6 to 7
6 + 13 = contour 15
6 + 45 = contour 10
6 + 77 = contour 5
Contours 7 to 8
7 + 20 = contour 0 shore line
Contours 8 to 9
8 + 64 = contour 0 shore line
Contours 9 to 10
9 + 08 = contour 5
9 + 55 = contour 10
Contours 10 to 11
10 + 05 = contour 15
10 + 93 = contour 15

22. Drawing the Contour Map.—The map shown in Fig.* 1 of this plate should be platted from the preceding level notes. The outlines, or boundary lines, of the area are first drawn in the form of a rectangle 11 by 8 inches in dimensions, representing an area 1,100 feet in length and 800 feet in width, to a scale of 100 feet to the inch. These boundary lines are then each divided into parts 1 inch in length, each part representing 100 feet to scale, as shown in the plate; the lines extending along the sides are numbered, and those extending along the ends are lettered, in consecutive order, as shown. The lines of division are then drawn in pencil, thus dividing the area into squares 100 feet on each side. The elevations of the intersections of the lines, as given in the notes, should be written in their respective positions with pencil, and the contour points then calculated and tabulated. The positions of the contour points on each line are calculated and then tabulated in any convenient form, or are located graphically, as may be desired. A simple form for

tabulating the contour distances is shown on the preceding page. The distances to contours given in this form are those calculated for the line FF' of Fig.* 1. The calculations are made as follows: From the notes, the elevation of Station F-0 is 39.1 feet, and that of Station F-1 is 47.2 feet, giving a rise equal to $47.2 - 39.1 = 8.1$ feet from the former station to the latter. Since the horizontal distance between the stations is 100 feet, the rate of slope is equal to $\frac{100}{8.1}$, or 12.3 feet horizontal for 1 foot rise. The contour interval is taken at 5 feet, and, consequently, the elevation of each contour is some multiple of 5 feet. Hence, the first contour above Station *F-0* is contour 40, and to locate this contour a rise of $40.0 - 39.1 = .9$ foot above this station must be made. Since the rate of slope is 12.3 feet horizontal for 1 foot rise, the horizontal distance from Station 0 on this line to contour 40 is equal to $12.3 \times .9 = 11.1$ feet. The rise from contour 40 to contour 45 is 5 feet. As the rate of slope continues the same, contour 45 will intersect line F at a distance of $12.3 \times 5 = 61.5$ feet from contour 40, or $61.5 + 11.1 = 72.6$ feet from Station 0. In the notes, these distances are given to the nearest whole foot.

From an inspection of the elevations, it is evident that contour 50 must occur between Stations 1 and 2, since the elevation of the former is 47.2 feet and that of the latter is 53 feet. The rise from Station 1 to Station 2 is equal to $53.0 - 47.2 = 5.8$ feet. Since the horizontal distance giving this rise is 100 feet, the rate of slope is equal to $\frac{100}{5.8} = 17.2$ feet horizontal for 1 foot rise. To locate contour 50, a rise of $50.0 - 47.2 = 2.8$ feet must be made, and since the rate of slope is 17.2 feet horizontal for 1 foot rise, contour 50 will intersect line F at a distance of $17.2 \times 2.8 = 48.2$ feet from Station 1.

Contour 55 will evidently occur between Stations 2 and 3 on this line, and is located in the same manner as just described for contours 40, 45, and 50. This operation is continued until all the contour points on the line FF' have been located. The contour points on all the lines parallel to FF' are located in a similar manner. Since the contour

lines intersect the lines *1–1*, *2–2*, etc., which are perpendicular to the line FF', as well as the lines that are parallel to it, their points of intersection must be located on these lines also. The positions of the contour points on these lines are calculated in substantially the same manner as those on the line FF', just described.

As the positions of the contour points are determined along the different lines, they are marked lightly in pencil with the elevation of the contour. The points of equal elevation, that is, all points having the elevation of any contour, are then joined by a continuous line, drawn lightly in pencil freehand, and curved in such manner as to represent the form of the surface as accurately as possible. These form the contour lines, and should afterwards be inked, either with a ruling pen* or with an ordinary writing pen, leaving gaps or spaces at suitable intervals in which to write the contour elevations. Usually, the lines dividing the area into squares and the elevations of the stations are only marked lightly in pencil and are erased after the contour lines have been inked and their respective elevations have been written in the gaps left at suitable intervals for that purpose.

The plate shows the division lines dotted, and also shows the elevations of the stations on the lines E, F, and G. In drawing this plate, however, the student will not show any of the division lines, except the line FF', and will not show the elevations of any stations whatever. These lines and elevations are shown merely as a guide in laying out the map, and should not appear on the finished drawing, except in the case of the line FF', which is shown because the profile of the surface along this line is constructed on the same drawing. The finished drawing will show the elevations of all contours, but will not show the elevations of any stations.

23. Platting Profiles From the Contour Map.—It occasionally becomes necessary to construct the profile of a

*A special ruling pen is made for this kind of work; it is commonly known as a **curve pen** or **contour pen**.

vertical section along some line of a given tract from a contour map of the tract that has previously been made. The profile is constructed in the following manner:

The contour map is placed on a drawing board, and the line along which the profile is required is then platted on the map. This line is, for convenience, here called the *line of profile*. If the profile is to be constructed on a separate sheet of paper, this is placed on the map with its edge parallel to the line of profile, and is fastened to the drawing board by thumbtacks or held in place by heavy paper weights. On this sheet a series of lines is drawn parallel to the line of profile as platted on the contour map, and are spaced at equal distances apart. The common distance between the parallel lines thus drawn will correspond to the contour interval, and, beginning at the lower one, the lines are numbered consecutively in multiples of the contour interval corresponding to the elevations of the required contours; the lowest parallel line usually has the elevation of the lowest contour intersecting the line of profile on the contour map. At each point where a contour line intersects the line of profile on the contour map, a projection line is drawn perpendicular to the line of profile. The intersection of this projection line with the parallel line corresponding in elevation to the contour from which it is drawn, is a point on the profile of the surface along the desired line.

In constructing *Profile F*, Fig.* 2, the lines *0*, *5*, *10*, etc. are drawn parallel to the line FF' on the contour map, and, to a scale of 20 feet to the inch, are spaced 5 feet apart, a distance equal to the contour interval. The lowest point on the line FF' is at the shore of the lake, the surface of which has been assumed as datum. The lowest line of the profile is therefore marked zero, and the lines above it are marked successively upwards in multiples of 5, until the point of highest elevation on the line FF' is reached. The points of intersection of the different contours with the line FF' are then projected on the lines of corresponding elevation on the profile sheet; the points so determined are points in the profile of the surface along the line FF'.

Thus, on the contour map, contour 40 intersects the line FF' at the point a, and from this point the line $a\,a'$ is drawn at right angles to $F\,F'$, intersecting the line on the profile sheet representing an elevation of 40 at the point a'. The point a' thus located is a point in the surface line of the profile of the line $F\,F'$, having an elevation of 40 feet. In a similar manner the lines $b\,b'$, $c\,c'$, $d\,d'$, $e\,e'$, $f\,f'$, etc. are drawn from the points b, c, d, e, f, etc., thus locating the points b', c', d', e', f', etc., whose elevations are 45, 50, 55, 55, 50, etc. feet, respectively, in the surface line of the profile of the line FF'. Beyond the point f, the slope becomes steeper and the projection lines are necessarily drawn nearer together; on this portion of the map and profile, the letters designating the projection lines are omitted for clearness. The points thus located on the surface line of the profile are joined by a line drawn freehand, as described in *Leveling*. This line will represent the surface line of the profile, or the surface of the ground along the line FF', referred to the lake as a datum. In constructing the profile, the projection lines $a\,a'$, $b\,b'$, $c\,c'$, etc., should be drawn lightly in pencil, and after the surface line of the profile is drawn and inked, they should be erased. They are shown on the plate merely for the purpose of explanation.

The profile along the line $A\,A'$ is constructed in a similar manner, but is shown on the plate without the construction lines. The student will draw both profiles, finishing them in the same general manner as this profile. In each case the finished profile should show the horizontal lines representing the elevations, with the elevations written on them, the surface line, and the horizontal and vertical scales of the profile, but should not show any of the construction lines that are shown dotted in the plate.

The map and profile should be so arranged on the sheet as to leave a margin 1 inch wide inside of the border line, which should enclose a surface 13 by 17 inches.

PLATE, TITLE: STADIA SURVEY AND HACHURES

24. Preliminary Remarks and Field Notes.—Fig.* 1 of this plate shows the plat of a small portion of a village, made from the notes that are described in *Topographic Surveying*. These notes are reprinted for convenient reference on pages 24 and 27.

Before beginning to plat the survey, the student should make all the calculations for reducing the notes, and verify the results given in the text. He should calculate also the ranges of all the courses in the traverse, as well as the latitudes and longitudes of all the courses, computed from Station 1.

The stadia traverse line is then platted; this can be done easily by means of latitudes and longitudes. The side shots from each stadia station are next platted by means of the protractor and scale. A circular protractor, graduated from 0° to 360°, is more accurate and expeditious than the ordinary semicircular protractor for platting azimuth readings, and the protractor sheet is much more expeditious than either. The points where positions are located by the side shots are marked on the plat by a dot and circle, and the elevation of each point is written close to it in pencil. The highways, streams, boundary lines, buildings, etc. are then platted from the points located and the measurements given in the sketches. After all the artificial features are platted, the contour lines are sketched in from the elevations of the different points located on the map, assuming the slope to be uniform between any two known elevations.

25. Platting the Stadia Notes.—For the map shown in Fig.* 1, Station 1 is the starting point of the survey. Since the latitudes and longitudes of all other stations have positive signs, these stations must lie to the north and east of Station 1. A point is chosen on the sheet for the location of Station 1, in the same relative position as in the plate, and Stations 2, 3, 4, 5, and 6 are located by their latitudes and longitudes, as explained in *Compass Surveying*,

STADIA SURVEY

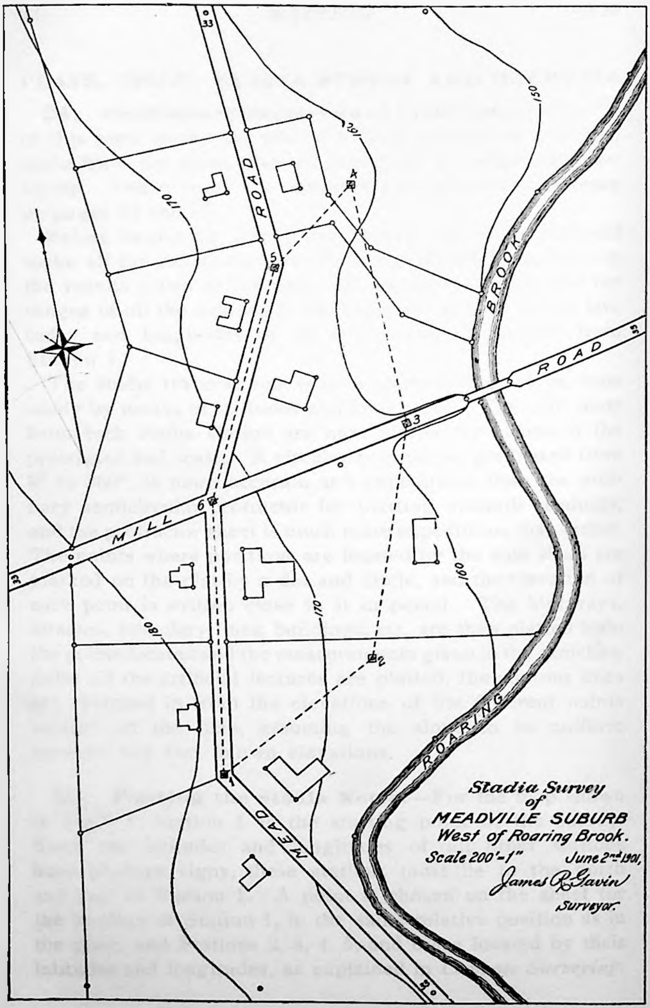


Fig. 1.

HACHURES

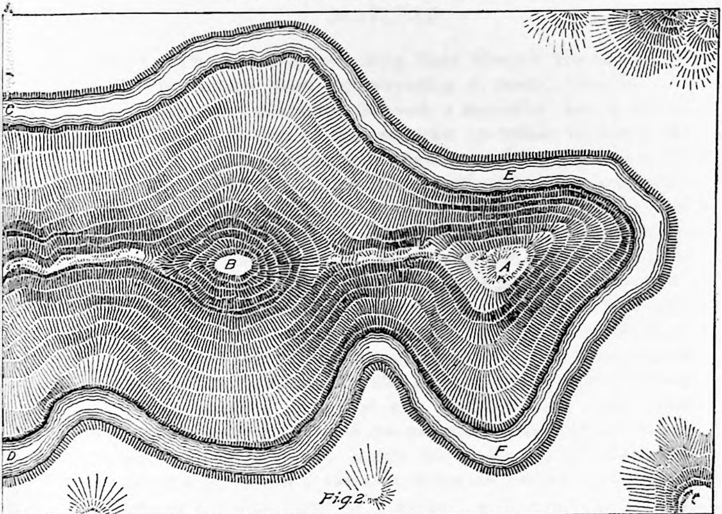


Fig. 2.

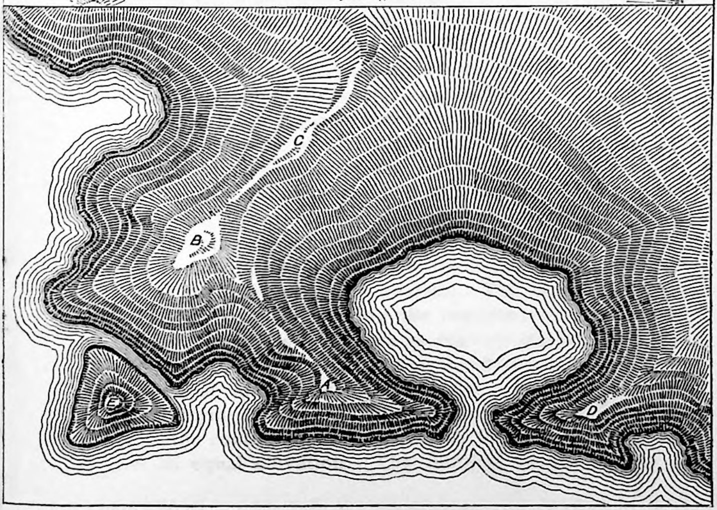


Fig. 3.

Part 2. The successive points thus located are then connected by dotted lines, representing a closed traverse.

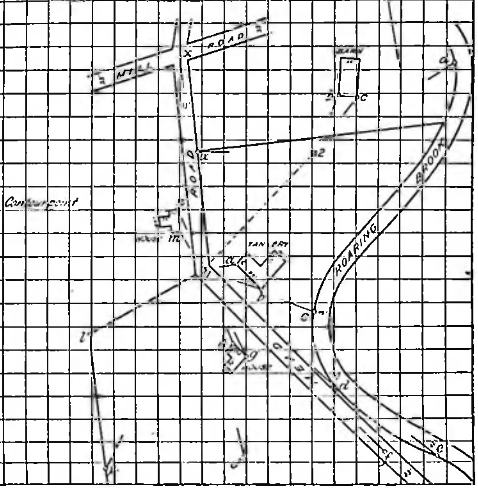
If a movable protractor is used, a meridian line is drawn through each station of the traverse, by which to orient the protractor. The protractor is then oriented at Station 1, by placing it in such a position that the 0° and 180° marks coincide with the meridian line, and the center of the protractor is directly over Station 1. The directions of the side shots from Station 1 are then laid off around the circumference of the protractor by making dots on the paper at points corresponding to the azimuths of the different pointings from this station. A pencil line is then drawn from Station 1 through each dot, and the stadia measurement for that pointing is scaled off from Station 1 on the line so drawn, thus locating the points marked *a*, *b*, *c*, *d*, etc. in the notes. The letter designating each point, and the corresponding elevations are marked lightly in pencil beside the point. The protractor is oriented at Station 2, the side shots are platted in the same manner as just described for Station 1, and each point platted is marked with the corresponding letter and elevation. The side shots from Stations 3, 4, 5, and 6 are platted in a similar manner, thus completing the platting of the side shots. The roads, streams, property lines, buildings, etc. are then platted from the measurements given in the sketches.

The next step is to plat the contour lines. In order to determine the positions of the contours, the slope of the surface is assumed to be uniform between the located points whose elevations are determined, and the positions of points on the contours between any two located points are determined by making the distances to the located points proportional to the differences of elevation. The calculations are the same as those for locating the contours on the contour map when the levels are taken at regular station intervals, as described in Art. 22, except that the distances between the points whose elevations are taken are varying instead of being uniformly 100 feet. The contour points having been located, the contour lines are sketched in pencil through the points of equal elevation. The platting of the survey is

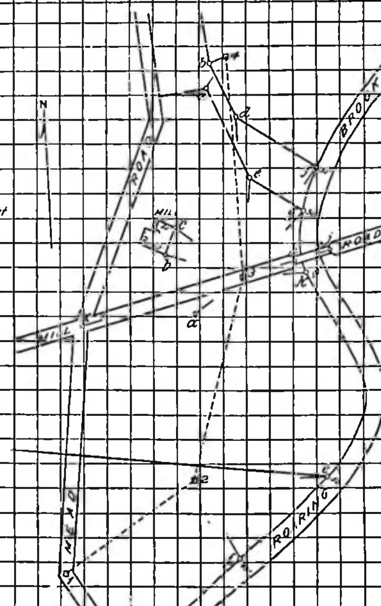
Stadia Survey of Meadville Suburb
May 27th 1901

Observer – J. R. Gavin
Recorder – Gonover
Rodmen – Griffith, Steele

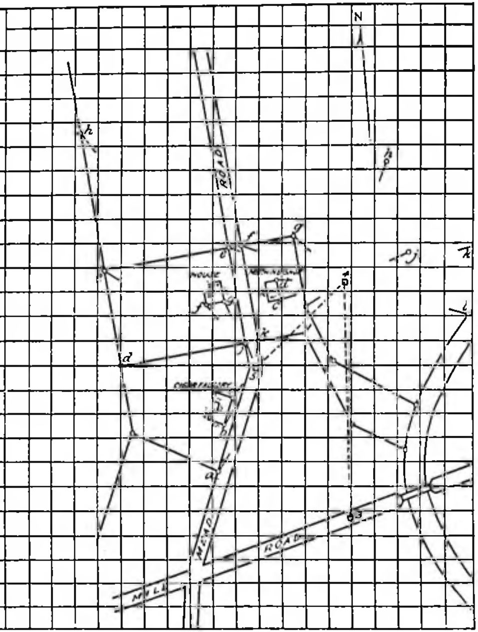
Sta.	Azimuth	Stadia	Vertical Angle	Horiz Distance	Elevation	
	Readings	from □1,	Elev. 177.42.			□1 Angle in Center Mead Road, South of Mill Road.
x	7° 30′	6.30	− 0° 23′	631	173.2	• Center Mill & Mead Roads as per Wilsons Survey
a	90° 43′	.91	− 2° 04′	92	174.1	
b	120° 18′	1.66	− 2° 12′	167	171.0	
c	126° 31′	3.15	− 2° 06′	316	165.8	
d	143° 43′	4.60	− 1° 25′	461	166.0	
e	141° 32′	7.85	− 0° 38′	786	168.7	
f	148° 04′	6.74	− 0° 34′	675	170.7	
g	167° 19′	2.47	− 0° 50′	248	173.8	
h	172° 22′	1.97	− 1° 20′	198	172.8	
j	181° 17′	4.99	+ 0° 12′	500	179.2	Contour point
k	221° 45′	5.79	+ 1° 02′	580	187.8	
l	256° 01′	3.47	+ 1° 50′	348	188.5	
m	342° 03′	1.17	+ 1° 16′	118	180.0	
n	350° 16′	1.71	+ 0° 52′	172	180.0	
□2	60° 00′	4.20	1° 33′	M. 422	165.77	
	Readings	from □2	Elev. 165.77			
□1		4.22	+ 1° 37′			
a	256° 01′	3.21	+ 2° 01′	322	177.1	
b	32° 02′	2.36	− 0° 48′	237	162.5	
c	41° 49′	2.60	− 1° 03′	261	161.0	
d	68° 32′	4.61	− 1° 22′	462	154.8	



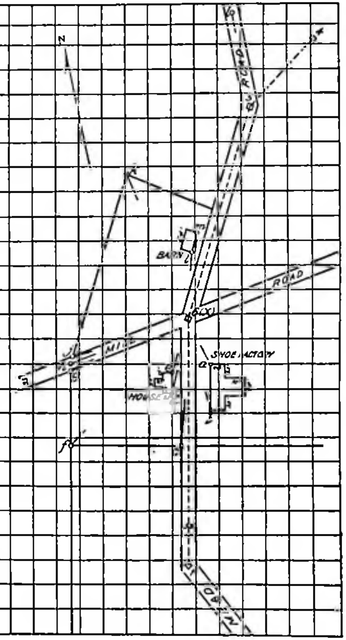
Sta	Azimuth	Stadia	Vertical Angle	Hor. Distance	Elevation	
	Readings	from ☐2	(cont.) Elev. =	165.77.		
e	90° 45′	3.59	− 1° 18′	360	157.6	
f	154° 53′	2.28	− 0° 54′	229	162.2	
☐3	17° 30′	5.47	+ 0° 17′	M = 545	168.61	
	Readings	from ☐3,	Elev. = 168.61			
☐2		5.42	− 0° 19′			
a	245° 30′	1.73	− 1° 53′	174	162.9	Contour Point
x	256° 45′	4.58	+ 0° 40′	459		Tie Line
b	287° 43′	2.19	− 1° 16′	220	163.7	
c	307° 24′	2.21	− 1° 41′	222	162.1	
d	359° 04′	4.06	− 1° 26′	407	158.4	
e	7° 33′	2.49	− 2° 50′	249	156.3	
f	38° 21′	3.39	− 3° 07′	339	150.1	
g	47° 20′	2.31	− 4° 18′	231	151.2	
h	76° 45′	2.39	− 0° 20′	240	167.2	
j	76° 46′	1.39	− 0° 34′	140	167.2	
k	82° 52′	1.66	− 5° 41′	165	152.9	
☐4	357° 35′	5.61	− 0° 46′	M = 559	161.29	
	Readings	from ☐4,	Elev. = 161.29			
☐3		5.55	+ 0° 44′			
a	215° 41′	0.96		97	161.3	
b	255° 10′	0.53	+ 1° 13′	54	162.5	



Sta.	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation
	Readings	from □4	(cont.) Elev. =	161.29	
c	275° 17′	1.09	+ 1° 19′	110	163.8
d	301° 11′	1.34	+ 0° 54′	135	163.4
e	303° 46′	2.84	+ 0° 47′	285	165.2
f	308° 36′	2.57	+ 0° 43′	258	164.5
g	339° 25′	1.79	+ 0° 08′	180	161.7
h	32° 19′	4.29	− 1° 24′	430	150.8
j	70° 03′	2.67	− 1° 38′	268	153.7
k	77° 32′	6.61	− 1° 21′	662	145.7
l	102° 12′	4.03	− 1° 54′	404	147.9
□5	230° 54′	2.53	+ 0° 44′	M = 252	164.51
	Readings	from □5,	Elev. = 164.51		
□4		2.49	− 0° 45′		
a	208° 02′	2.73	+ 0° 28′	274	166.7
b	218° 53′	1.63	+ 0° 38′	164	166.3
c	228° 02′	0.96	+ 0° 53′	97	166.0
d	277° 15′	3.68	+ 1° 59′	369	177.3
e	307° 47′	4.87	+ 1° 04′	488	173.6
f	326° 20′	2.18	+ 1° 02′	219	168.5
g	338° 43′	1.99	+ 0° 48′	200	167.3
h	327° 47′	7.71	− 0° 15′	772	161.1
j	343° 48′	0.71	+ 0° 19′	72	164.9
k	13° 31′	0.72	− 0° 34′	73	163.8



Sta.	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation	
	Readings from □5, (cont) Elev. = 164.51					
l	355° 45′	6.02	+ 0° 27′	603	169.2	
x = ⊡6					173.35	Should be 173.20
						as per Wilson's Survey
	Readings from ⊡6, Elev. = 173.20					= Original B.M.
□5		5.55	− 0° 59′			
a	159° 12′	1.27	− 0° 14′	128	172.7	
b	172° 20′	2.34	+ 0° 22′	235	174.7	
c	190° 14′	3.22	+ 0° 53′	323	178.2	
d	201° 46′	2.02	+ 1° 12′	203	177.5	
e	207° 24′	1.51	+ 1° 13′	152	176.4	
f	232° 18′	4.52	+ 1° 25′	453	184.4	
g	254° 03′	3.44	+ 1° 18′	345	181.0	
h	256° 45′	3.39	+ 0° 50′	340	178.1	
j	260° 02′	3.29	+ 1° 14′	330	180.3	
k	342° 47′	3.96	+ 0° 04′	397	173.7	
l	10° 14′	1.65	− 1° 07′	166	170.0	
m	13° 11′	2.18	− 1° 07′	219	168.9	
⊡1	187° 30′	6.27	+ 0° 25′			
	Survey finished May 28th 1901					
	Stadia Readings reduced by Gonaver – June 1st 1901					
	Notes platted by Griffith – June 1st 1901					



now completed in pencil. The lines defining the natural and artificial features of the surface surveyed, including the contour lines, are inked in, and the elevations of the contour lines are written in gaps left in the lines at suitable intervals for that purpose; all pencil lines are then erased. The plate is completed by lettering it in a neat and plain manner.

From this map, as shown on the plate, the title and the names of property owners are omitted for the sake of clearness, and in order that the more important matters relating to the platting of the survey can be shown plainly and be readily understood. After the map is completed, the student may show a title and the names of property owners, if he so desires, using fictitious names and being careful to place them in such positions on the map as not to interfere with any of the features platted from the notes of the survey.

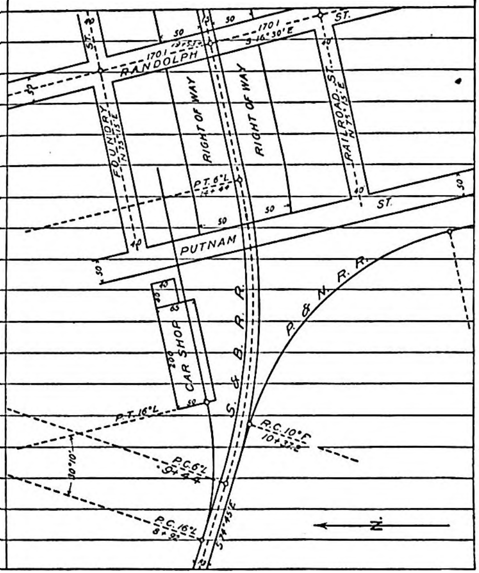
26. Representing Slopes by Hachures.—An example of this system is given in Fig.* 2. The figure represents an abrupt promontory whose base marks the channel of a river. The ground on the opposite side of the river is generally level with occasional undulations. The degree of the slope is indicated by the spacing of the contours and the corresponding lengths and numbers of hatchings. The more abrupt the slope, the closer together are the contours and hachures. The preliminary work necessary for such a topographical map is as follows: A traverse or meander line is run, defining the windings of the stream. The topographer sketches this meander line and stream in his notebook; he then sketches the main features of the surface from the promontory itself. A hand level is of great service in determining relative elevations. From these notes the final map is made up, the work being done in the office. Fine topographical drafting should not be attempted in camp. The facilities of a well-equipped office are necessary for rapid and satisfactory work. The student is not expected to reproduce the exact outline of the figure, but it is expected that his work will show a proper understanding of the subject. Having drawn the outline of the river, he

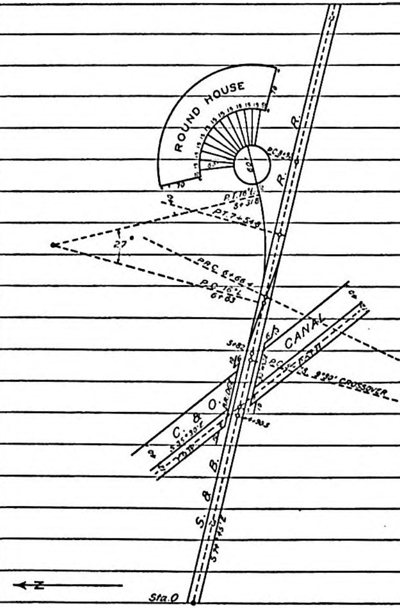
should draw the contours in light pencil lines, spacing them to conform to the different slopes. It will be evident that within the space represented by Fig.* 2, the surface of the river at C and D will be practically the same. Hence, if the distance from the summit A to the river at E is but half the distance from A to F, the slope $A\,E$ must be twice as abrupt as the slope $A\,F$, and the contours that mark equal heights will be twice as far apart on the slope $A\,F$ as on the slope $A\,E$. The student should draw all the contours, outlining the summits at A and B, before commencing the hachures. The short hachures on each side of the river mark its banks. On the promontory side they are shorter than on the opposite side, as the former has the more abrupt banks.

Fig.* 3 represents an irregular and abrupt sea coast. The survey for such an area would embrace a traverse of the entire shore line, including the shore line of the island, as well as that of the mainland. This traverse line should be used as a base line for auxiliary traverse or triangulation lines, by means of which the summits A, B, C, D, and E, and any other important objects could be located. The heights of these summits could be determined either by triangulation or by the aneroid barometer. With this information as a basis, the shore line is located, the contours are sketched with pencil, and the hachures drawn. As in the case of Fig.* 2, the student is not expected to produce an exact copy, but to show his proficiency by furnishing a clear and finished drawing.

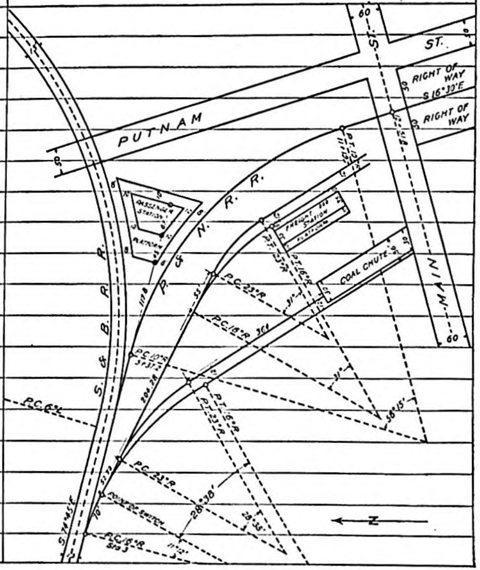
Figs.* 2 and 3 should each be $5\frac{3}{4}$ by $7\frac{3}{4}$ inches in dimensions, and be arranged side by side, as shown on the plate, thus occupying a rectangular space $11\frac{1}{2}$ by $7\frac{3}{4}$ inches on the drawing. The border lines should enclose a rectangular space 13 by 17 inches, as usual.

Sta.	Degree of Curve	Intersection Angle	Tangent	Radius	Magnetic Bearing
19+93.4	Center of Randolph Street				
14+44	P.T. 6°L				N 75°15′ E
10+37.2	P.C. 10°R	58°15′	319.64	573.7	
9+44	P.C. 6°L	30°	255.9	955.3	
8+92	16° turnout to left				

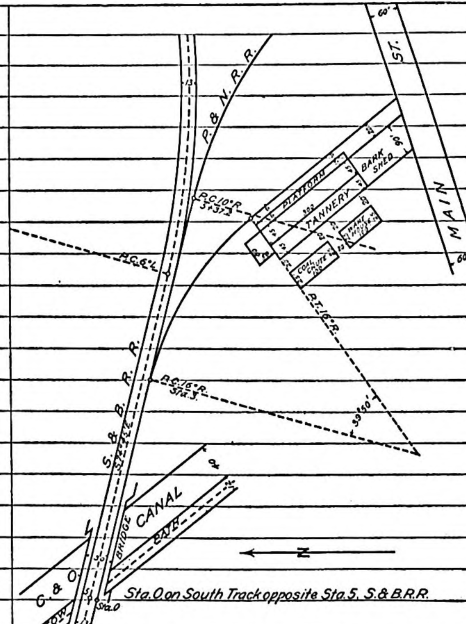


8+92	P.C. 16° turnout to Left					
7+54.8	P.T. 9°30'	crossover				
6+66.4	P.R.C.	crossover				
6+63	P.C. 16° turnout to Left					
5+82	East End of Bridge		(30 ft. wide)			
5+78	P.C. 9°30'	crossover				
5	Center of Towpath, C. & O. Canal					
4+90.5	West End of Bridge		(30 ft. wide)			
0	Center Line S. & B. R.R.				S 74°45' E	

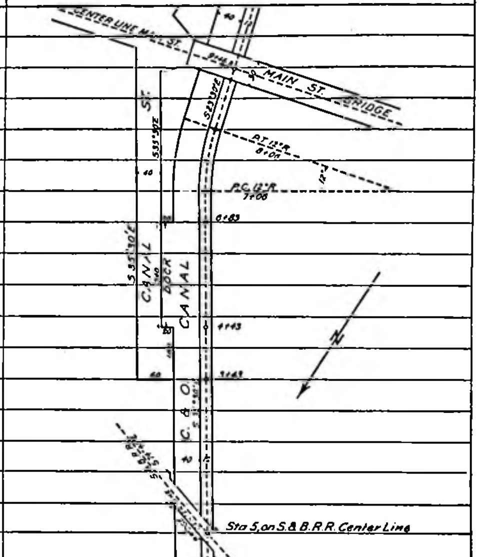
Sta.	Degree of Curve	Intersection Angle	Tangent	Radius	Magnetic Bearing
12+51.8	Center of Main Street				
11+19.7	P.T. 10°R				S 16°30'E
5+37.2	P.C. 10°R	58°15'	319.64	573.7	
3	P.C. 16° turnout to Right				



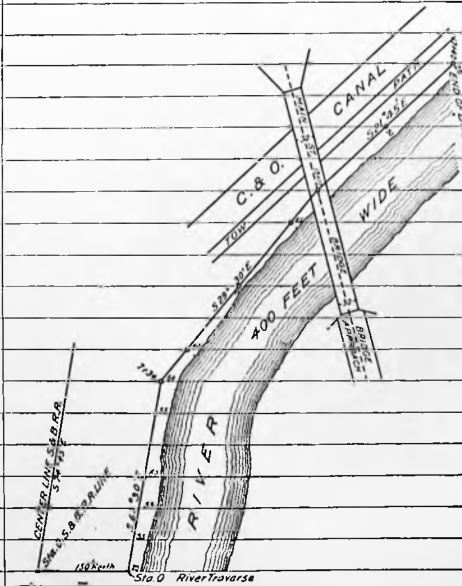
5+37.2	P.C.10°R	58°15'	319.64	573.7	
3	P.C.16° turnout to Right				
0					S 74°45'E



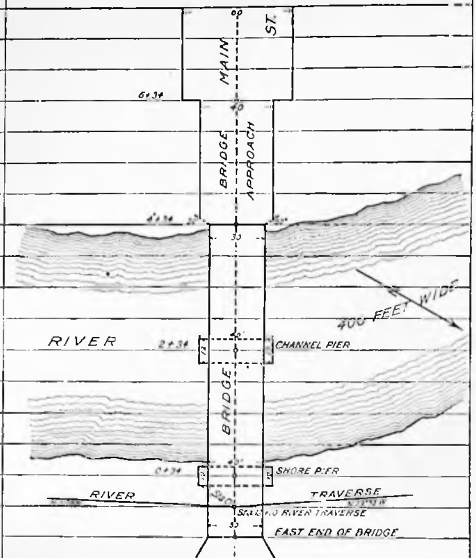
Sta.	Magnetic Bearing	Remarks
9+46.3		Center of Main Street
8+06	S 23°30'E	P.T. 12° Curve to Right
7+06		P.C. 12° Curve to Right for 12°
6+83		South End of Dock
4+43		North End of Dock
3+43		End of Canal Street
0	S 35°30'E	Sta. 5, on the S.& B. R.R. Center Line



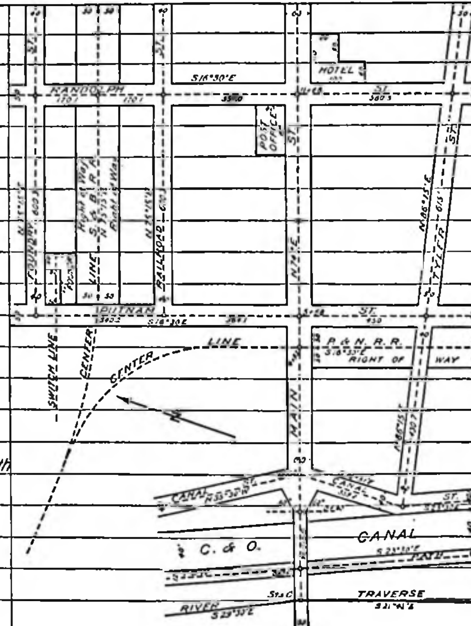
17+40		26' to Right, River Bank, End of Line
15		30' to Right, River Bank
13+10	S 21°45'E	Center of Main Street
12		25 to Right, River Bank
10		10' to Right, River Bank
8		40' to Right, River Bank
7+34	S 29°30'E	64' to Right, River Bank
7		55' to Right, River Bank
5		60' to Right, River Bank
3+50		55' to Right, River Bank
1+80		35' to Right, River Bank
0	S 67°30'E	25' to Right, River Bank



Sta.	Magnetic Bearing	Remarks
6+34		West End of Approach
4+34		West End of Bridge
2+34		Center of Channel Pier
0+34		Center of Shore Pier
0	S 74° W	Center of Main St. and Sta. 13+10 River Traverse



11+68		Center of Randolph Street	
5+68		Center of Putnam Street	
4+93		Center Line P. & N. R. R.	
1+70.7		Center of Canal St. leading North & South	
0+90		East End of River & Canal Bridge	
0+36.2		Center of Towpath C. & O. Canal	
0	N 74° E	Center of Main St. & Sta. 13+10 River	
		Traverse	



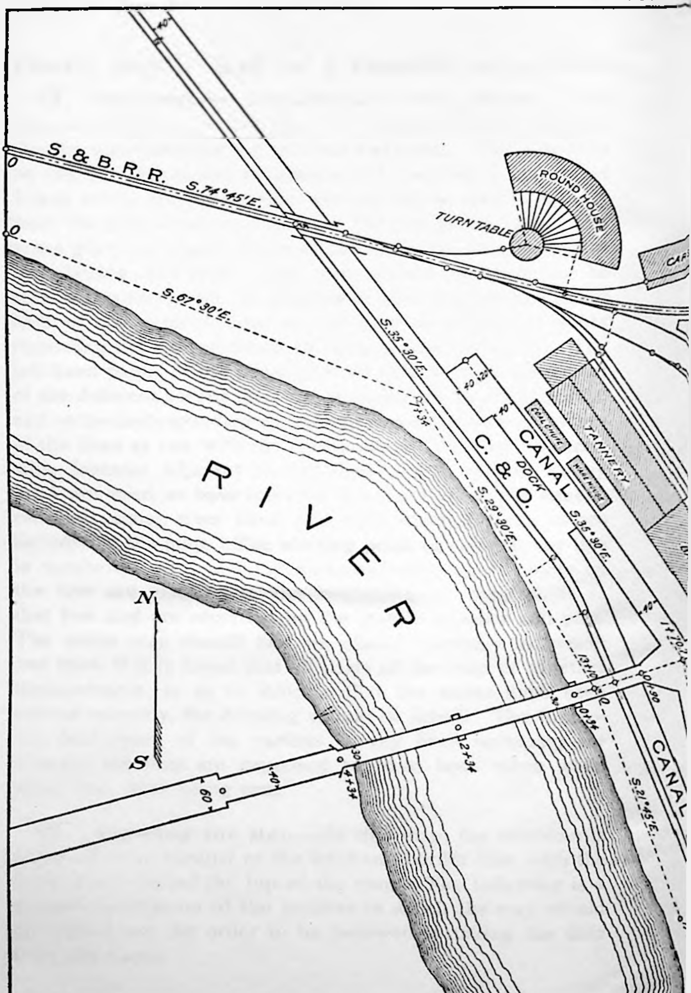
PLATE, TITLE: MAP OF A PORTION OF A TOWN

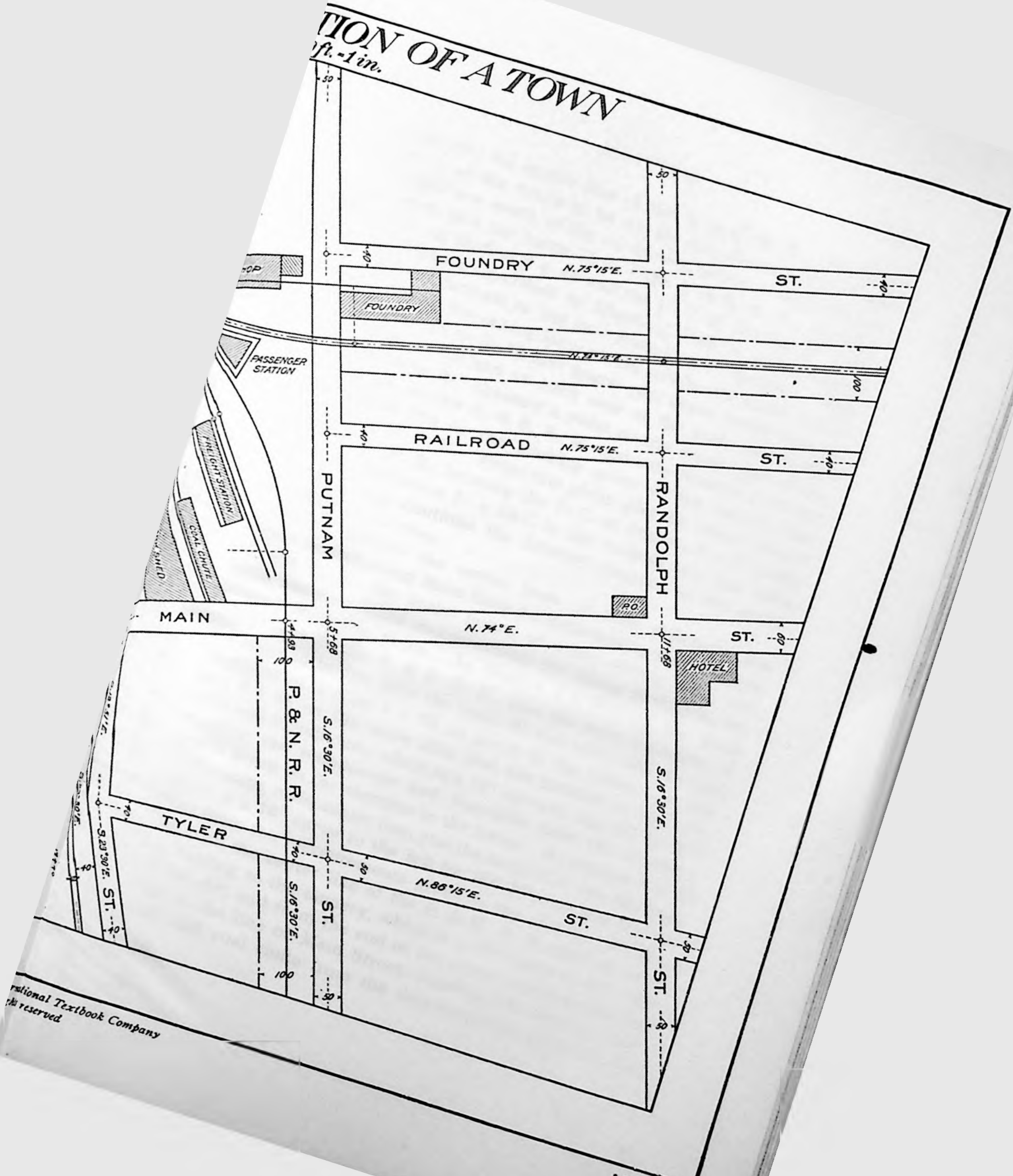
27. Preliminary Remarks and Field Notes.—This map represents a portion of a town, together with its facilities for transportation by railroad and canal. The map is to be $11\frac{1}{2}$ by $15\frac{1}{2}$ inches in dimensions, leaving a margin of $\frac{3}{4}$ inch inside the border line, and should be platted entirely from the field notes, to a scale of 150 feet to the inch. The notes given on pages 30 to 37 show all the data necessary for platting this map. The plate should therefore not be referred to except for the purpose of obtaining an intelligent idea of the general form of the map or in case of doubt regarding the interpretation of some part of the notes. The left-hand pages of the notes give all details of the alinement of the different survey lines run to locate the required points, and on the corresponding right-hand pages are shown sketches of the lines as run, with the measurements locating all accessible features adjacent to the lines. The several survey lines are used as base lines for the location of the streets, railroads, canal, river bank, and such other features as are included in the map. The starting point of each survey line is numbered zero, and the measurements to all points on the line are referred to the beginning or zero point of that line and are recorded by the station number and plus. The entire map should first be platted carefully in pencil, and then, if it is found that all parts of the map fit together harmoniously, so as to indicate that the notes have been platted correctly, the drawing should be inked. The foregoing field notes of the various survey lines necessary for drawing the map are supposed to have been taken while those lines were being run.

28. Drawing the Map.—In this map, the meridian is assumed to be parallel to the left-hand border line, with the north point toward the top of the map. The following is a general description of the manner in which the map should be platted and the order to be followed in taking the data from the notes:

MAP OF A POR

Scale




TION OF A TOWN
ft.-1in.
FOUNDRY
N.75°15'E.
ST.
FOUNDRY
PASSENGER STATION
RAILROAD
N.75°15'E.
ST.
FREIGHT STATION
COAL CHUTE
PUTNAM
RANDOLPH
MAIN
N.74°E.
ST.
P.O.
HOTEL
5+68
11+68
P. & N. R. R.
S.16°30'E.
S.16°30'E.
TYLER
N.86°15'E.
ST.
S.23°30'E.
ST.
ST.
ST.
100
100
50
40
60
30

First, plat the center line of the S. & B. R. R., assuming Station 0 of the notes to be on the west border line of the plate, 250 feet south of the northwest corner. Beginning at this point, plat the center line of the S. & B. R. R., locating the P. C. at Station 9 + 44 and the P. T. at Station 14 + 44, in the manner described in *Mapping*, Part 1, and continue the forward tangent to the east boundary of the map. The sketch shows the center lines of the north and south tracks of this railroad to be 13 feet apart; plat these center lines at a distance of 6.5 feet on each side of the main center line and parallel to it. Assume a point on the center line of the south track on the S. & B. R. R. opposite Station 5 on the main center line as the starting point of the survey line of the P. & N. R. R. From this point, plat the center line of the P. & N. R. R., locating the P. C. at Station 5 + 37.2 and the P. T. at Station 11 + 19.7, in the manner that has been described, and continue the forward tangent to the south boundary of the map.

Having platted the center lines of both railroads, the points located along these lines for determining the positions of objects, for the initial points of other survey lines, and from which to make measurements for filling in details, are platted from the notes. At Station 4 + 90.5 on the main center line of the S. & B. R. R., plat the west abutment of the railroad bridge over the canal 30 feet wide, and the east abutment at Station 5 + 82, as given in the notes. At Station 6 + 63 on the same line, plat the turnout to the turntable and roundhouse, which is a 16° curve to the left for 27°, locating the roundhouse and turntable from the measurements given on the sketches in the notes. At Station 8 + 92 of the same main center line, plat the turnout to the car shop, which is a 16° curve to the left for 30° 10′, locating the car shop from the measurements given in the sketch. At Station 3 on the center line of the P. & N. R. R. plat the turnout leading to the tannery, which is a 16° curve to the right for 39° 50′, and from the end of the curve continue the tangent to the line of Main Street, locating the tannery, bark shed, and coal chute from the measurements given in the

sketches. From a point on this turnout curve at the distance of 70 feet from the P. C., draw a line tangent to the curve, thus giving the direction of the straight portion of the track leading to the freight station, which makes an angle of 11° 12′ with the preceding tangent of the S. & B. R. R. From a point on this tangent at the distance of 52.72 feet from its initial point, plat as the turnout leading to the coal chute, a 23° curve to the right for 28° 38′, terminating in a tangent 364 feet long; this tangent should be parallel to the tangent of the tannery track and spaced 12 feet from it. The coal chute at the end of this tangent is platted from the measurements given in the sketches. At the distances of 257 feet and 312 feet, respectively, from the initial point of the track leading to the freight station, locate the P. C's. of two curves, the first a 16° curve to the right for 31°, and the second a 23° curve to the right for 31°, both curves terminating in parallel tangents spaced 12 feet apart. The freight station is next platted from the measurement given in the sketch.

From the notes, it is known that Station 5 on the main center line of the S. & B. R. R. is the starting point of the traverse line along the towpath of the C. & O. canal. Beginning at this point, plat the center line of the towpath, locating the P. C. of a 12° curve to the right for 12° at Station 7 + 6 and the P. T. at Station 8 + 6. Continue the forward and backward tangents to the south and north boundaries of the map, respectively. Plat the canal, canal dock, and towpath from the measurements given in the sketch. The west border line, being parallel to the meridian as established, bears directly north and south. On this line at the distance of 150 feet south of Station 0 of the S. & B. R. R. is the starting point of the traverse line along the river bank. Beginning at this point, plat this traverse line and locate both banks of the river from the offset measurements given in the notes.

Station 13 + 10 of this traverse line is in the center of the Main Street and is Station 0 of the Main Street traverse. Beginning at this point, plat the center line of Main Street and continue it to the east and west boundaries of the map.

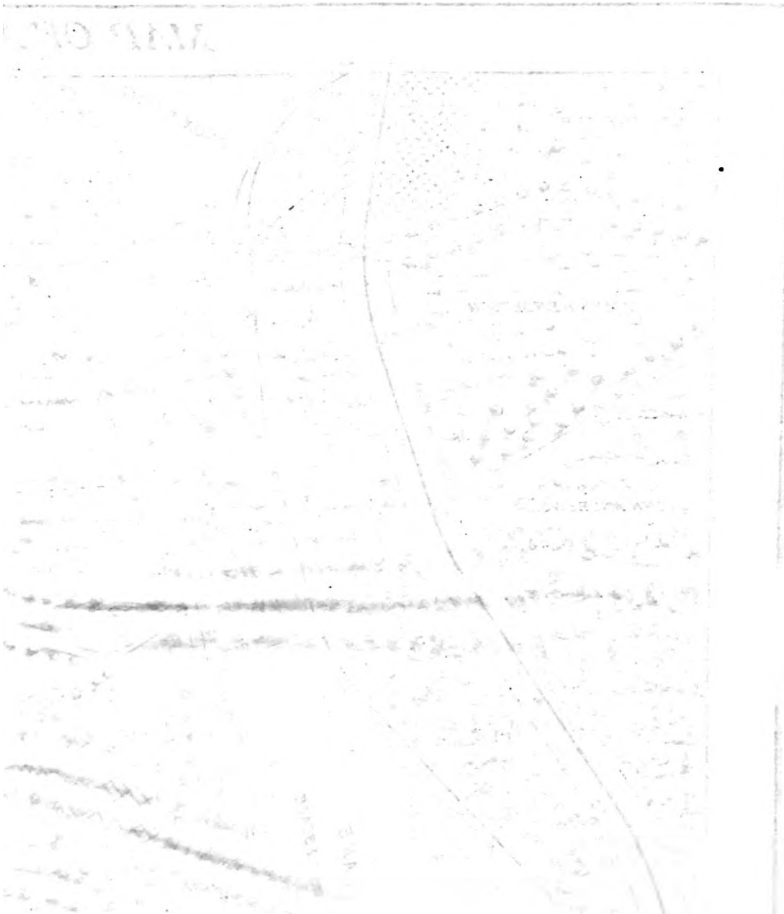
On this line, to the east of Station 0, locate the center of the canal towpath at Station 0 + 36.2 and the east end of the river and canal bridge at Station 0 + 90, the abutment wing walls of which diverge at angles of 30° with the center line of the street. Also, locate the center of Canal Street at Station 1 + 70.7, the center of the P. & N. R. R. at Station 4 + 93, the center of Putnam Street at Station 5 + 68, and the center of Randolph Street at Station 11 + 68. On this same line to the west of Station 0, locate the center of the shore pier of the bridge at Station 0 + 34, the center of the channel pier at 2 + 34, the west end of the bridge at 4 + 34, the abutment wing walls of which diverge at angles of 60° with the center line of the street or 30° with the face of the abutment, and the west end of the approach at Station 6 + 34. From the points thus fixed, locate the centers of Canal Street, Putnam Street, and Randolph Street, plat the center lines of these streets, and on them locate the center lines of Tyler Street, Railroad Street, and Foundry Street from the measurements and courses given in the sketches. The lines thus drawn will represent the center lines of the streets shown in the sketch.

The boundary lines of the streets, the bridge, and its approach, are next platted parallel to their respective center lines and of the widths indicated in the sketches, and the bridge piers are platted of the sizes and in the positions there shown. The post office and hotel are next platted at the northwest and southeast corners, respectively, of Main and Randolph Streets, and at the southeast corner of Foundry and Putnam Streets the foundry is platted, all dimensions being taken from the sketches. The right of way of the P. & N. R. R. is next platted; this extends 50 feet each side of the center line and from the south side of the Main Street to the south boundary of the map. The right of way of the S. & B. R. R. is then platted; this also extends 50 feet each side of the center line, and from the east side of Putnam Street to the east boundary of the map.

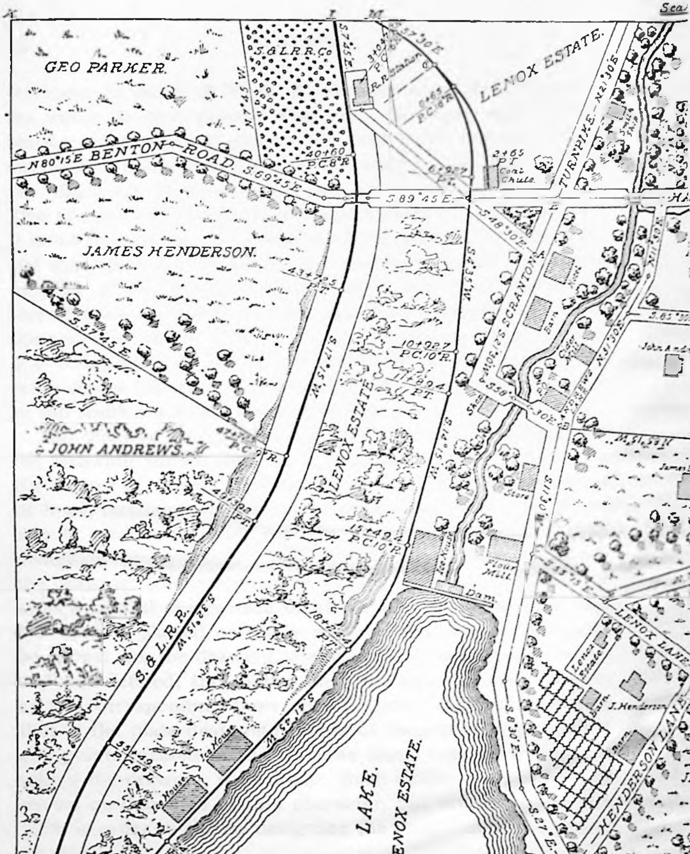
The passenger station is next located; its westerly end is at the distance of 162.8 feet from the P. C. of the 10° curve to

the right on the P. & N. R. R., measured on the tangent to the curve at that point. The east and west walls of the station are at right angles to the prolonged tangent of the curve, and the dimensions of the station and platforms in the direction of the tangent are as shown in the sketch. The positions of the north and south walls of the station are located by placing the corners of the station 10 feet from the corresponding edges of the platform, which edges are parallel to the adjacent tracks and 8 feet from the center line of each, as shown in the sketches. Returning now to the notes for the S. & B. R. R., the crossover from Station 5 + 78 is platted as a 9° 30′ curve to the right, reversing at the main center line to a 9° 30′ curve to the left, and terminating in the south track and tangent thereto.

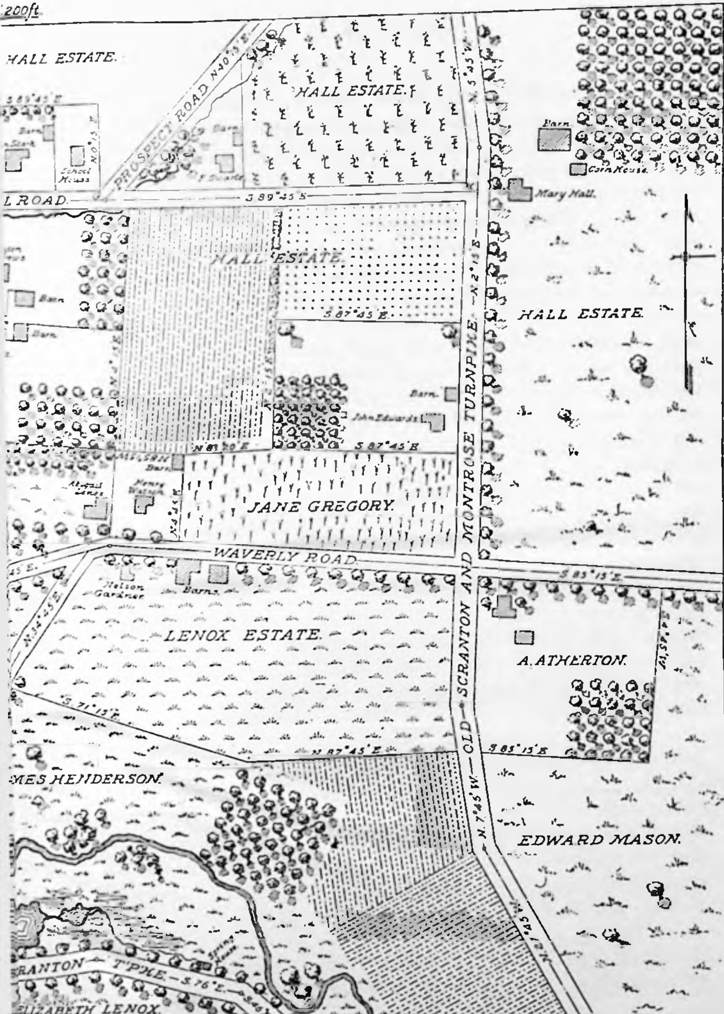
The platting of the map should now be complete, and the lettering should next be done neatly in pencil, including the names of streets, railroads, buildings, etc., the bearings of lines, numbers of stations at important points, widths of streets, and such other matters as it may be essential to designate by lettering. The student should then compare the drawing, as constructed in pencil, with the plate, and if, after a careful comparison of all details, he is satisfied that his work is platted correctly and the drawing is complete, he should ink the drawing, making the lines smooth, sharp, and well defined, and giving them the same relative weights as the corresponding lines on the plate. The small circles that in the plate designate points of tangency, angle points and points located in the traverse lines, may be omitted in inking the drawing, if desired. Such circles are not usually shown on drawings of this character, but are shown on the plate in order to clearly designate the positions of the points located.



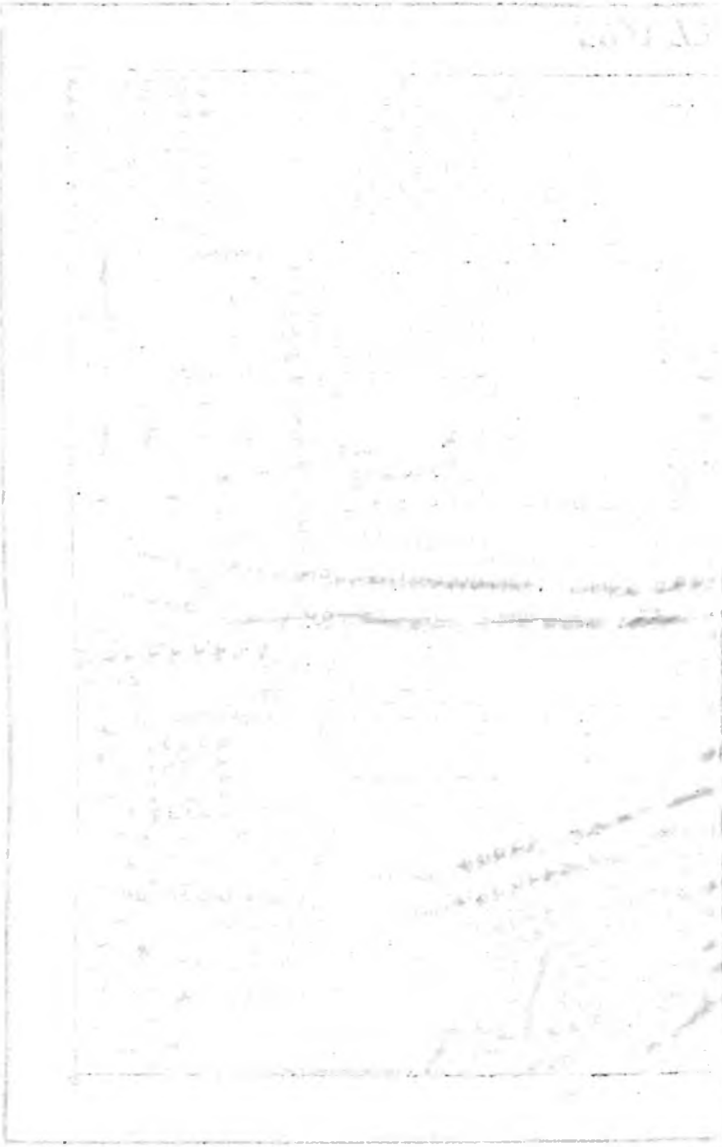
MAP OF


GEO PARKER.
S. & L. R. R. Co
LENOX ESTATE.
BENTON ROAD
N 80°15 E
S. 89°45 E.
JAMES HENDERSON.
JOHN ANDREWS.
TURNPIKE
SCRANTON
LENOX ESTATE
S. & L. R. R.
To Scranton.
STANTON LAKE.
LENOX ESTATE.
Ice House
Flour Mill
Dam
Store
Barn
Coal Chute
LENOX LANE
HENDERSON LANE
J. Henderson
NEWTON ROAD

VILLAGE



200ft
HALL ESTATE.
PROSPECT ROAD N40°5'E
HALL ESTATE.
School House
Barn
Mary Hall.
Corn House.
Barn
L ROAD.
S 89°45'E
HALL ESTATE.
S 87°45'E
HALL ESTATE.
Barn
John Edwards
S 87°45'E
JANE GREGORY.
WAVERLY ROAD
S 85°15'E
Barns
Nelson Gardner
LENOX ESTATE.
A. ATHERTON.
N 87°45'E
S 85°15'E
OLD—SCRANTON AND MONTROSE TURNPIKE.
MES HENDERSON.
EDWARD MASON.
SCRANTON T'PKE.
ELIZABETH LENOX.



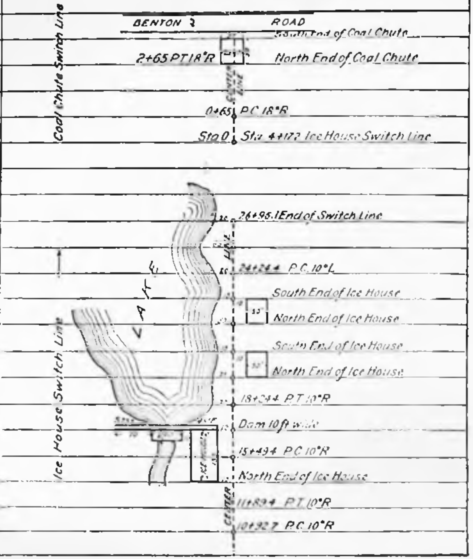
PLATE, TITLE: MAP OF A VILLAGE

29. Preliminary Remarks and Field Notes.—This plate represents a topographical map of a village. In making a survey of this description, some well-defined landmark is selected for a starting point. As there are usually a number of points to choose from, the choice will depend on the judgment of the surveyor. The intersection of the center lines of two highways or of a railroad and highway, or the head-block of a railroad switch, are excellent points from which to commence a survey. The center lines of highways and railroads are the base lines from which the minor details, such as houses and other buildings, are located. The quickest and best method of locating a building is to set a temporary plug on the survey line near the building, then set up the transit at this point and measure the angles between the survey line and two consecutive corners of the building, measuring the distances from the instrument point to the corners of the building. These angles and distances will locate one side of the building. A small freehand sketch is then made, giving the survey line, the station of the plug, or its distance from some known point, and the angles and distances to the side of the building. The remaining sides of the building are added to the sketch and their several lengths measured in consecutive order and marked on the sketch. Such notes are quickly made and as quickly platted, but are omitted here in order to avoid too great an amount of detail.

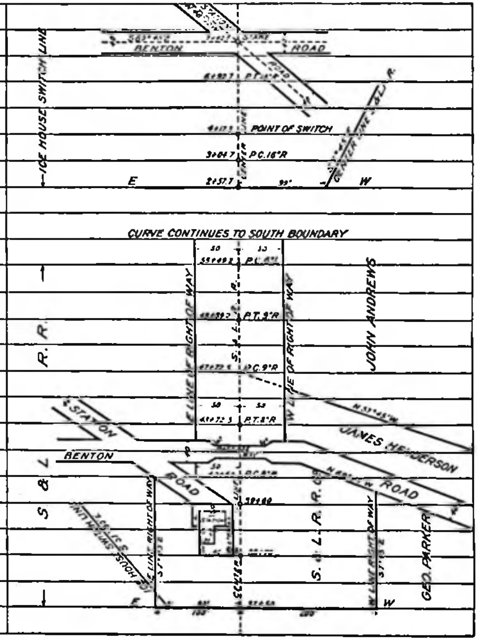
Sketches are of the greatest value in taking topographical notes. They can be made in much less time than is required for writing full descriptions, and are always more intelligible to the draftsman. Each surveyor has his individual methods, both in order of work and form of notes, and often one will consume much less time than another in performing the same work; but expedition is of no value if had at the cost of accuracy.

In drawing this map, the center lines of the railroads and highways are platted from the bearings and distances of the

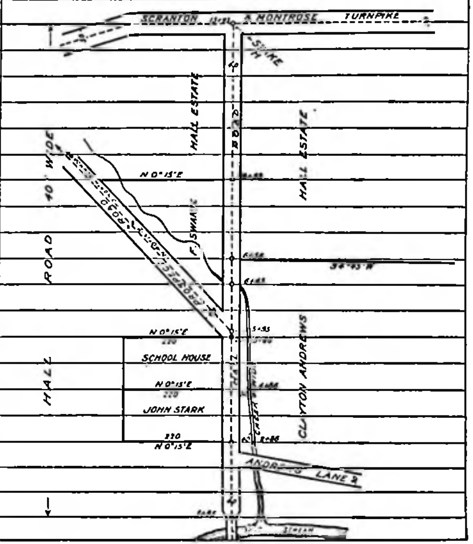
Sta.	Degree of Curve	Intersection Angle	Tangent	Radius	Magnetic Bearing
3+15					
2+65	P.T. 18°R				
0+65	P.C. 18°R	36°	103.8	319.6	
0	P.T.				S 39° 30'E
26+96.1					
25+14.4					
24+24.4	P.C. 10°L			573.7	
23+74					
22+74					
22+24					
21+24					
18+24.4	P.T. 10°R				S 41° 45'W
16+39.4					
15+49.4	P.C. 10°R	27° 30'	140.4	573.7	
15+04.4					
11+89.4	P.T. 10°R				S 14° 15'W
10+92.7	P.C. 10°R	9° 40'	46.5	573.7	



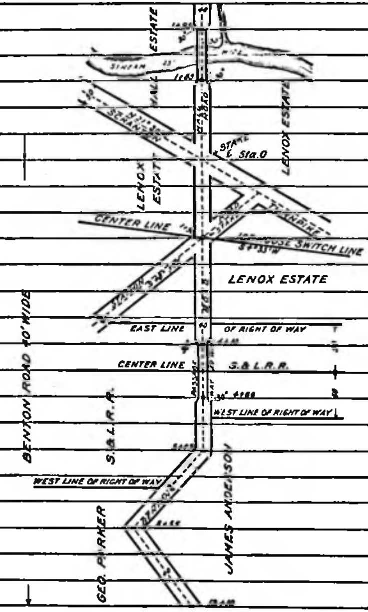
7+42.7	Center Benton Road				
6+92.7	P.T. 16°R				S 4°35'W
4+17.2	Point of Switch to Coal Chute				
3+04.7	P.C. 16°R	62° 05'	216.2	359.3	
2+57.7					S 57°30'E
55+49.2	P.C. 6°L			955.4	
49+39.2	P.T. 9°R				S 32°15'W
47+72.5	P.C. 9°R	15°	83.9	637.3	
43+72.5	P.T. 8°R				S 17°15'W
40+60	P.C. 8°R	25°	158.9	716.8	
39+60	South End of Platform				
38+70	North End of Platform				
37+50	Center Line S&L.R.R. beginning point				S 7°45'E



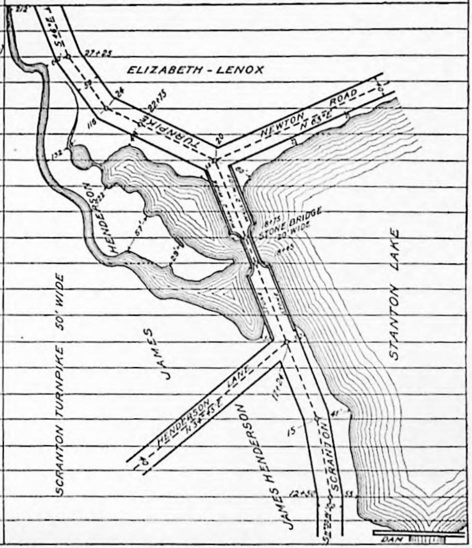
Sta.	Magnetic Bearing	Remarks
13+93	S 89° 45' E	Center of Hall Road & Scranton & Montrose Turnpike
8+93		Division between F. Swartz & Hall Estate
6+56		Division between Clayton Andrews & Hall Estate
6+45		Center of Creek Crossing
5+95		Center of Prospect Road (40' wide)
5+86		East Boundary School House Lot, Creek 25' R-of Center Line.
4+86		Division between School House Lot & John Stark, Creek 30' R-of Center Line.
2+86		West Boundary John Stark's Lot, Creek 40' R-of Center Line.



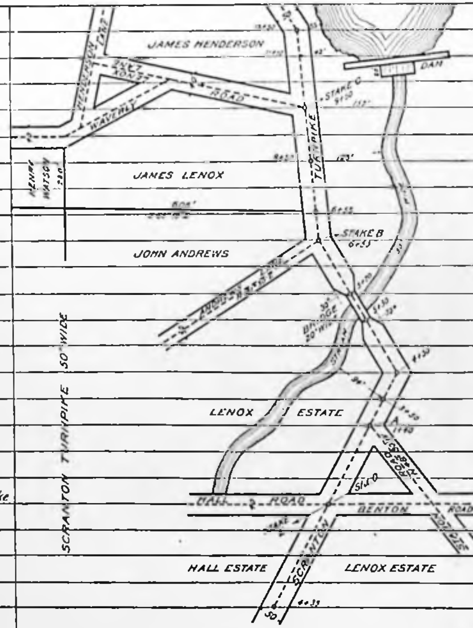
1+95		East End of Bridge
		Mouth of Creek 30' R-of Center Line
1+65		West End of Bridge
	S 89° 45' E	
0		Center of Hall Road, Benton Road & Scranton Turnpike
	N 89° 45' W	
1+90		Station 7+427 Ice House Switch Line (Stake)
4+10		East End of Passage Way
4+60		West End of Passage Way
5+05	N 69° 45' W	Angle in Benton Road
8+55	S 80° 15' W	Angle in Benton Road
12+10		End of Line



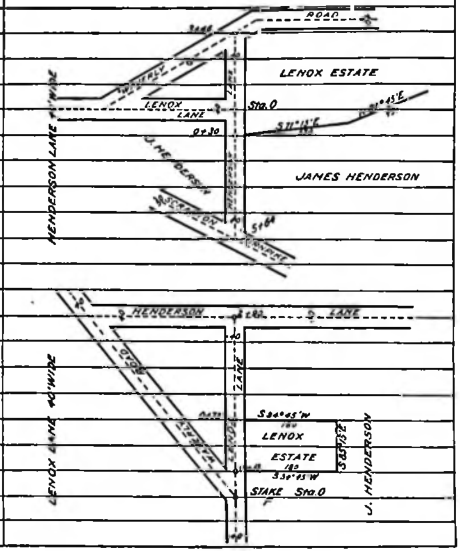
Sta.	Magnetic Bearing	Remarks
28+10	End of Line	Shore of Stream 212' left of Center Line
27+25	S 46° E	Angle in Scranton Turnpike, Shore of Stream (14' wide)
		60' left of Center Line
24	S 76° E	Angle in Scranton Turnpike, Shore of Stream (10' wide) 116' left of Center Line.
22+75		Shore of Lake 48' left of Center Line
20	N 83° E	Center of Newton Road, Angle in Scranton Turnpike
18+75		South End of Stone Bridge
18+45		North End of Stone Bridge
17+24		Center of Henderson Lane, Shore of Lake 50' left
		of Center Line and 32' Right of Center Line
15	S 27° E	Angle in Scranton Turnpike, Shore of Lake 41'
		Right of Center Line
	S 8°30' E	
	S 11°30' W	



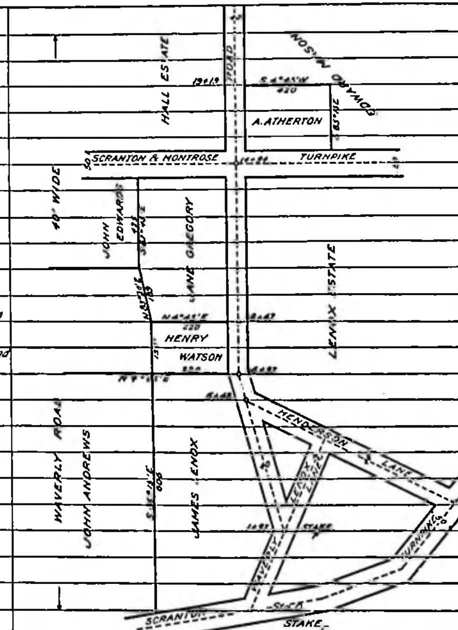
12+50	S 8° 30' E	Angle in Scranton Turnpike, Shore of Lake 55' R of Center Line
11+10		Shore of Lake 42' R of Center Line
9+50		Center of Waverly Road, Shore of Stream 152' R of Center Line
8+50		Shore of Stream 123' R of Center Line
6+55		Division Line between James Lenox & John Andrews
6+50	S 11° 30' W	Angle in Scranton Turnpike & Center Andrews Lane
5+70		East End of Bridge
5+30		West End of Bridge
4+50	S 58° 30' E	Angle in Scranton Turnpike
3+50		Shore of Stream 94' L of Center Line
1+40		Center of Station Road (40' wide)
	S 21° 30' W	
0		Center of Hall Road, Benton Road & Scranton Turnpike
	N 21° 30' E	
4+35		End of Line



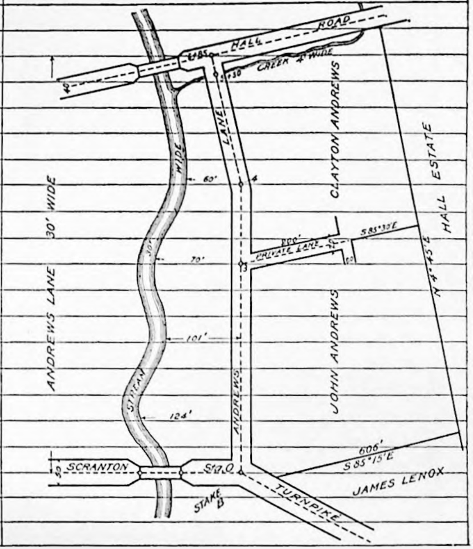
Sta.	Magnetic Bearing	Remarks
3+46		Center of Waverly Road
	↑ N 34° 45′ E	
0		Center of Henderson Lane & Lenox Lane
0+30	↓ S 34° 45′ W	Division Line between Lenox Estate & J. Henderson
5+64		Center of Scranton Turnpike
2+90		Center of Henderson Lane & Lenox Lane
0+73		Division Line between Lenox Estate & J. Henderson
0+13		Division Line between Lenox Estate & J. Henderson
0	S 55° 15′ E	Center of Waverly Road & Lenox Lane



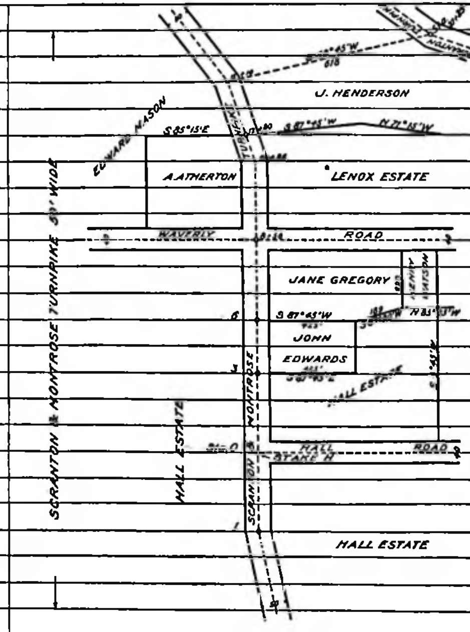
		East Boundary	
19 + 19		Division Line between A. Atherton & Edward Mason	
14 + 94		Center of Scranton & Montrose Turnpike	
8 + 47		Division Line between Jane Gregory & Henry Watson	
6 + 97	S 85° 15' E	Angle in Waverly Road Division Line between Henry Watson & James Lenox,	
6 + 48		Center of Henderson Lane	
1 + 97	N 74° 45' E	Angle in Waverly Road, Center of Lenox Lane	
0	S 55° 15' E	Center of Scranton Turnpike & Waverly Road	



Sta.	Magnetic Bearing	Remarks
5+85		Center of Hall Road
5+50		Center of Creek, 4' wide
4	N 11° 30'E	Angle in Andrews Lane Stream 60' Left of Center Line
3		Center of Private Lane, Division Line between John & Clayton Andrews, Stream 70' left of Center Line
2		Stream 101' left of Center Line
1		Stream 124' left of Center Line
0	N 31° 30'E	Center of Andrews Lane & Scranton Turnpike



19+48		South End of Line
15+10	S 27° 45' E	Angle in Scranton & Montrose Turnpike
		Division Line between A Atherton & Edward Mason
12+80		Division Line between Lenox Estate & J. Henderson
11+86	S 7° 45' E	Angle in Scranton & Montrose Turnpike
8+56		Center of Waverly Road
6		Division Line between Jane Gregory & John Edwards
3		Division Line between John Edwards & Hall Estate
	S 2° 15' W	
0		Center of Hall Road & Scranton & Montrose Turnpike
	N 2° 15' E	
1	N 5° 45' W	Angle in Scranton & Montrose Turnpike
4+10		North End of Line



traverse lines, as recorded in the notes. The property lines are located at the points where they intersect the center lines of the highways. The directions of these lines are platted from the bearings given in the notes, and their lengths are platted from the measurements shown in the sketches, the measurements being taken from the center lines of the highways. In platting the various courses of the highway traverse lines, it would be well first to plat each course by the method of tangents, as described in *Mapping*, Part 1, and then check it by the protractor. The right of way of the railroad and the boundary lines of the highways are then platted to their respective widths, as indicated in the sketches, thus completing the framework of the map.

The student will then sketch in the topography, such as the water courses, buildings, etc., from observation, giving them the same relative forms and positions as are shown in the plate. The lettering should then be done in a neat and uniform manner. The conventional signs are the last to be put on the map. The student should bear in mind that the worth of a drawing depends nearly as much on its neatness and uniformity as on its accuracy, and that, therefore, nearly as much care should be exercised in the sketching and lettering as in the platting.

The field notes of the survey made for the purpose of obtaining the data necessary for drawing this map are given on pages 44 to 53.

30. Drawing the Map.—From the preceding notes of the survey, draw the map to a scale of 200 feet to the inch, making the dimensions $11\frac{1}{2}$ by $15\frac{1}{2}$ inches for the map itself and 13 by 17 inches within the border line. The magnetic meridian is assumed to be parallel to the right and left border lines of the plate, with the north point toward the top of the map. The following is a description of the general order in which the map should be drawn:

From the northwest corner of the map, marked *K* on the plate, scale off 700 feet to the scale of the map along the

north boundary line, locating the point L at the extremity of the measurement. Assume the point thus located to be Station 37 + 50 on the center line of the S. & L. R. R., and the starting point of the platting. From this point, draw a line having a magnetic bearing S 7° 45′ E; the line thus drawn represents the direction of the center line of the S. & L. R. R. at this point. The entire center line of this railroad should now be platted from the notes in the manner described in *Mapping*, Part 1. The notes show that the point M on the center line of the ice-house switch line is at a distance of 90 feet due east of Station 37 + 50 of the S. & L. R. R. Locate this point and assume it as Station 2 + 57.7 on the center line of the ice-house switch line. From this point draw a line whose magnetic bearing is S 57° 30′ E; the line so drawn represents the direction of the center line of the ice-house switch line at this point. This entire line is now platted from the notes in the manner that has been described.

At Station 4 + 17.2 of the ice-house switch line is the point of switch to the coal chute. This point can be located in the curve by means of the central angle of that portion of the curve preceding the point. From the rule for the length of curve given in *Circular Curves*, the central angle of a curve, which is equal to the angle of intersection, is known to be equal to the length of the curve, expressed in stations of 100 feet, multiplied by the degree of curve. Since the P. C. of the 16° curve is at Station 3 + 04.7, and the point of switch is at Station 4 + 17.2, the length of that portion of the curve preceding the point of the switch to the coal chute is equal to $4.172 - 3.047 = 1.125$ stations, which, when multiplied by the degree of curve, gives a value of $1.125 \times 16 = 18°$ for the corresponding central angle. This central angle is laid off at the center of the curve, to the right of the radial line drawn to the P. C., thus giving the direction of the radial line to the switch point; the intersection of this radial line with the curve is the zero point of the switch line to the coal chute. At this point, draw a line tangent to the curve, that is, at right angles to the radius.

This line has a bearing S 39° 30′ E and is the direction of the short tangent in the first part of the switch line leading to the coal chute. The remainder of this line is platted from the notes in the usual manner.

A sketch in the notebook shows that at Station 7 + 42.7 of the ice-house switch line a stake was driven in the center of Benton Road. By referring to the notes of the Benton Road traverse, this stake is seen to be marked as Station 1 + 90 of that line, and the magnetic bearing of Benton Road at this point is N 89° 45′ W. Therefore, draw a line through this point having a bearing of N 89° 45′ W, for the center line of the Benton Road. On this line, scale off 190 feet toward the east; the extremity of this measurement locates a point that is in the center line of the Hall Road, the Benton Road, and the Scranton Turnpike, and is also Station 0 of the Benton Road traverse. Mark this point *E*. Beginning at the point *E*, plat the center line of Benton Road westwards, locating the angle points at Stations 5 + 05 and 8 + 55. The last course is continued to the west boundary of the map.

Beginning at the same point *E*, plat the center line of the Hall Road traverse eastwards, locating the center of Prospect Road at Station 5 + 95 and the center of the Scranton and Montrose Turnpike at Station 13 + 93, and also locating the property lines at Stations 2 + 86, 4 + 86, 5 + 86, 6 + 56, and 8 + 93. At Station 5 + 95, plat the center line of the Prospect Road by its magnetic bearing, as given in the sketch, and continue this line to the north boundary of the map. Mark Station 13 + 93 with the letter *H*. Returning to the same point *E*, which is also taken as Station 0 of the Scranton Turnpike traverse, plat the center line of this turnpike from the notes, locating the center and terminus of the station road at Station 1 + 40, the angle point at Station 4 + 50, the second angle point and center of Andrews Lane at Station 6 + 50, the center of Waverly Road at Station 9 + 50, the angle points at Stations 12 + 50 and 15, the center of Henderson Lane at Station 17 + 24, the angle and center of Newton Road at Station 20, the angle points at Stations 24 and 27 + 25, and continue the last course to the south

boundary of the map. Mark the Stations at 1 + 40, 6 + 50, and 9 + 50 by the letters *A*, *B*, and *C*, respectively. At Station 20 of this traverse, plat the center line of Newton Road from the magnetic bearing given in the sketch, and continue this line to the south boundary of the map.

The point *C*, in the Scranton Turnpike traverse, is Station 0 of the Waverly Road traverse. Beginning at this point, plat the center line of Waverly Road, locating the angle and the center of Lenox Lane at Station 1 + 97, the center of Henderson Lane at Station 6 + 48, the angle at Station 6 + 97, the center of the Scranton and Montrose Turnpike at Station 14 + 94, and continue the center line of Waverly Road to the east boundary of the map. Mark the angle at Station 1 + 97 by the letter *F*; this point is Station 0 of the Lenox Lane traverse. Beginning at the point *F*, plat the center line of Lenox Lane, locating the terminus of Lenox Lane, and the center of Henderson Lane at Station 2 + 90. The last point is Station 0 of the Henderson Lane traverse. From this point, plat the center line of Henderson Lane leading north into Waverly Road and also leading south into the Scranton Turnpike.

The point *H*, located on the Hall Road traverse, is Station 0 of the Scranton and Montrose Turnpike traverse. Beginning at this point, plat, from the notes, the center line of this turnpike northwards, locating the angle at Station 1, and then continue the center line to the north boundary of the map. From the same point *H*, plat the center line of the turnpike southwards, intersecting the center of Waverly Road at Station 8 + 56, and locating the angles at Stations 11 + 86 and 15 + 10; continue the last course to the south boundary of the map. The point *B*, located on the Scranton Turnpike, is Station 0 of the Andrews Lane traverse. Beginning at this point, plat, from the notes, the center line of Andrews Lane, locating the center of a private lane leading to the east, at Station 3, an angle at Station 4, and the terminus of the lane at Station 5 + 85, which is in the center of Hall Road.

Now return to the notes for the S. & L. R. R. and plat the station and platform as located from the center line. Then

plat the right-of-way line on each side of the center line at the distances indicated in the sketch. From the bearing given in the sketch, plat the property line between James Henderson and John Andrews extending west from the railroad right-of-way line.. From the notes for the ice-house switch line, plat the ice house and dam on the east side of the line, and plat the two ice houses on the west side of the line from the measurements given in the sketches. Then plat the shore line of the lake on the east side of the center line from the offset measurements given in the sketch. The coal chute is next platted at the terminus of the switch line to the coal chute from the measurements given in the sketch.

Beginning at the point *E* of the Benton Road traverse running westwards, plat the retaining walls and abutments of a passageway 20 feet wide under the railroad, locating the east end at Station 4 + 10 and the west end at Station 4 + 60, as given in the notes, platting the wing walls at an angle of 30° with the direction of the passageway. Beginning at the same point *E*, plat the bridge between Stations 1 + 65 and 1 + 95 of the Hall Road traverse, as shown in the notes, platting the wing walls at an angle of 30° with the direction of the bridge. The property lines should next be platted, giving them the directions as indicated by the magnetic bearings in the sketches, and scaling the depths of lots where these are given. Beginning at the same point *E*, plat the bridge extending from Station 5 + 30 to Station 5 + 70 of the Scranton Turnpike traverse, as located in the notes, platting the wing walls of the bridge at an angle of 30° with the direction of the bridge. Also, plat the property line at Station 6 + 55, giving it the direction and depth as indicated in the sketch. The bridge extending from Station 18 + 45 to Station 18 + 75 should next be platted as shown in the sketch.

Beginning at the point *C*, which is the starting point of the Waverly Road traverse, the property lines at Stations 6 + 97, 8 + 47, and 19 + 19 of this traverse should be platted, giving them their respective depths and directions as indicated in the sketch. From the point *F*, which is the

beginning of the Lenox Lane traverse, the property lines at Station 0 + 13 and 0 + 73 of this traverse should be platted, giving them their respective depths and directions as indicated in the sketch. At Station 0 + 30 of the Henderson Lane traverse running south, the property line between the Lenox estate and James Henderson's land should be platted to the angle in the line, and its continuation then platted of the length and in the direction indicated in the notes. This line should terminate in the center of the Scranton and Montrose Turnpike. The point *B* of the Scranton Turnpike traverse is Station 0 of the Andrews Lane traverse. At Station 3 of the latter traverse, the property line and the private lane 10 feet wide each side of this line should be platted of the depth and in the direction indicated in the sketch. As has been stated, the point *H* is Station 0 of the traverse of the Scranton and Montrose Turnpike running southwards. At Stations 3, 6, and 12 + 80 of this traverse, the property lines should be platted of the depths and in the directions indicated in the sketch. When all the property lines have been platted according to the measurements in the notes and sketches, the boundaries of the various roads should be drawn, making their respective widths as indicated in the sketches and terminating them as there shown.

The framework of the map is now finished, having been platted entirely from field notes, and the map is to be completed by sketching in the details, such as water courses, shore lines, and buildings from the plate, putting in the names of property owners, bearings, etc., and the title, and making the conventional signs. After the traverse lines and property lines have been platted, the buildings should be drawn. They should be drawn in the same relative positions and of substantially the same form and dimensions as on the plate. The positions of important buildings are usually located by measurements made from convenient points in the surveys, and the buildings are platted on the map from the measurements, as recorded in the notes. For this map, however, the measurements locating buildings have in most cases been omitted, in order that the notes will not be too voluminous

and that the more important portions of the platting will not be obscured by too many details. After the buildings are drawn, the names of the various property owners should be written neatly in their proper places. The names and bearings of the highways, the alinement of the railroad, the bearings of property lines, and other necessary or important details should also be written. A neat, plain system of lettering should be used. In a really complete map that is drawn to a sufficiently large scale, the lengths of all courses along highways and property lines, as well as their bearings, should be shown on the map. But in the present case the distances are omitted for brevity.

All this work should be done lightly in pencil, and all details completed. Then, if, after a careful inspection, it is considered to be correctly drawn, the lines should be inked in. The letters should then be carefully gone over in ink, and finally the conventional signs should be made and the entire map finished substantially as shown on the plate.

MATTERS RELATING TO MAPPING

31. Tinting and Coloring.—All conventional signs thus far described are made with a pen. Where surveys cover extensive areas, it sometimes happens that the labor and time for pen work cannot be spared, and colors applied with a brush are used instead. With a skilful hand, work of this character may be rapid and very effective. However, owing to the fact that colors cannot be reproduced by blueprinting or any similar cheap and expeditious process for reproducing drawings, the tinting and coloring of maps and drawings is not now practiced extensively. For this reason, the subject is not treated at length here, and no examples are given. But the process is described sufficiently in detail, it is believed, to give the student a clear understanding of the subject, and enable him to do work of this character if he so desires.

Only a few colors besides India ink are required; the most essential colors are gamboge (yellow), indigo (deep blue),

new blue (light blue), burnt sienna (brown), and scarlet lake. By mixing these colors, almost any color, shade, or tint desired can be obtained. Any shade of green can be produced by mixing blue and yellow colors, such as indigo and gamboge. A pleasing brown can be obtained by mixing scarlet lake, gamboge, and a very small amount of India ink, and a very dark brown by increasing the amount of India ink. Purple can be made by mixing scarlet lake and indigo. Orange tints can be produced by adding a small amount of scarlet lake to gamboge; and so on in almost endless variety.

The colors used in the drafting room are of two kinds, namely, dry colors and moist colors. **Dry colors** are sold in the form of rectangular cakes, wrapped in tin-foil. **Moist colors** are packed in small dishes of porcelain, rectangular in form and open at the top. The surface of the color is covered with wax and the entire dish wrapped in tin-foil.

In using moist colors, the cake of color is rubbed lightly with a moistened brush, which will take up sufficient color, and the color is then diluted in water to the proper tint, which should always be light and delicate. A different dish, preferably shallow and saucer-shaped, should be provided for each color used. For this purpose what is known as a **nest of color saucers** is very convenient. This consists of a number, usually half a dozen, of saucers of the same size that fit on top of each other. A dish for water should also be provided; an ordinary glass tumbler, or a teacup, serves well for this purpose.

To cover a surface with a uniform tint, a camel's hair or sable brush should be used, and a separate brush used for each tint, or the brush should be washed thoroughly in clear water when changing from one tint to another. Confusion in the use of brushes is sure to spoil a tint. For large masses of the same tint, a large brush should be used, but for small details, small brushes are indispensable. Heavy and labored strokes should be avoided. Light and rapid strokes produce smooth and pleasing effects. The map should be pinned to a light drawing board, so that it can readily be inclined at an angle. Keep the brush well filled with color and begin

at the top of the surface, inclining the board toward you. If the outline is very irregular, moisten the edge with water. Apply the tint the full length of the surface and continue it down the surface, never allowing the edge to dry.

In representing topography by colors, woods are commonly colored yellow; grass land, green; cultivated land, brown; brushwood, marbled green and yellow; vineyards, purple; lakes and rivers, light blue with a darker tint at the shore line; seas, dark blue with a little yellow added; marshes, the water blue, with patches of green applied horizontally; and roads, dark brown. Woods may be made very effective by drawing the trees, coloring the angle toward the light (the upper left hand) with a touch of yellow, and the opposite, or lower right-hand, side with indigo.

Skill and judgment in mixing and applying colors can be acquired only by practice. When a combination tint, such as brown, is required, the draftsman must estimate how much coloring is required and provide accordingly. He is likely to use too much color, thus producing a heavy tint that is almost certain to become streaked when applied. If sufficient brushes are available, a separate brush should be used to take up each color, and a separate tinting dish or color saucer should be provided for each color, with sufficient water in it for mixing the required amount. A dish of either glass or porcelain contains the water. The brush is first moistened by dipping it in the water, without, however, letting it take up too much water, and then it is rubbed on the cake of color; a very small amount of color is sufficient. The brush is next dipped into the water in the color saucer, giving off its proportion of color. This water is then stirred until every particle of color is dissolved. If the tint is too light, add more color; if too heavy—a common fault—add more water, until the proper shade is obtained.

The tint should then be applied rapidly, beginning at the top of the paper and not allowing the edge of the tinted surface to become dry before carrying it further down the paper, until the bottom of the surface to be tinted is reached. If the edge of the tinted surface should become dry before

the tint is extended, when the tint is applied to the adjacent surface, a streak of deeper tint will appear where the two tinted surfaces unite. Should this occur unavoidably, however, the streak can be mostly removed and the tint rendered reasonably uniform by a careful application of a sponge rubber after the paper is thoroughly dry. Any tint is deepened by repeating the application of it. The paper must not be allowed to dry between the successive applications of the tint. If from any cause it should become dry, the entire surface must be moistened with clear water before another application of the tint. Careful practice will enable the student to produce a smooth tint.

When a marbled effect is desired, the entire surface is first covered with one tint, and then the other is applied in shorter or longer strokes of the brush, according to the effect that it is desired to produce. In tinting shore lines, the outline of the shore is first traced with a brush moistened with clear water, extending the wash as far as the tint is to be used. A dark-blue color having been prepared, a fine brush is dipped in the color and the outline of the shore is traced. The adjoining paper being moist will cause the color to run. Then a brush is moistened in clear water and the shore line thus traced is washed, the strokes of the brush being drawn from the shore. The effect will be a dark-blue shore line shaded to light blue. For roads, a dark-brown color is used.

32. Scales.—The scale of a topographical map should depend on the character of the work involved, but should always be large enough to clearly admit all necessary details without making the map unwieldy. The work should be so well done that dimensions may be accurately scaled from the map without any calculation. For small plats, such as public squares and small parks, 50 feet to the inch would be a suitable scale, admitting the smallest detail. For larger areas, such as town sites, extensive parks, suburban resorts, etc., scales of from 100 to 400 feet to the inch are commonly used, according to the amount of detail to be shown. The

scale must be reduced in proportion as the area is increased. Published topographical maps are usually made to scales varying from about 1 mile to the inch to 800 feet to the inch, according to the amount of detail represented. The smaller scale permits only the location of cities and towns and the prominent geographical features to be shown, while the larger scale permits the representation of all towns, villages, farms, woods, isolated buildings, roads, foot-paths, streams, and hills of 500 or 600 feet in horizontal extent. On a scale of 2 inches to the mile, the various features of the ground can be clearly represented. In all cases, the character of the surface, the amount of detail included in the survey, and the purpose of the map should determine the scale.

33. Size of Maps.—Maps for use in the field may vary in size from 18 by 24 inches to 24 by 30 inches. Both sizes are

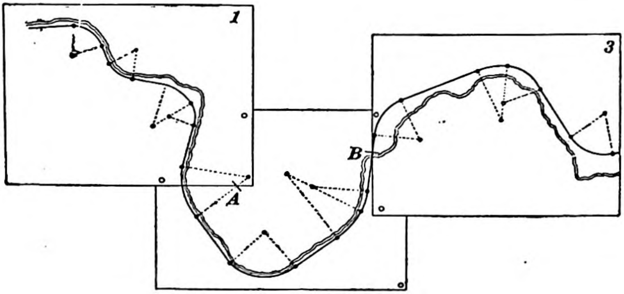


FIG. 22

suitable for railroad work. The line should be so arranged on the different sheets that they can be fitted together, making a continuous map of the line of survey, and the sheets should be numbered in regular succession, appearing somewhat as shown in Fig. 22.

When possible, the sheets should be arranged so that each curve with its center and limiting radii will come on the same sheet. Sometimes this cannot be done. The points where the different sheets join each other should be fixed

by a line drawn at right angles to the center line or radial line at the point of junction, as at *A* or *B*. This simplifies the work of fitting the sheets and greatly increases accuracy.

If the entire map is to be contained on a single sheet, judgment must be exercised in fixing the direction of the first course so as to attain that result, that is, so that the platting will not run off the edge of the sheet. When possible so to arrange the map on the sheet, the points of the compass should be in their natural order, namely, north at the top and south at the bottom of the map. Very fine lines on a map are a blemish rather than a merit, and heavy lines are likewise to be avoided, except when used for shading or boundaries. Boundaries of private property are represented by bold, full lines, and those of states, counties, or municipalities by heavy broken and dotted lines.

34. Lettering.—Legibility and uniformity are the chief requisites for good lettering. Ornamental letters are entirely out of place on a map, except for titles, and they are suitable for the titles of very elaborate maps only. All lettering in the body of the map or on details should preferably be in Italics. Small letters should be two-thirds the height of capitals. Ordinary capitals should usually be $\frac{1}{8}$ inch in height, and the small letters two-thirds of $\frac{1}{8}$ or $\frac{1}{12}$ inch in height. Uniformity in the spacing and slant of letters is as important as uniformity in size. All dimensions should be expressed in figures, and all important lines and objects briefly, but accurately described. So far as possible, without disadvantage to the map otherwise, the tops of the letters should be toward the north and west sides of the map. There is no work where practice is more essential, if skill is to be acquired, and nothing adds more to the finish of a drawing, than good lettering, while poor and slovenly lettering will rob of all merit an otherwise perfect drawing. The outline of a map and its position on the sheet will determine the position of the title, which should usually be in plain, bold lettering, but may be in ornamental lettering when the character of the map is such as to justify it.

PRACTICAL ASTRONOMY

(PART 1)

INTRODUCTION

THE GREEK ALPHABET

1. In works on astronomy, several letters of the Greek alphabet are used very commonly, both to represent the different stars of a constellation and to represent certain quantities. The letters of this alphabet and their names are as follows:

α alpha	ι iota	ρ rho
β beta	κ kappa	σ sigma
γ gamma	λ lambda	τ tau
δ delta	μ mu	υ upsilon
ε epsilon	ν nu	ϕ phi
ζ zeta	ξ xi	χ chi
η eta	o omicron	ψ psi
θ theta	π pi	ω omega

The brightest star in a constellation is generally denoted by α, the next brightest by β, and so on.

DEFINITIONS

NOTE.—A few of the subjects contained in this Section were treated in *Geometry* and in *Transit Surveying*. Their treatment is repeated here both for convenience of reference and for completeness.

2. **Astronomy** is the science that treats of the heavenly bodies.

This science is divided into five branches, namely:

(*a*) **Descriptive astronomy,** which is an orderly statement of astronomical facts ascertained by systematic observation, and of the laws derived from those facts.

(*b*) **Spherical astronomy,** which is an application of geometry and trigonometry to the determination of the relative positions of the heavenly bodies (the earth included).

(*c*) **Practical astronomy,** which treats of the methods of making astronomical observations, and of deducing from them the values of certain important quantities used in navigation and surveying.

(*d*) **Celestial mechanics,** which treats of the motions of the heavenly bodies as depending on the laws of dynamics.

(*e*) **Astrophysics,** called also **physical astronomy,** which treats of the physical condition and chemical constitution of the heavenly bodies.

3. The accurate determination of the bearing of lines on the earth's surface and of the latitude and longitude of points depends on the principles of practical astronomy. These principles will therefore be developed quite fully in the present text. But before proceeding to do so, it will be necessary to state some elementary geometrical rules and principles of which astronomy makes frequent use.

MEASURES OF AN ANGLE

4. An angle is measured by the arc of a circle whose center is at the vertex and whose ends are on the sides of the angle. This arc is usually graduated, or measured, in one of the three ways explained below.

5. Arc, or Degree, Measure.—In this method of measuring an angle, the whole circumference of which the measuring arc is a part is divided into 360 equal parts, called **degrees;** each degree is subdivided into 60 equal parts, called **minutes;** and each minute into 60 equal parts, called **seconds.** The magnitude of an angle is then expressed by the number of degrees, minutes, and seconds that its measuring arc contains. This is the familiar method explained in

geometry. An angle so expressed is frequently said to be expressed **in arc**, or **in degrees** (minutes and seconds being included).

6. Time Measure.—In this method, which is much employed in astronomy, the circumference is divided into 24 equal parts, called **hours**; each hour into 60 equal parts, called **minutes**; and each minute into 60 equal parts, called **seconds**. An angle expressed in hours, minutes, and seconds is frequently said to be expressed **in time**. Hours, minutes, and seconds of time are denoted, respectively, by the abbreviations, hr., min., sec., or, more simply, h., m., s.

7. Circular Measure.—In the third method of measuring an angle, the magnitude of the angle is expressed by stating how many times the measuring arc contains the radius; that is, the angle is measured by the ratio of the length of the arc that subtends it to the length of the radius of the arc. An angle so expressed is said to be expressed **in circular measure**, or **in radians**. The name **radian** is given to an angle the length of whose subtending arc is equal to that of the radius. The circular measure of such an angle is equal to unity, and for this reason the radian is said to be the unit of circular angle measure.

CONVERSION OF ARC MEASURE INTO TIME MEASURE, AND VICE VERSA

8. By Direct Multiplication and Division.—Since the circumference of a circle contains 360° when expressed in arc, and 24 hours when expressed in time, it follows that an arc of 1 hour contains $\frac{1}{24}$ of 360°, or 15°. Also, 1 minute of time contains $\frac{1}{60}$ of 15°, or 15′; and 1 second of time contains $\frac{1}{60}$ of 15′, or 15″. From these relations, time can be converted into arc, and arc into time, by a simple process of multiplication and division. This process was fully explained and illustrated in *Transit Surveying*, Part 2, and need not be repeated here.

9. By Means of Tables.—Instead of performing the arithmetical operations just referred to, which are somewhat

laborious, Table I at the end of this Section may be used. This table contains the value, in hours and minutes, of any number of degrees, from 1 to 360, and also the value, in time measure, of any number of minutes of arc. The seconds of arc must be reduced to seconds of time by dividing by 15. The use of this table is illustrated by the following examples:

EXAMPLE 1.—To change 278° 18′ 42″ to time measure.

SOLUTION.—Using Table I, we find

Opposite 278° in column 5	$18^h\ 32^m\ 0.0^s$
Opposite 18′ in column 1	1 12.0
Dividing 42″ by 15	2.8
Hence, adding the three numbers	$18^h\ 33^m\ 14.8^s$ Ans.

EXAMPLE 2.—To change $7^h\ 40^m\ 55^s$ to arc measure.

SOLUTION.—Using Table I, we find

Opposite $7^h\ 40^m$ in column 2	115° 0′ 0″
Opposite 52^s in column 1	13 0
Multiplying the remaining 3^s by 15	45
Hence, adding the three numbers	115° 13′ 45″ Ans.

EXAMPLES FOR PRACTICE

1. Change 18° 10′ 45″ to time measure. Ans. $1^h\ 12^m\ 43^s$
2. Change 351° 0′ 30″ to time measure. Ans. $23^h\ 24^m\ 2^s$
3. Change $3^h\ 20^m\ 40^s$ to arc measure. Ans. 50° 10′ 0″
4. Change $23^h\ 52^m\ 10^s$ to arc measure. Ans. 358° 2′ 30″

CONVERSION OF ARC MEASURE INTO CIRCULAR MEASURE, AND VICE VERSA

10. Since the whole circumference of a circle is approximately 6.28318 times the radius, it follows that the 360° of the measuring circumference are equivalent to 6.28318 radians. Hence, 1 radian contains 360 ÷ 6.28318 = 57.29578°, or, near enough for most practical purposes, 57.3°. Hence, if

A_d = angle expressed in degrees, and

A_r = same angle expressed in radians,

then
$$A_d = 57.3\ A_r \qquad (1)$$
$$A_r = \frac{A_d}{57.3} = .0175\ A_d \qquad (2)$$

In applying formula **2**, minutes and seconds must be reduced to decimals of a degree.

As radians are frequently used in connection with the very small angles that enter into astronomy, it is useful to remember that the value of 1 radian, expressed in seconds of arc, is $3,600 \times 57.29578 = 206,264.8''$.

EXAMPLE 1.—To change the angle .0234 radian to arc measure.

SOLUTION.—Here $A_r = .0234$. By formula **1**,

$$A_d = .0234 \times 57.3^\circ = 1.3408^\circ = 1^\circ\ 20'\ 26.9''. \quad \text{Ans.}$$

EXAMPLE 2.—To change 59° 52′ 30″ to circular measure.

SOLUTION.—Here $A_d = 59^\circ\ 52'\ 30'' = 59.875^\circ$. By formula **2**,

$$A_r = 59.875 \div 57.3 = 1.045 \text{ radians.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Change 1.25 radians to arc measure. Ans. 71° 37′ 30″
2. Change $\frac{1}{12}$ radian to arc measure. Ans. 4° 46′ 30″
3. Change 3° 10′ 30″ to circular measure. Ans. 0.0554 radian, nearly
4. Change 15° 24′ to circular measure. Ans. 0.2688 radian, nearly

THE SPHERE

DEFINITIONS

11. In geometry, a **sphere** is defined as a solid bounded by a surface every point of which is equidistant from a point within, called the **center**. The surface of such a solid is properly called a **spherical surface**, but for the sake of brevity a spherical surface is often called a sphere, just as the word circle is used instead of the longer word circumference.

12. A **radius** of a sphere is a straight line drawn from the center to the surface. A straight line passing through the center and terminated at both ends by the surface is called a **diameter** of the sphere.

13. A sphere may be generated by the revolution of a semicircle about its diameter. For, if the semicircle ACB,

Fig. 1, is turned about its diameter AB, any point P on the semicircle is at a constant distance oP from the center o. Consequently, during the revolution, every point on the semicircle lies on a sphere whose center is o and whose radius is oP.

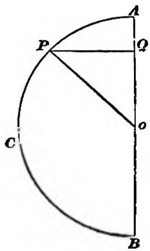


FIG. 1

14. Let Q, Fig. 1, be a fixed point in the diameter AB, and let QP be drawn perpendicular to AB. Then, during the revolution of the semicircle, QP lies always in the same plane, and the point P describes a circle whose center is Q. Hence, every plane section of a sphere is a circle.

15. A section of a sphere made by a plane passing through the center is called a **great circle.** A section made by a plane not passing through the center is called a **small circle.** Thus, $ABA'B'$ and $BPB'P'$, Fig. 2, are great circles, because their planes pass through the center C of the sphere; while $aba'b'$ and cc' are small circles, because their planes do not pass through C. A great circle divides the sphere into two equal parts, called **hemispheres.**

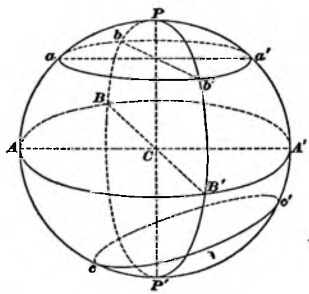


FIG. 2

16. A straight line through the center of either a great or a small circle, and perpendicular to the plane of the circle, is called the **axis** of the circle. The points where the axis of a circle meets the sphere are called the **poles** of the circle. Thus, PP', Fig. 2, is the axis of the great circle $ABA'B'$ and of the small circle $aba'b'$; the points P and P' are the poles of these circles. The axis of any circle of a sphere passes through the center of the sphere.

17. If any great circle of a sphere is taken as a **primary, or fundamental,** circle, great circles passing through its poles are called its **secondaries.** Thus, if $A\,B\,A'\,B'$, Fig. 2, is taken as a primary circle, $P\,B\,P'\,B'$, a great circle passing through the poles P and P', is one of its secondaries.

It is evident that the plane of a great circle is perpendicular to the plane of each of its secondaries; it follows that, if one circle is a secondary to another, the latter is also a secondary to the former. Thus, the circle $A\,B\,A'\,B'$, Fig. 2, is a common secondary to the two circles $A\,P\,A'\,P'$ and $B\,P\,B'\,P'$.

18. The angle between two great circles, called a **spherical angle,** is the angle between their planes, and is measured by the angle between two lines drawn, one in each plane, perpendicular to the line in which the planes intersect. Each of these lines must meet the line in which the planes intersect at the same point. Thus, the angle $A'\,P\,B$ between the great circles $A\,P\,A'\,P'$ and $B\,P\,B'\,P'$, Fig. 2, is measured by the angle $A'\,C\,B$, in which $C\,A'$ lies in the plane of $A\,P\,A'\,P'$ and is perpendicular to the intersection $C\,P$ of the two planes at C; while $C\,B$ lies in the plane of $B\,P\,B'\,P'$, and is also perpendicular to that intersection at the same point C. Now, the angle $A'\,C\,B$ is measured by its intercepted arc $A'\,B$. Hence, the angle between the circles $A\,P\,A'\,P'$ and $B\,P\,B'P'$ is measured by the arc $A'\,B$ that they intercept on their common secondary $A\,B\,A'\,B'$. This may be stated as a general principle as follows:

The angle between two great circles is measured by the arc that they intercept on their common secondary.

19. The **angular distance** between two points on a sphere is the angle that they subtend at the center of the sphere.

20. A **spherical triangle** is a portion of a sphere bounded by three arcs of great circles.

21. **Parallel circles** of a sphere are those whose planes are parallel.

PROPERTIES OF SPHERICAL CIRCLES

22. Every point on a circle of a sphere is at a constant angular distance from either pole of the circle. For, during the revolution of the generating semicircle, Fig. 1, the arc AP remains constant, and A is the pole of the circle described by the point P.

23. The shortest distance, measured on the surface, between two points on a sphere, is the arc of the great circle joining the two points.

24. Through two points on a sphere that are not the extremities of a diameter of the sphere, one and only one great circle can be described.

25. Through two points on a sphere that are not the extremities of a diameter of the sphere, an infinite number of small circles can be described.

POSITION OF A POINT ON A SPHERE

26. Let AOA', Fig. 3, be a fixed great circle whose axis is PP', and let O be a fixed point on this circle. If we conceive the sphere to be generated by the revolution of a semicircle about its diameter PP', it is evident that, during a complete revolution, the generating semicircle passes once and only once through every point on the sphere. Let $PXBP'$ be the position of the generating semicircle in which it passes through the point X. This position of the generating semicircle may be fixed, with respect to O, by measuring the arc OB. Let QXQ' be a plane parallel to the plane AOA'; then the plane QXQ' and the semicircle PXP' can intersect only in the

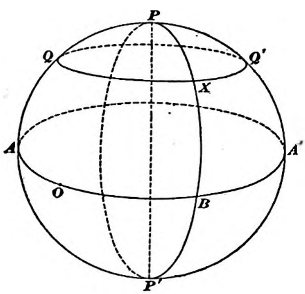


FIG. 3

one point X. Hence, the position of the point X can be determined by fixing the position of the plane QXQ' and the position of the semicircle PXP'. The position of the semicircle PXP' is fixed by measuring the arc OB; the position of the plane QXQ' can be fixed by measuring how far it is above or below the plane AOA'. Manifestly, if the arc BX is measured, it will fix the height of the plane QXQ' above the plane AOA'. Thus, the posifion of the point X with respect to O may be fixed by measuring the arcs OB and BX.

Suppose that X and X' are two points on a sphere, the position of X being fixed by the arcs OB and BX, while the position of X' is fixed by the arcs OB' and $B'X'$. Then, if OB is equal to OB', the points X and X' lie on the same circle passing through P and P'; that is, on the same secondary to AOA'. If BX is equal to $B'X'$, the points X and X' are on the same small circle parallel to AOA'.

The position of a point, then, is fixed by specifying: (1) on which of the secondaries to a fixed circle AOA' it lies, and on which of the halves of that secondary it lies; and (2) on which of the parallels to AOA' it lies.

27. The method here described is employed for fixing the position of a place on the earth's surface, the points P and P' being the poles of the earth. The great circle $AOBA'$, perpendicular to PP', is called the **equator**, and the arc POP' (not shown) is called the **principal meridian**, or **reference meridian**. The arcs OB and BX are called the **longitude** and **latitude**, respectively, of the point X. The meridian that passes through the Royal Observatory at Greenwich, near London, England, is adopted as a principal meridian for some purposes; that passing through the dome of the new observatory at Washington is the principal meridian for the United States.

THE CELESTIAL SPHERE

28. Apparent Conditions.—To one who observes the heavens at night, the celestial bodies appear to be bright points attached to the inner surface of a vast hollow spherical dome, whose center is at the observer's eye.

A little reflection, however, is sufficient to establish the fact that the heavenly bodies are not all equidistant from the observer's eye, and are not attached to any surface. Indeed, we have no direct means of estimating the distances of all these bodies; all that we can directly observe is their relative directions. Most astronomical instruments determine merely the relative directions of the heavenly bodies. It is very important, therefore, to have a convenient mode of representing these relative directions.

29. Assumed Conditions.—Imagine a vast spherical surface to be described enclosing all the heavenly bodies, and having its center at the observer's eye; this imaginary surface is called the **celestial sphere.** The heavenly bodies are so enormously remote that in comparison with this sphere the whole earth is a mere dot or point. Any point on the earth may therefore be regarded as the center of the celestial sphere, since from all such points the apparent directions of the heavenly bodies are practically the same.

30. The Stars.—To any observer at the center, this sphere appears to be covered with numerous **stars,** some of which rise and set regularly, while others are nearly stationary. The stars are aggregated into more or less definite groups called **constellations.** A very little watching will convince the observer of the important fact that the forms of the constellations, or the relative positions of the stars on the celestial sphere, do not change. Maps of the constellations made centuries ago do not materially differ from those made at present.

The stars are fixed in their positions on the celestial sphere, and it is the apparent rotation of this sphere as a whole that causes them to appear to rise and set.

31. There is but one-half of the celestial sphere that is visible to the observer at one time, since the earth under foot intercepts the other half from view; but if we imagine the earth to become transparent and the light of the sun to be extinguished, the observer would behold the heavens entirely surrounding him, below as well as above, and covering the entire surface, all the constellations of the sky would shine out, each immovably fixed in its proper position on the sphere.

32. If the observer could cause the earth's axis to become a real visible line produced in both directions until it met the celestial sphere, he would perceive that the entire spherical surface bearing the constellations was apparently turning slowly about this axis from east to west, completing one revolution in about 24 hours. Further knowledge would teach him that it is not the enormous sphere that is turning about the earth's axis, but that the earth itself, by a uniform rotation from west to east, brings different points of the celestial sphere successively overhead.

33. Unfortunately, the observer cannot see the whole surrounding sphere at one time. In the night-time, the earth cuts off one-half of it; and in the day-time, the overpowering brilliancy of the sun renders the stars invisible to the unaided eye. It should be remembered, however, that the sphere is about us no less during the day than during the night, and that at any hour of the day, could the light of the sun be blotted out, the constellations would appear to us each in its proper place.

POSITION OF A BODY ON THE CELESTIAL SPHERE

34. The exact position of any point on the earth's surface is known when the latitude and longitude of the point are known (Art. 27). For the purpose of defining latitude

and longitude, certain imaginary circles on the earth's surface, of which the most important are the equator and the principal meridian, are used as reference circles. In exactly the same way, certain reference circles are conceived to be drawn on the celestial sphere, and the positions of heavenly bodies are referred to them.

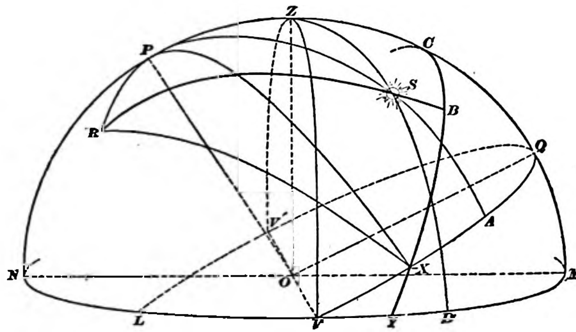


FIG. 4

Fig. 4 represents the celestial hemisphere, all the reference circles that are used in astronomy being drawn on it. The definitions of these circles should be learned and thoroughly understood, as they are indispensable in the study of practical astronomy.

THE EQUINOCTIAL SYSTEM OF CIRCLES

35. The earth's axis, when produced indefinitely in both directions, is called the **axis** of the celestial sphere, or of the heavens. The point in which the axis of the earth produced toward the north pierces the celestial sphere is called the **north** pole of the sphere, or of the heavens. The point in which the earth's axis produced toward the south pierces the celestial sphere is called the **south pole** of the sphere, or of the heavens. In Fig. 4, OP is one-half of the earth's axis, and P is the north pole of the sphere. The south pole is not shown in the figure.

36. The **celestial equator** is the great circle in which the plane of the earth's equator intersects the celestial sphere. In Fig. 4, $VQV'L$ is part of the celestial equator.

37. All great circles passing through the north and the south pole of the celestial sphere are called **hour circles.** It follows that hour circles are secondaries to the celestial equator; also, that the poles of the celestial equator coincide with the poles of the celestial sphere. Thus, in Fig. 4, PX, PA, and PQ are hour circles.

38. The celestial equator divides the celestial sphere into two hemispheres: that containing the north pole is called the **northern hemisphere;** the other, the **southern hemisphere.**

39. The sun in its apparent motion over the celestial sphere crosses the equator, passing from the southern to the northern hemisphere, on the 21st of March. The point at which the sun appears to cross the equator as it passes from the southern to the northern hemisphere is called the **vernal equinox.** In Fig. 4, $YXBC$ is a part of the apparent path of the sun over the celestial sphere; the point X is the vernal equinox. The position of the vernal equinox will be clearly understood from the following explanation.

The sun appears to move slowly over the celestial sphere among the stars from west to east, and takes just 1 year to move entirely around the sphere. The celestial equator is a circle drawn on the celestial sphere 90° from the poles, just as the earth's equator is a circle on the earth 90° distant from the north and south poles. Now, in passing around the sphere, the sun crosses this equator twice each year; on the 21st of March it crosses it in passing from the southern to the northern hemisphere; and on the 22d of September, in passing from the northern to the southern hemisphere. In Fig. 5, AB represents a part of the celestial equator; the whole figure shows a little band of the celestial sphere lying along the equator. The line CDE shows the path of the sun among the stars between March 21 and September 22. On March 21, the sun crosses the celestial equator

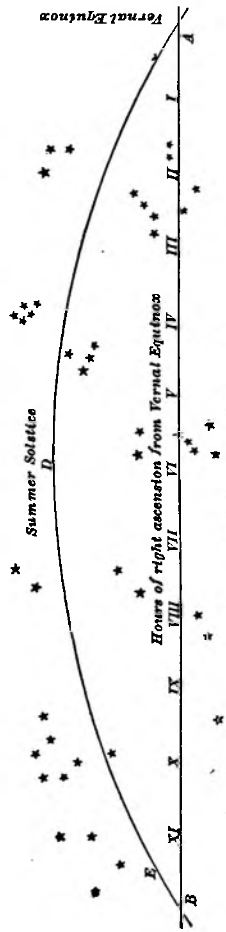


FIG. 5. THE SUN'S APPARENT PATH IN SUMMER

at A, and this point, which is the intersection of the path of the sun with the equator, is the vernal equinox.

40. The hour circle passing through the vernal equinox is called the **equinoctial colure.** In Fig. 4, PX is the equinoctial colure.

41. The **right ascension** of a celestial body is the arc of the celestial equator measured eastward from the vernal equinox to the hour circle passing through the body. Thus, in Fig. 4, XA is the right ascension of the body S.

42. The **declination** of a heavenly body is its angular distance north or south of the celestial equator, and is measured by the arc of the hour circle passing through the body and intercepted between it and the equator. Thus, in Fig. 4, AS is the declination of the body S. The declination of a heavenly body is considered positive or negative (+ or −) according as the body is north or south of the celestial equator.

43. Right ascension and declination are exactly similar to terrestrial longitude and latitude, respectively; the celestial equator corresponds to the earth's equator, and the equinoctial colure to

the principal meridian. When the right ascension and declination of a heavenly body are known, its position on the celestial sphere is definitely fixed.

44. Except for very minute motions, the stars do not change their positions on the celestial sphere; it therefore follows that the right ascension and declination of every star remains sensibly constant during long periods of time. The sun, moon, and planets, however, are continually in motion on the sphere; hence, their right ascension and declination are constantly changing.

45. The **polar distance** of a heavenly body is its angular distance from the nearer pole, and is measured by the arc of the hour circle intercepted between the pole and the body. The polar distance is, therefore, the complement of the declination. In Fig. 4, SP is the polar distance of S.

THE DIURNAL MOTION

46. An observer standing at night in the middle of a great prairie or on a ship on the ocean sees about him an apparently level surface surmounted by the visible hemisphere of the sky. The great circle in which this surface and the sky appear to meet is the **apparent horizon,** which for brevity is usually called *the horizon.* We know, however, that the surfaces of the prairie and ocean are not really plane surfaces, but a portion of the curved spherical surface of the earth. For astronomical purposes, the **plane of the horizon** is a plane tangent to the earth's surface at the point occupied by the observer.

47. The **celestial horizon** is the great circle in which the plane of the horizon intersects the celestial sphere. In astronomy, the term *horizon* is generally understood to mean the celestial horizon. In Fig. 4, the circle NVM is the celestial horizon.

48. The **zenith** of a point on the earth's surface is a point in which a line passing through the center of the earth and the given point pierces the celestial sphere; in Fig. 4,

Z is the zenith. The point in which the same line pierces the celestial sphere below the given point is called the **nadir** of that point. The zenith and nadir of a point occupied by an observer are referred to, respectively, as the zenith and nadir of the observer. The zenith and nadir are the poles of the horizon. The prolongation of the plumb-line is the axis of the horizon, and meets the celestial sphere in the zenith directly overhead, and in the nadir underfoot.

The position of the zenith and that of the horizon on the celestial sphere depend on the position of the observer on the earth's surface, and are, therefore, different for observers differently located.

49. If the axis about which the celestial sphere revolves were a luminous straight line, the observer, standing on the horizon plane and facing north, would see this line piercing the plane of the horizon and extending to the north pole of the heavens. On account of the apparent rotation of the whole sphere about this axis, the various constellations would appear above the horizon plane in the east, pass across the sky in small circles parallel to the equator, and finally disappear below the horizon plane in the west.

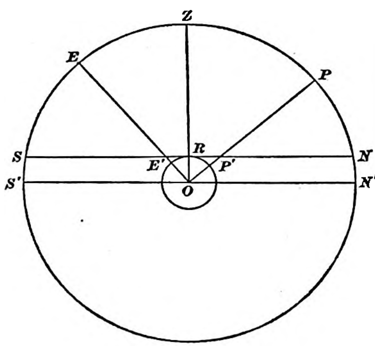


FIG. 6

50. Fig. 6 represents a section of the earth and the celestial sphere made by a plane passing through the north and south poles of the earth and the observer's station R; it is therefore a meridian plane. SZN is an arc of the celestial sphere; Z is the zenith; and SN, the horizon. Since the distance between

O and R is inappreciable compared with the distances of these two points from the surface of the celestial sphere, the observer's position R may be taken as coinciding with the center O of the earth; and the plane $S'\,O\,N'$, passed through O parallel to the plane $S\,R\,N$, as coinciding with the horizon plane (Art. 29). It is assumed that $O\,E'\,E$ is the equator and $O\,P'\,P$ the axis both of the earth and of the celestial sphere. The angle $E'O\,R$ is the latitude of the observer, or his angular distance from the equator, and since it equals $Z\,O\,E$, which is measured by the arc $E\,Z$, it follows that the arc of the meridian intercepted between the zenith and the celestial equator measures the observer's latitude. The angle $P\,O\,N'$, measured by the arc $N'P$, or, practically, by $N\,P$, equals the angle $Z\,O\,E$, for each of these angles is the complement of $P\,O\,Z$; hence, the angular elevation of the north pole of the heavens above the horizon plane is equal to the latitude of the observer. This angular elevation is called the **altitude** of the pole. We have, then, the following important principle:

The latitude of an observer is equal to the altitude of the pole above the observer's horizon. (See *Transit Surveying*, Part 2.)

51. The apparent motion of the stars will be understood by referring to Fig. 7, which represents the celestial sphere. The earth is represented by the point O at the center of the sphere; $S\,E\,N$ is the horizon plane; $P\,P'$, the polar axis; $E\,Q\,W\,Q'$, the celestial equator; and $N\,K$, $B\,A$, $E\,Q$, and $G\,F$, the apparent paths pursued by the stars as the apparent rotation of the celestial sphere about $P\,P'$ carries them around the sky. A star between the pole and the equator will be

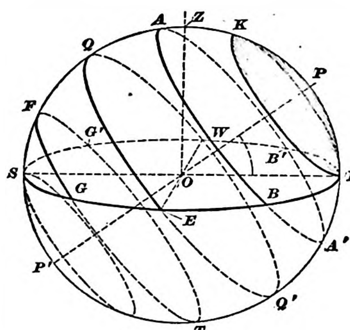


FIG. 7

brought above the horizon at some point B, and after being carried across the sky along the path BAB', will finally set at B'. A star whose polar distance is PN will, by the rotation of the celestial sphere, be carried about the axis on a small circle NK that just touches the horizon plane at N, and hence it will never rise nor set. Similarly, all stars whose polar distance is less than PN will remain always above the horizon. A star exactly at P would have no motion at all. There is no star exactly at either pole. The closest star to the north pole is **Polaris,** often called the **pole star** and the **north star,** whose polar distance is about 1° 13′.

Any star south of the equator will rise above the horizon at some point, as G, and remaining visible but a short time, will be carried across the sky on the small circle GFG'. A star whose south polar distance is less than $P'S$ will never appear above the horizon.

52. The small circle KN is called the **circle of perpetual apparition,** while the small circle $P'S$ is called the **circle of perpetual occultation.** Since the angle PON, measured by the arc NP, is equal to the elevation of the pole above the horizon, which is equal to the latitude of the observer, it follows that the polar distance of any point in the circle of perpetual apparition, or in the circle of perpetual occultation, is equal to the latitude of the observer.

53. It is very important that this apparent motion of the stars should be clearly understood. It should be borne in mind that the stars are immovably fixed on the celestial sphere, and that this sphere appears to turn as a whole from east to west. If possible, a few evenings should be spent in studying the motion from the sky itself, for when it has been observed actually taking place in the heavens, it is not easily forgotten. For this purpose, go out on some clear evening and, facing toward the north, trace out the constellation of the Great Dipper, called by astronomers Ursa Major, Fig. 8. If a line is drawn through the first two stars in the bowl of the dipper and produced as shown in the figure, it will pass nearly through the north star, or Polaris. Next

make a diagram of several of the brighter stars about the pole, and then, after an interval of 3 or 4 hours, compare this diagram with the stars in the sky. It will at once be per-

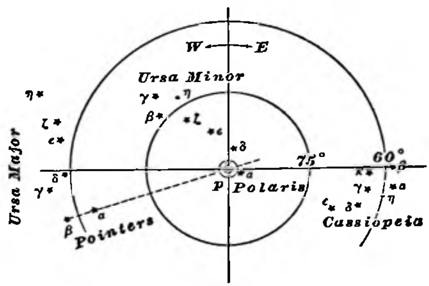


FIG. 8

ceived that the stars have moved about the pole in the manner described above. A few evenings spent in observation will suffice to fix the nature of this motion clearly in the mind.

THE HORIZON SYSTEM OF CIRCLES

54. It has been explained (Art. 43) that the position of a body on the celestial sphere may be exactly indicated by referring it to the celestial equator and the vernal equinox. For many purposes, however, it is more convenient to refer the body to the horizon as a primary circle.

55. The **celestial meridian** of an observer is the great circle of the celestial sphere that passes through the poles and the zenith of the observer. This circle passes also through the nadir, and is secondary to the horizon. In Fig. 4, $NPZCQM$ is one-half of the celestial meridian.

56. The celestial meridian cuts the horizon in two points called, respectively, the **north point** and the **south point** of the horizon. The points on the horizon midway between the north and the south point are called the **east point** and the **west point**, respectively. In Fig. 4, N is the north

point; M, the south point; V', the east point; and V, the west point of the horizon.

57. A **vertical circle** is any circle passing through both the zenith and the nadir. It follows that all vertical circles are secondaries to the horizon. In Fig. 4, ZN, ZV', ZQM, ZD, and ZV are portions of vertical circles.

58. The vertical circle that passes through the east and the west point is called the **prime vertical**. The prime vertical is at right angles to the meridian. In Fig. 4, $V'VZ$ is the prime vertical.

59. The **altitude** of a heavenly body is its angular distance from the horizon, and is measured along the vertical circle passing through the body. In Fig. 9, in which SEN is the horizon and SZN the meridian, HM is the altitude of the star M. In Fig. 4, DS is the altitude of S.

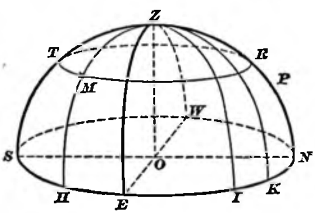


FIG. 9

60. The **zenith distance** of a celestial body is the angular distance from the body to the zenith, and is measured on the vertical circle passing through the body. The zenith distance is the complement of the altitude. In Fig. 9, in which Z is the zenith, MZ is the zenith distance of M. In Fig. 4, SZ is the zenith distance of S.

61. The **azimuth** of a celestial body is the angle between the plane of the meridian and the plane of a vertical circle passing through the body. It is measured either from the south point toward the west, or from the north point toward the east, along the horizon. In Fig. 9, in which N and S are, respectively, the north and south points, NH is the azimuth of M, reckoned from the north toward the east, and $SWNEH$ is the azimuth of M, reckoned from the south toward the west. Similarly, in

Fig. 4, MD is the azimuth of S measured from the south toward the west.

62. It is evident that the zenith, meridian, and horizon are fixed with reference to the observer, but are not fixed with reference to the celestial sphere. The zenith traces out a small circle on the celestial sphere as the sphere turns on its axis, and hence the altitudes and azimuths of all heavenly bodies are constantly changing. The altitude and azimuth serve to indicate the apparent direction of a body with reference to the observer at any given instant.

ANOTHER METHOD OF FIXING THE POSITION OF A HEAVENLY BODY

63. In this method, one circle from each of the preceding systems is employed as a circle of reference.

64. The **hour angle** of a star is the arc intercepted on the equator between the meridian and the foot of the hour circle passing through the star. It is measured from the meridian toward the west. Hour angles are usually expressed in time measure. In Fig. 4, QA is the hour angle of the body S.

The position of a star is indicated in this method by stating the values of the hour angle and declination.

65. On account of the uniform rotation of the celestial sphere, the hour angle increases uniformly while the declination remains constant. The hour circle passing through the star makes a complete revolution around the sky once in every 24 hours, sidereal time (Art. **67**). It follows that the hour angle of the star is simply the time that has elapsed since the star was on the meridian. This affords an easy way of determining the hour angle by observation, as will be explained elsewhere.

COMPARISON OF THE THREE SYSTEMS

66. Fig. 4 represents the celestial hemisphere visible to an observer whose horizon is NDM: Z is the zenith,

	Equinoctial System	Horizon System	Hour-Angle System
Primary circle of reference	Equator VXQ.	Horizon $NVDM$.	Equator VXQ.
Secondary circle of reference	Equinoctial colure PX.	Meridian $PZQM$.	Meridian $PZQM$.
Poles of the primary circle	North pole P and south pole.	Zenith Z and nadir.	North pole P and south pole.
The position is fixed by	Right ascension XA, + toward the east.	Azimuth MD, + toward the west.	Hour angle QA, + toward the west.
	Declination AS, + above the equator; or polar distance PS.	Altitude SD; or zenith distance ZS.	Declination AS, + above the equator; or polar distance PS.
	The declination and right ascension of stars do not change.	The altitude and azimuth change continually and not uniformly.	The hour angle increases uniformly with the time.

N and M are the north and the south point, respectively; $V Q V'$ is the equator; P, the north pole; and S, any star; $Y X B C$ is the path followed by the center of the sun as it moves over the celestial sphere; hence, X is the vernal equinox (Art. 39). Referring to Fig. 4, the three systems of locating the body S on the celestial sphere may be tabulated as shown on the opposite page.

NOTE.—Before making observations in the field, the student should provide himself with a copy of The American Ephemeris and Nautical Almanac. This book is published yearly by authority of Congress. It contains the positions of the principal stars and of the sun, moon, and planets for every night of the year, and is indispensable to the geodetist and practical astronomer. It can be obtained from the Navy Department, Washington, D. C., for $1. In ordering this book, be careful to state the year desired, since the Ephemeris is of value for only a single year. For this reason also, the student will probably prefer not to buy a copy until he actually begins observations in the field. Specimen portions of all tables required in the solution of the examples in this Section are printed at the end of the Section. By means of these, the student can thoroughly learn the nature and use of the Ephemeris without buying a copy until he needs it in his practical work.

EXAMPLES FOR PRACTICE

1. What is the zenith distance of the north pole in latitude $+30°$?

Ans. 60°

2. What is the azimuth of: (*a*) the north point? (*b*) the south point? (*c*) the east point?

Ans. $\begin{cases} (a)\ 180° \\ (b)\ 0° \\ (c)\ 270° \end{cases}$

3. The latitude of Philadelphia, Pa., is $+39° 58' 02''$. (*a*) What is the declination of a star that passes directly overhead at Philadelphia? (*b*) What is the zenith distance of Polaris at Philadelphia when Polaris is on the meridian, the declination of Polaris being $+88° 47'$?

Ans. $\begin{cases} (a)\ +39° 58' 02'' \\ (b)\ 48° 48' 58'' \end{cases}$

4. What is the hour angle: (*a*) of the zenith? (*b*) of the sun at noon? (*c*) of the sun at 8 o'clock in the morning? (*d*) of the sun at midnight?

Ans. $\begin{cases} (a)\ 0° \\ (b)\ 0° \\ (c)\ -4^h \text{ or } -60° \\ (d)\ 12^h \text{ or } 180° \end{cases}$

TIME

SIDEREAL TIME

67. Clocks throughout the world are regulated by comparing them, directly or indirectly, with the clocks of the great national observatories; and these observatory clocks are regulated by means of astronomical observations. To the astronomer, therefore, belongs the duty of the regulation and measurement of time; and this is one of the most important problems of practical astronomy.

The celestial sphere, which turns with an absolutely uniform motion about its axis, is itself a great clock. It carries the stars one after another across the meridian, and if we could select convenient stars on the equator distant 1 hour, or 15°, from one another, we might number them 1 hour, 2 hours, etc., and read the time from the celestial sphere exactly as from a clock on which the dial revolved and the hour hand (or meridian) remained motionless. It is more convenient, however, to use one single point of the celestial sphere, and for this purpose the vernal equinox is selected (Art. **39**). When this point is on the meridian, the time is said to be 0 hr. 0 min. 0 sec. When it has moved on from the meridian 1 hour, or 15°, the time is 1 o'clock, and so on. Time so estimated is called **sidereal time,** and the interval of time occupied by the celestial sphere in completing one revolution on its axis is called a **sidereal day.**

68. The passage of a celestial body across the celestial meridian is called its **transit,** or **culmination.** During one complete revolution of the celestial sphere every celestial body crosses the meridian twice. The passage of a body across that branch of the meridian that contains the observer's zenith is called the **upper transit,** or **upper culmination;**

the passage across that branch of the meridian that contains the observer's nadir is called the **lower transit, or lower culmination.**

A sidereal day is the interval between two successive upper transits of the vernal equinox, and begins when the vernal equinox is on the meridian.

69. From Art. **68**, it follows that the sidereal time is the hour angle of the vernal equinox. The sidereal clock used in observatories shows sidereal time, provided that it is perfectly regulated. The hands point to $0^h\ 0^m\ 0^s$ when the vernal equinox is on the meridian, and the hours are reckoned from 0^h up to 24^h, when the vernal equinox is again on the meridian. In Fig. 4, the sidereal time is measured by the arc XQ of the equator, or by the hour angle XPQ.

70. Suppose that, in Fig. 4, we had a star in transit, that is, on the meridian between P and M; the right ascension of this star would be the arc XQ (Art. **41**). Hence, the right ascension of a star, when expressed in time, is equal to the sidereal time of the star's transit.

Since the arc XQ is the right ascension of the meridian, it follows that the sidereal time is equal to the right ascension of the meridian.

71. For any star S, Fig. 4, not on the meridian, $QX = QA + AX$; that is, sidereal time = star's hour angle + star's right ascension. Let

$$\theta = \text{sidereal time;}$$
$$t = \text{hour angle;}$$
$$\alpha = \text{right ascension.}$$

Then, $$\theta = t + \alpha$$

In applying this formula, the algebraic signs of the different quantities should be carefully observed. Since θ expresses time, the hour angle t and the right ascension α should be expressed in time measure. When the hour angle of a heavenly body is reckoned toward the east, it is negative.

EXAMPLE 1.—The hour angle of Sirius on December 25, 1903, at Washington was observed to be $+ 2^h\ 7^m\ 18.1^s$. The right ascension of Sirius, as found from the American Ephemeris (Art. **89**), was

$6^h\ 40^m\ 56.4^s$. What was the sidereal time at which the observation was made?

SOLUTION.—

$$\begin{aligned} t &= +2^h\ \ 7^m\ 18.1^s \\ \alpha &= +6\ \ 40\ \ 56.4 \\ \hline \theta &= +8^h\ 48^m\ 14.5^s. \quad \text{Ans.} \end{aligned}$$

EXAMPLE 2.—The star Vega was observed 4 hours to the east of the meridian at Washington on May 20, 1903. The instant of the observation as shown by the sidereal clock was $14^h\ 33^m\ 58.9^s$. Was the clock fast or slow, and how much?

SOLUTION.—Since the star was east of the meridian, t is $-$:

$$t = -\ \ 4^h\ \ 0^m\ \ 0^s$$

From Ephemeris (Art. 89), $\alpha = +18\ \ 33\ \ 41.6$

$$\theta = \ \ 14^h\ 33^m\ 41.6^s$$

Hence, the clock at the time of this observation was 17.3 sec. fast. Ans.

APPARENT AND MEAN SOLAR TIME

72. The hour angle of the sun, instead of the hour angle of the vernal equinox, may be employed to determine time. The center of the sun crosses the upper branch of the meridian at **apparent noon**; therefore, the **apparent solar time** at any instant is simply the hour angle of the center of the sun at that instant.

73. An **apparent solar day** is the interval between two successive apparent noons.

74. The sun, unlike the vernal equinox and the stars, is not a fixed point on the celestial sphere. It is continually moving around the heavens toward the east, completely circling the sky in 1 year. In this motion, the sun's right ascension is constantly changing. Hence, the apparent solar day is not of the same length as the sidereal day; nor are all apparent solar days of equal duration. The apparent solar day is longer than the sidereal day by the amount of the sun's daily increase in right ascension.

75. Disadvantage of Sidereal Time.—The sidereal time of apparent noon on any day is equal to the sun's right ascension on that day, and, consequently, it gets later by 24 hours during the year. Thus, the sidereal time

of apparent noon on March 21 is 0^h; on June 21, 6^h; on September 23, 12^h; on December 22, 18^h. It is seen that sidereal time bears no simple relation to the phenomena of day and night, and is therefore unsuitable for every-day use.

76. Disadvantage of Apparent Solar Time.—All apparent solar days are not of equal duration, because the sun in its journey around the sky moves faster at some times than it does at others. They cannot therefore be measured by a clock whose rate is uniform, and for this reason apparent solar time is unsatisfactory for scientific and practical purposes.

77. Mean Time.—Another kind of time, called **mean time**, is generally used for practical purposes. It is defined by reference to what is called the **mean sun.** The mean sun is not a body of any kind, but merely a point that is imagined to move with a uniform speed around the celestial equator completing the entire circuit in the same time that the true sun does. Sometimes the mean sun is ahead of the true sun, and sometimes behind it.

Mean noon is the time of the mean sun's upper transit.

A mean solar day is the interval between two successive mean noons. **Mean solar time**, or **mean time**, is measured by the hour angle of the mean sun. It is the time shown by clocks, and is now used for all practical and scientific purposes, except in some kinds of astronomical work.

78. Astronomical and Civil Time.—When mean time is employed in astronomical work, it is called **astronomical mean time**, and is reckoned continuously up to 24 hours, the astronomical day beginning at mean noon. When mean time is employed in the ordinary affairs of life, it is called **civil time**, and the **civil day** begins at midnight, 12 hours earlier than the astronomical day. Thus, we have the following rules:

Rule I.—*To convert civil time into astronomical time: if the civil time is marked A. M., take 1 from the day and add 12 to the hours; if marked P. M., simply drop the letters P. M.*

Rule II.—*To convert astronomical time into civil time: if the astronomical time is less than 12 hours, simply write P. M. after it; if greater than 12 hours, subtract 12 hours from it, mark the result A. M., and add 1 to the date.*

EXAMPLE 1.—To change January 3, 23^h astronomical time to civil time.

SOLUTION.—According to rule II, the result is, January 4, 11 A. M., civil time.

EXAMPLE 2.—To change June 7, 9 A. M. into astronomical time.

SOLUTION.—According to rule I, the result is, June 6, 21^h astronomical time.

79. Standard Time.—The United States, which lies between longitudes 65° and 125° west of Greenwich, is divided into four **time sections**, in each of which the time used for ordinary purposes is the mean local time of places lying on the meridian passing near the center of the section. These times are called **standard times.** The meridians adopted for this purpose are the 75th, the 90th, the 105th, and the 120th. It will be noticed that these meridians differ by 15°, or 1 hour. The time shown by ordinary clocks and watches at places within $7\frac{1}{2}°$ on either side of one of these meridians is not exactly local time for those places; it is local time only for places lying exactly on that meridian. To find the local time at any given place, it is necessary to subtract, algebraically, from the standard time the difference between the longitude of the place and that of the standard-time meridian. This is a very important fact, and should be constantly borne in mind. Thus, if the standard 75th-meridian time at a place whose longitude is 78° $(= 5^h\,12^m)$ is $9^h\,13^m$, the local time is $9^h\,13^m - (5^h\,12^m - 5^h) = 9^h\,01^m$. (See also *Transit Surveying*, Part 2.)

Standard times are called by the following names: 75th-meridian time is called **eastern time**; 90th-meridian time is called **central time**; 105th-meridian time is called **mountain time**; 120th-meridian time is called **Pacific time.**

80. The **equation of time** is the amount that must be added algebraically to the apparent solar time to obtain the corresponding mean time. We have, therefore,

$$(\text{mean time}) = (\text{apparent time}) + (\text{equation of time})$$

It is to be noted that the equation of time is not an equation in the ordinary sense of the word, but simply a quantity. The value of the equation of time is given in the American Ephemeris for each day of the year.

CONVERSION OF TIME

81. When observations are made on stars for the determination of time, it is the sidereal time that is obtained. Though time may be determined more accurately in this way than in any other, it is the mean solar time that is in common use, and hence it is important that one should know the method of changing the time shown by a sidereal clock into mean solar time. On the other hand, when the sun is observed in the field, it is the apparent solar time that is obtained directly from the observation, and from this the mean solar time is obtained by simply adding, algebraically, the equation of time. For many kinds of observations it is absolutely necessary that the sidereal time be known.

82. The **civil local time** of any place on the earth's surface, at any instant, is the time elapsed since the mean sun's last transit, either upper or lower, over the meridian of that place. The 12 hours elapsed between the mean sun's lower and upper transit are A. M. time; the other 12, P. M. time.

83. The **astronomical local time** of any place on the earth's surface, at any instant, is the time elapsed since the mean sun's upper transit over the meridian of that place.

84. To Change a Mean Solar Interval Into Its Equivalent Sidereal Interval.—A mean solar day is longer than a sidereal day by 3 sidereal minutes and 56.555 sidereal seconds. That is, an interval of 24 hours, mean solar time, is equivalent to $24^h\ 3^m\ 56.555^s$, sidereal time.

Mean solar time is reduced to sidereal time by means of Table III of the Ephemeris. This table, a part of which is given at the end of this Section (Table II), contains the

amount to be added to any number of mean solar hours, minutes, and seconds in order to obtain the corresponding number of sidereal hours, minutes, and seconds. The numbers at the heads of the columns express mean solar hours, and those in the column at the extreme left of the page, mean solar minutes. The correction corresponding to any given number of mean solar hours and minutes is found in the column headed by the given number of hours, and in the same horizontal line as the given number of minutes in the left-hand column. The part of the correction corresponding to the mean solar seconds in the given interval is found in the column headed For Seconds at the right of the page. The two numbers taken from the table must be added together, and their sum added to the given number of mean solar hours, minutes, and seconds. The result will be the corresponding number of sidereal hours, minutes, and seconds.

EXAMPLE.—How many sidereal hours, minutes, and seconds are there in the mean solar interval $2^h\ 4^m\ 6.5^s$?

SOLUTION.—From Table II, looking in the column headed 2^h and in the horizontal row containing 4^m, we find

Correction corresponding to $2^h\ 4^m$	$+0^m\ 20.370^s$
From the column headed For Seconds, we find for 6.5^s	$+.017$
Total correction	$+0^m\ 20.387^s$
Mean solar interval	2 4 6.500
Required sidereal interval	$2^h\ 4^m\ 26.887^s$ Ans.

EXAMPLES FOR PRACTICE

Change the following mean solar hours, minutes, and seconds into sidereal hours, minutes, and seconds:

(a) $0^h\ 2^m\ 10^s$.	Ans.	(a) $0^h\ 2^m\ 10.356^s$
(b) $2^h\ 10^m\ 2^s$.		(b) $2^h\ 10^m\ 23.361^s$
(c) $2^h\ 1^m\ 1.4^s$.		(c) $2^h\ 1^m\ 21.281^s$
(d) $1^h\ 3^m\ 2.5^s$.		(d) $1^h\ 3^m\ 12.856^s$

85. To Change a Sidereal Interval Into Its Equivalent Mean Solar Interval.—Sidereal time is converted into mean solar time by means of Table II of the Ephemeris. This table, a part of which is given at the end of

this Section (Table III), is used exactly as the preceding, except that the corrections taken from it are to be subtracted from the given sidereal hours, minutes, and seconds, since any interval contains fewer mean solar than sidereal units.

EXAMPLE.—A sidereal interval contains $2^h\ 4^m\ 6.8^s$. To find its value expressed in mean solar hours, minutes, and seconds.

SOLUTION.—From Table III, we find

For $2^h\ 4^m$	$-0^m\ 20.314^s$
For 6.8^s	$-.018$
Total correction	$-0^m\ 20.332$
Sidereal interval	2 4 6.800
Required mean solar interval	$2^h\ 3^m\ 46.468^s$ Ans.

EXAMPLES FOR PRACTICE

Change the following sidereal hours, minutes, and seconds into mean solar hours, minutes, and seconds:

(*a*) $2^h\ 5^m\ 2^s$.	Ans.	(*a*) $2^h\ 4^m\ 41.517^s$
(*b*) $1^h\ 1^m\ 9^s$.		(*b*) $1^h\ 0^m\ 58.982^s$
(*c*) $2^h\ 10^m\ 10^s$.		(*c*) $2^h\ 9^m\ 48.676^s$
(*d*) $0^h\ 9^m\ 2.24^s$.		(*d*) $0^h\ 9^m\ 0.760^s$

86. The Sidereal Time of Mean Noon.—The sidereal time at any instant is equal to the right ascension of the meridian (Art. 70). Mean noon is the time of the mean sun's upper transit (Art. 77). Therefore, the sidereal time at the instant that the center of the mean sun is on the meridian, or, in other words, the sidereal time of mean noon, is equal to the right ascension of the center of the mean sun at that instant. The sidereal time of mean noon is given in the Ephemeris for every day, the quantity there tabulated being the value of the right ascension of the mean sun at the instant that it crosses the meridian of Washington. (A part of this table will be found at the end of this Section, Table IV.) When the sun is on the meridian of a place west of the Washington observatory, the corresponding Washington time will be a number of hours, minutes, and seconds after noon exactly equal to the difference between the longitude of that place and that of the Washington observatory. In other words, if *d* is the longitude (west of Washington) of a place, the mean sun will occupy *d* hours

(minutes and seconds included) in passing from the Washington meridian to the meridian of the place in question. During these d hours, the mean sun's right ascension is increasing uniformly, so that at a station whose longitude is d the right ascension of the mean sun, or, what is the same thing, the sidereal time of mean noon, is greater than the Washington sidereal time of mean noon. The difference is simply the amount by which the right ascension of the mean sun has increased during d hours. This increase may be found from Table III of the Ephemeris (see Table II at the end of this Section). Hence, the following rule to find the sidereal time of mean noon at any place whose longitude *west* of Washington is d, or whose longitude *east* of Washington is $-d$:

Rule.—*Take the sidereal time of mean noon at Washington from the Ephemeris* (*or Table IV of this Section*), *and add to it the correction corresponding to d hours, minutes, and seconds taken from Table III of the Ephemeris* (*or Table II of this Section*), *if d is* $+$; *if d is* $-$, *subtract this correction.*

EXAMPLE.—To find the sidereal time of mean noon on January 5, 1903, at an observatory whose longitude is $-2^h\ 10^m\ 8.5^s$.

SOLUTION.—From Table IV, we find

Sidereal time of mean noon at Washington, January 5, 1903, ...	$18^h\ 56^m\ 27.800^s$
From Table II, correction for $-2^h\ 10^m$ . . .	-21.356
Correction for -8.5^s	$-.024$
Hence, the sidereal time of mean noon is . .	$18^h\ 56^m\ 6.420^s$ Ans.

The corrections are subtracted, since d is $-$.

87. Conversion Rules.—Rules for finding mean solar time from sidereal time, and conversely, may now be stated.

Rule I.—*To change sidereal time into mean solar time: from the given sidereal time subtract the sidereal time of the preceding local mean noon* (*Art. 86*); *change the remainder, which is the sidereal interval of time elapsed since local mean noon, into mean solar time* (*Art. 85*).

Rule II.—*To change mean solar time into sidereal time: change the mean solar interval of time elapsed since the*

preceding local mean noon into sidereal time (Art. 84), and add the result to the sidereal time of local mean noon (Art. 86).

EXAMPLE 1.—The sidereal time on the afternoon of January 5, 1903, at a station whose longitude is $+1^h\ 10^m$, was found to be $20^h\ 58^m\ 40^s$. To find the corresponding mean solar time.

SOLUTION.—Here rule I is to be applied.

From Table IV, sidereal time of Washington, mean noon, January 5, 1903	$18^h\ 56^m\ 27.80^s$
From Table II, correction for d	$+11.499$
Hence, sidereal time of mean noon at the station	18 56 39.299
Observed sidereal time	20 58 40.000
Sidereal interval past mean noon	$2^h\ 2^m\ 0.701^s$
Correction to reduce sidereal to mean solar interval, Table III	-19.987
The desired mean solar time is	$2^h\ 1^m\ 40.714^s$ Ans.

EXAMPLE 2.—At a station whose longitude is $+1^h\ 10^m$, the mean solar time on January 5, 1903, was observed to be $2^h\ 1^m\ 0.876^s$, P. M. To find the corresponding sidereal time.

SOLUTION.—Here rule II will be applied.

Mean solar interval since preceding mean noon	$2^h\ 1^m\ 0.876^s$
Correction to reduce mean solar to sidereal interval, Table II	$+19.880$
Sidereal interval since preceding mean noon	$2^h\ 1^m\ 20.756^s$
Sidereal time of mean noon on January 5, 1903, at a longitude of $1^h\ 10^m$, as in example 1	18 56 39.299
Sidereal time	$20^h\ 58^m\ 0.055^s$ Ans.

EXAMPLES FOR PRACTICE

1. At a station whose longitude is $+2^h\ 10^m$, the sidereal time on January 2, 1903, was $19^h\ 45^m$. What was the corresponding mean solar time? Ans. $0^h\ 59^m\ 50.692^s$ P. M.

2. Find the sidereal time corresponding to the instant of 5 minutes past noon, on January 6, 1903, at a station whose longitude is $-1^h\ 1^m\ 10^s$. Ans. $19^h\ 5^m\ 15.123^s$

3. What is the sidereal time corresponding to the mean time $2^h\ 8^m\ 5^s$ P. M. on January 10, 1903, at a station whose longitude is $+7^m\ 4^s$? Ans. $21^h\ 24^m\ 37.782^s$

THE AMERICAN EPHEMERIS

88. Explanation.—The book to be relied on for the fundamental positions of all the heavenly bodies observed is The American Ephemeris and Nautical Almanac, which is published yearly by the United States government. From innumerable observations, the exact nature of the motions of the sun, moon, and planets across the sky is ascertained, and the right ascensions and declinations of these bodies, as well as those of the fixed stars, are predicted for every night of the year, and published in the above work for the use of mariners and astronomers. For the determination of latitude, longitude, and time, and, in short, for any of the observations described in the following articles, this work is indispensable. It is divided into four parts, with an appendix and tables.

89. As all portions of the Ephemeris give the positions of the heavenly bodies at various Washington mean solar times, it is necessary, in order to make use of it, to know how to find the Washington time corresponding to the local time at any place of observation. This is done by adding, algebraically, to the local time, the longitude of the place, counted from Washington.

EXAMPLE.—When it is 8 P. M. in Paris, what is the Washington time, the longitude of Paris being $-5^h\ 18^m\ 36.75^s$?

SOLUTION.—

Local time	$8^h\ 0^m\ 0.00^s$
Longitude of Paris	$-5\ \ 18\ \ 36.75$
Corresponding Washington time . . .	$2^h\ 41^m\ 23.25^s$ P. M. Ans.

EXAMPLES FOR PRACTICE

1. The longitude of Cairo, Egypt, is $-7^h\ 13^m\ 24.69^s$. When it is 6 A. M., January 10, at Cairo, what is the time at Washington?

Ans. $10^h\ 46^m\ 35.31^s$ P. M., January 9

2. Find the Washington time of Philadelphia noon, the longitude of Philadelphia being $-7^m\ 37.27^s$. Ans. $11^h\ 52^m\ 22.73^s$ A. M.

3. Find the Washington time corresponding to 4 P. M. at Rome, the longitude of Rome being $-5^h\ 58^m\ 5.25^s$. Ans. $10^h\ 1^m\ 54.75^s$ A. M.

4. When it is 11 A. M. at San Francisco, what is the Washington time, the longitude of San Francisco being $+3^h\ 1^m\ 27.08^s$?

Ans. $2^h\ 1^m\ 27.08^s$ P. M.

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90. To Find the Right Ascension and Declination of a Star From the Ephemeris.—*First method* (approximate).—Look for the name of the star in the first column of Table V of this Section, or of the more complete table found in the Ephemeris, from which Table V is taken. The star's right ascension and declination for the beginning of the year will be found given opposite the name. For example, the right ascension and declination of Polaris are found to be $1^h\ 23^m\ 49.77^s$ and $+88°\ 47'\ 22.84''$, respectively.

This method is sufficiently accurate for nearly all observations made with the engineers' transit.

Second method.—As stated in Art. **30**, the stars are nearly fixed on the celestial sphere, and their right ascensions and declinations do not change materially from year to year. This is nearly the case, yet their positions undergo certain very minute changes, the amounts of which are indicated in the fourth and the sixth column of Table V. Thus, the right ascensions of most of the stars of the table increase about 3^s during the year, and the declinations increase about $20''$.

If it is desired to know the position of a star with great accuracy, the tables of the Ephemeris similar to Table VI, of this Section, but more complete, may be used. This table gives the positions of the stars for successive astronomical dates (Art. **78**). The times are Washington mean solar times; hence the following rule:

Rule.—*To find the right ascension and declination of a star for any given instant at any place, change the civil date to astronomical date, and the resulting local time to Washington*

time (Art. 89). Convert the resulting hours, minutes, and seconds into decimals of a day, and take from Table VI, or from the Ephemeris, the corresponding value of the right ascension and declination.

EXAMPLE.—To find the right ascension and declination of Polaris at 11 P. M., January 4, 1903, San Francisco mean time, the longitude of San Francisco being $3^h\ 1^m$ west of the Washington meridian.

SOLUTION.—Jan. 4, 11 P. M., is Jan. 4, $11^h\ 0^m$ astronomical time

Longitude of San Francisco . .	+ 3 1	(Art. 78)
Corresponding Washington time	Jan. 4, $14^h\ 1^m$ = Jan. 4.58	

From Table VI, the right ascension and declination of Polaris corresponding to January 4.58 are found to be,

Right ascension, $1^h\ 24^m\ 30.05^s + \frac{2.8}{100} \times (-1.01^s) = 1^h\ 24^m\ 29.77^s$

Declination, $+88°\ 47'\ 42.4'' + \frac{2.8}{100} \times (+0.1'') = +88°\ 47'\ 42.4''$

Ans.

EXAMPLE FOR PRACTICE

The longitude of Denver, Colo., is $+1^h\ 52^m$. Find the right ascensions and declinations of the following stars at the Denver mean times specified (see Art. 77): (*a*) of Polaris, at noon, January 4, 1903. (*b*) of Vega, at 9 P. M., February 1, 1903.

Ans. $\begin{cases} (a)\ 1^h\ 24^m\ 30.28^s;\ +88°\ 47'\ 42.4'' \\ (b)\ 18^h\ 33^m\ 38.35^s;\ +38°\ 41'\ 34.1'' \end{cases}$

91. The Solar Ephemeris.—This part of the Ephemeris, the first few lines of which are given as Table IV at the end of this Section, gives the right ascension and declination of the mean sun for every day in the year, at the instant of Washington mean noon, and also the hourly increase or decrease in the sun's right ascension and declination. It also contains the equation of time, the apparent angular semi-diameter of the sun, and the sidereal time of mean noon at Washington. All these quantities will be required in the calculations to be explained in this Section.

92. To Find the Right Ascension and Declination of the Sun at Any Instant.

Rule.—*Change the local time to Washington time (Art. 89). Take from the table the right ascension and declination corresponding to the preceding Washington mean noon, and add algebraically to these quantities the product of the corresponding*

hourly motions by the number of hours elapsed since Washington mean noon.

EXAMPLE 1.—What is the right ascension and declination of the sun at 9 A. M., January 3, 1903, Ann Arbor mean time, the longitude of Ann Arbor being $+26^m\ 39.41^s$?

SOLUTION.—

Astronomical local time	Jan. 2, $21^h\ 0^m\ 0.00^s$
Longitude of Ann Arbor	$+26\ \ 39.41$
Washington mean time	Jan. 2, $21^h\ 26^m\ 39.41^s$ = Jan. 2, 21.444^h

Right ascension of sun at Washington mean noon, January 2	$18^h\ 48^m\ 30.52^s$
Increase of right ascension during 21.444 hr, = hourly motion (11.037^s) × 21.444 . . .	$+3\ \ 56.68$
Hence, the desired right ascension	$18^h\ 52^m\ 27.20^s$ Ans.
Declination of sun at Washington mean noon, January 2	$-22°\ 58'\ 51.00''$
Increase of declination during 21.444 hr. = hourly motion (12.78″) × 21.444	$+4\ \ 34.05$
Hence, the desired declination is	$-22°\ 54'\ 16.95''$ Ans.

EXAMPLE 2.—What is the right ascension and declination of the sun when it has an hour angle of 1 hour at an observatory near Philadelphia, January 5, 1903, the longitude of this observatory being $-7^m\ 37.27^s$?

SOLUTION.—The local time is 1^h after apparent noon (Art. 72).

Local time after apparent noon . . .	$1^h\ 0^m\ 0.00^s$
Longitude of Philadelphia	$-7\ \ 37.27$
Washington time after apparent noon	$0^h\ 52^m\ 22.73^s$ = $.873^h$

Right ascension of sun for Washington apparent noon, January 5 (Table IV)	$19^h\ 1^m\ 44.41^s$
Increase of the right ascension = $+10.987^s$ × .873	$+0\ \ 9.59$
Hence, the desired right ascension	$19^h\ 1^m\ 54.00^s$ Ans.
Declination of sun for Washington apparent noon, January 5 (Table IV)	$-22°\ 41'\ 26.5''$
Increase of the declination, +16.18″ × 0.873	$+14.1$
Hence, the desired declination	$-22°\ 41'\ 12.4''$ Ans.

Example 2 may also be solved by changing the apparent time into mean time (Art. 80), and proceeding as in example 1. Precisely the same results will be obtained, but the solution will involve an unnecessary amount of labor.

EXAMPLE FOR PRACTICE

The longitude of Chicago is $+42^m\ 11^s$. Find the right ascension and declination of the sun at the following Chicago times: (*a*) January 2, $11^h\ 18^m\ 49^s$ A. M. (*b*) January 5, apparent noon.

Ans. $\begin{cases} (a)\ 18^h\ 48^m\ 30.52^s;\ -22°\ 58'\ 51.0'' \\ (b)\ 19^h\ 1^m\ 52.13^s;\ -22°\ 41'\ 15.1'' \end{cases}$

93. To Find the Equation of Time at Any Instant. This may be taken directly from Table IV, after reducing local time to Washington time.

Rule.—*Find the difference between the values of the equation of time corresponding to the preceding and the following Washington noon, and multiply it by the decimal part of a day elapsed since Washington noon; add the result algebraically to the equation of time corresponding to the preceding Washington noon.*

The difference between two successive values of the equation of time is the daily change; it is additive if these values are increasing, and subtractive if they are decreasing.

EXAMPLE.—To find the value of the equation of time at 9 A. M., January 3, Ann Arbor mean time, the longitude of Ann Arbor being $+26^m\ 39.41^s$.

SOLUTION.—

Ann Arbor local time	Jan. 2, $21^h\ 0^m\ 0.00^s$
Longitude of Ann Arbor	$+26\ \ 39.41$
Washington mean time	Jan. 2, $21^h\ 26^m\ 39.41^s$
	Jan. 2, $+0.894$ da.
Equation of time for noon, Jan. 2 . .	$+3^m\ 52.48^s$
Equation of time for noon, Jan. 3 . .	$+4\ \ 20.63$
Change in equation of time for 1 da.	$+28.15^s$
Total change since Washington noon	
$= +28.15^s \times 0.894$	$+25.17^s$
	$+3\ \ 52.48$
Hence, the desired equation of time	$+4^m\ 17.65^s$
	Ans.

EXAMPLES FOR PRACTICE

The longitude of Cincinnati is $+29^m\ 26^s$. Find the values of the equation of time at the following Cincinnati mean times:

(*a*) January 2, 1903, 9 A. M.
(*b*) January 5, noon.
(*c*) January 4, 6 P. M.

Ans. (*a*) $+3^m\ 49.49^s$; (*b*) $+5^m\ 16.29^s$; (*c*) $+4^m\ 55.78^s$

94. To Find the Angular Semi-Diameter of the Sun.—This is the angle subtended at the eye of the observer by the sun's radius, and may be taken from the table in exactly the same manner as the equation of time.

Rule.—*Take the value corresponding to the preceding Washington noon and add to it the product of the daily change by the decimal part of a day elapsed since that Washington noon.*

EXAMPLE.—To find the semi-diameter of the sun at 4 P. M., January 1, 1903, local mean time, the longitude of the observer being $-7^m\ 37.27^s$.

SOLUTION.—

Local mean time	Jan. 1, $4^h\ 0^m\ 0.00^s$
Longitude	$-7\ 37.27$
Washington mean time	Jan. 1, $3^h\ 52^m\ 22.73^s$
	Jan. 1, +0.161 da.
Semi-diameter, January 1	16′ 17.81″
Semi-diameter, January 2	16′ 17.83″
Daily change	−.02
Increase of semi-diameter since Washington noon = (−.02″) × 0.161	−.00″
	16 17.81
Hence, the desired semi-diameter is	16′ 17.81″

The method of finding the sidereal time of mean noon at any place of observation was explained in Art. 86.

NOTE.—For the purpose of ordinary calculations, the semi-diameter of the sun may be taken from the following short table:

Time of year (approx.) . . .	Jan. 1,	Apr. 1,	July 1,	Oct. 1
Sun's semi-diameter	16′ 18″	16′ 2″	15′ 45″	16′ 2″

EXAMPLES FOR PRACTICE

The longitude of San Francisco is $+3^h\ 1^m\ 27^s$. Find the semidiameter of the sun at the following San Francisco mean times:

(*a*) January 1, 10 P. M.
(*b*) January 5, 6 A. M.
(*c*) January 2, 8 A. M.

Ans. (*a*) 16′ 17.82″; (*b*) 16′ 17.81″; (*c*) 16′ 17.83″

DETERMINATION OF ALTITUDE

95. The **altitude** of a heavenly body is the angle between the plane of the observer's horizon and a line from the observer's station to that body. The instruments most frequently employed for measuring this angle are the sextant and the engineers' transit.

USE OF THE SEXTANT

NOTE.—For a description of the sextant, its theory, adjustments, and uses, see *Hydrographic Surveying*.

96. The Artificial Horizon.—For observing altitudes on land, an artificial horizon similar to the one shown in Fig. 10 must be used. This is a shallow basin cc' about 3 by 5 inches for holding mercury. It is provided with a

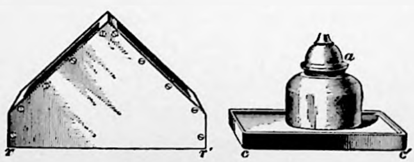

FIG. 10

roof rr' formed of two pieces of plate glass set at right angles to each other in a metal frame, for protecting the mercury from agitation by the wind. The surface of the mercury forms a mirror from which the image of the sun or star is reflected; and as it is perfectly horizontal, the

reflected image will appear at an angular distance below the horizon equal to the altitude of the body itself above the horizon.

If the image of a star reflected from the mirrors of the sextant is brought into contact with the image reflected from the mercury, the angle that will be measured is twice the altitude of the star. This is explained by means of

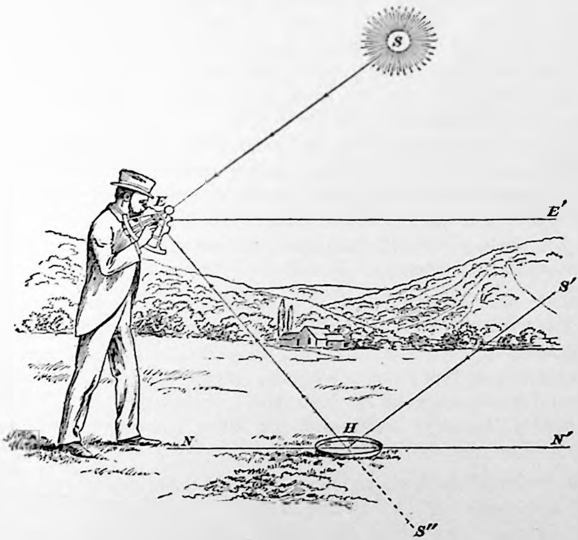

FIG. 11

Fig. 11, where either SEE' or $S'HN'$ may be considered as the altitude of the body, EE' and NN' being horizontal lines, and $S'H$ being the line from H to the star, which line may be considered to be parallel to ES, on account of the remoteness of the star. Now, $S'HS'' = SES''$ is the angle measured, since the object seen at H will appear to be in the direction EHS''. But, from the physical law of

reflection, $S' H N' = E H N = N' H S''$. Hence, the angle measured $S' H S''$ equals twice the altitude $S' H N'$.

In connection with the use of an artificial horizon, it is well to bear in mind that, before any observations are made, the mercury must be freed from the particles of dust and impurities that will generally be found floating on its surface. A good method is to strain the mercury through a piece of chamois skin or through a funnel of paper brought down to a fine point at the end. Another method is to add a small amount of tin-foil to the mercury, when the amalgam that will be formed will rise to the top and may be drawn to one side with a card, leaving the surface entirely free from specks of any kind.

When observing, it will be found much more convenient to sit on a box or a very low chair, or even on the ground, so as to bring the telescope as near to the mercury as possible. An artificial horizon may be formed by pouring oil or syrup into a shallow vessel; but when mercury is available, it is always to be preferred.

97. The Index Error.—The index error should always be determined both before and after any observations with the sextant. At least four pointings should be made, preferably on a fixed star, and the mean of these readings should be taken (see *Hydrographic Surveying*).

When the sun is observed, the index correction must be determined as follows:

Rule.—*Measure the apparent diameter of the sun by bringing the direct and reflected images tangent to each other, and read the vernier; then bring the opposite limbs into the position of tangency and again read the vernier. Prefix to the readings their proper signs, as explained in Hydrographic Surveying, add them together algebraically, and divide the sum by 2. The result will be the index correction.*

More than one pointing is usually made on each limb, and the mean taken.

EXAMPLE.—To determine the index error from the following observations, which were made on the sun in the manner described:

MINUS READINGS	PLUS READINGS
32′ 20″	30′ 60″
32′ 20″	30′ 60″
32′ 25″	30′ 50″
32′ 20″	30′ 50″
129′ 25″	123′ 40″

SOLUTION.—The mean of each series is found by dividing the sum of the four single readings by 4. We thus find:

Mean of negative readings	−32′ 21.2″
Mean of positive readings	+30′ 55.0″
Algebraic sum	−1′ 26.2″
Index correction, ½ (algebraic sum)	−0′ 43.1″

USE OF THE ENGINEERS' TRANSIT

98. Essential Conditions.—The engineers' transit is fully described in *Transit Surveying*, Part 1. If the instrument is provided with a vertical circle, it may be used either with or without an artificial horizon for directly measuring altitudes. The bubbles on the plate must be very carefully adjusted; especially the bubble parallel to the vertical circle, for it is on this bubble that the accuracy of the result wholly depends. The telescope bubble must also be in accurate adjustment, parallel to the line of sight.

99. The Vertical Circle.—The circle should read zero when the line of sight is horizontal, if there is no index error. The reading, if any, of the vertical circle when the plate is leveled and the telescope bubble is brought to the middle of its tube, is the index error. If the vernier on the vertical circle is adjustable, the reading on the limb may be made zero when the line of sight is horizontal, and then no correction for index error will be required.

The adjustment of the telescope bubble, of the plate bubbles, and of the vernier of the vertical circle should always be tested just before and just after any series of observations with the transit. If the plate bubble parallel to the vertical circle, the telescope bubble, and the vernier of the vertical circle have been accurately adjusted, then, whenever these bubbles are brought to the centers of their tubes, the reading on the vertical circle should either be zero or the index

correction previously determined, according as the vernier is or is not adjustable. This test can always be very quickly applied.

In case the vertical limb is a complete circle, the error of adjustment of the plate bubbles and of the vernier can be eliminated by first reading the altitude, then revolving the instrument in azimuth 180°, releveling the instrument (but without readjusting any of the levels), reading the vertical angle again with the telescope in its reversed position, and taking the mean of the two readings. Every altitude should be measured in this way when the vertical circle is complete; no correction for index error is then required.

100. To Measure an Altitude by Using the Artificial Horizon.—When the vertical limb is not a complete circle, the index error can be eliminated by using an artificial or mercury horizon. This must first be put in the proper position with reference to the transit, since, unlike the sextant, the transit cannot be conveniently moved about. To accomplish this, the transit is set up and pointed on the star, and the altitude is read from the vertical circle. The telescope is then depressed through an angle equal to twice the approximate altitude, and sighting through it, the point on the ground is noted at which the intersection of the cross-wires seems to fall. The mercury horizon should be set at that point.

The telescope is then directed to the star whose altitude is required, and the vertical circle is read. A second reading is taken by directing the telescope toward the image reflected in the mercury. One-half of the total angle turned through, or of the sum of the two readings, will be the altitude of the star (Art. 96).

No correction for index error need be applied, since the use of the mercury horizon eliminates the index error. This is the only simple method by which this error can be eliminated when the vertical limb is not a complete circle.

101. Observation of Stars With a Transit.—When a transit is used for observing the stars, it is necessary to

illuminate the cross-wires. The simplest method of doing this is by holding a bull's-eye lantern so as to throw the light down the telescope tube through the objective, care being taken not to obstruct the line of sight. A little practice will enable one to do this very easily: the lantern is held in front and a little to one side of the object end of the telescope with the left hand, and the instrument is manipulated with the right. It is more convenient, however, to have some kind of reflector fitted to the object end of the telescope, so that the lantern may be turned from the eyes of the observer rather than toward them.

A very good reflector may be made from a piece of new tin, cut and bent as in Fig. 12. The straight strip is bent about the object end of the telescope tube, leaving the annular elliptic piece projecting over in front. This is then bent to any desired angle, preferably about 45°, and turned so that the light can be thrown down the tube by illuminating the disk from a convenient position. If the reflecting side of the disk is whitened, the effect is very good. The opening should be about $\frac{3}{4}$ inch or $\frac{7}{8}$ inch in its shorter diameter, the longer diameter being such as to make its normal projection equal to the shorter one. There is, of course, no necessity for limiting the outer edges of the disk.

FIG. 12

CORRECTIONS TO THE MEASURED ALTITUDE

102. It has been stated that to every reading from the circle of either the sextant or the transit instrument, the index correction must be added. This is a purely instrumental error. The apparent altitude of a heavenly body is also always affected by the refraction of the rays of light from the body in passing through the atmosphere. In case the sun or moon is observed, further corrections for semi-diameter and parallax must be applied; while the altitudes of all bodies observed at sea must be corrected for the dip of the horizon. The methods of computing and applying these corrections will now be explained.

REFRACTION

103. A ray of light travels in a straight line so long as its path is in a medium of uniform density; but when it passes obliquely from one medium into another of different density, it undergoes a change of direction at the surface of separation. This change of direction or bending of a ray of light is called **refraction.** Thus, if EE', Fig. 13, represents the surface of the earth on which an observer stands at O, and if the surface EE' is surmounted by an atmosphere $EMNE'$ of uniform density and of a definite height, a ray of light from any celestial body S will be bent downwards on reaching the upper surface MN of the atmosphere. The ray will move along the broken line SAO instead of traveling in a straight path.

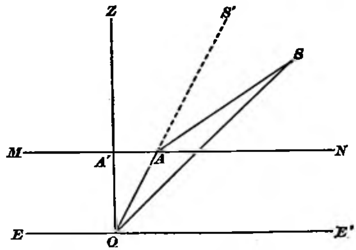

FIG. 13

The apparent position of a body depends on the direction in which its light enters the observer's eye; hence, the celestial body S will appear to be at S' instead of in its true position S. The difference between the direction OS in which the body would be were there no refraction, and the direction OS' in which it appears to be, is the astronomical refraction. It is thus seen that the effect of refraction is to increase the altitude of all heavenly bodies. The altitude SOE' is called the **true altitude**; the altitude $S'OE'$ is called the **apparent altitude.** To find the true altitude when the apparent altitude has been measured, the amount of the refraction SOS' must be subtracted from the apparent altitude $S'OE'$.

This explanation assumes the space above MN in the figure to be entirely empty, and the earth's atmosphere $MNE'E$ to be equally dense throughout. In fact, however, the earth's atmosphere is most dense at the surface of the

earth, and gradually diminishes in density to its exterior boundary. The direction of a ray of light traveling in such medium is constantly changing, and so the path of the ray is a curved line. This curve is represented by line $e\,d\,c\,b\,a\,A$, Fig. 14. The ray of light from the star S first meets the upper surface of the atmosphere at e and is then successively refracted as it passes into layers of greater and greater density, until it finally enters the eye of the observer at A in the direction of $A\,a\,S'$, making S' the apparent position of the star.

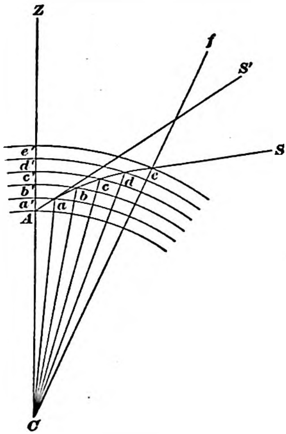

Fig. 14

104. The exact determination of the amount of refraction corresponding to any altitude is a problem that cannot be completely solved, since the refraction depends on the temperature and density of the air, not only at the surface of the earth, but also along the whole path of the ray $A\,a\,b\,c\,d\,e$, Fig. 14. Approximate tables may be constructed, however, that will give the amount of the correction within 1 second or less, provided that the body is not too near the horizon. When the altitude of a body is small, the rays of light from it pass nearly along the earth's surface through many hundred miles of comparatively dense air and the refraction cannot be accurately determined by any method. Hence, when the measured altitude of any body is less than 8° or 10°, *the refraction becomes so uncertain that the measurement is of no value for any kind of accurate work.*

Table VII, at the end of this Section, gives the amount of refraction corresponding to different altitudes. It is founded on the investigations of a celebrated German

astronomer named Bessel, and is known as *Bessel's table of refractions.*

105. Correcting a Measured Altitude for Refraction.

Rule.—*Look in Table VII for the refraction corresponding to the measured altitude, and subtract this correction from that altitude.*

EXAMPLE.—The altitude of Sirius was observed to be 18° 04′ 10″. It is required to correct this altitude for refraction.

SOLUTION.—

Observed altitude	18° 04′ 10″
Refraction	−02 53
Corrected or true altitude	18° 01′ 17″
	Ans.

EXAMPLES FOR PRACTICE

Correct the following measured altitudes for refraction:

(*a*) 39° 48′ 10″.		(*a*) 39° 47′ 02″
(*b*) 15° 10′ 20″.	Ans.	(*b*) 15° 06′ 52″
(*c*) 22° 11′ 05″		(*c*) 22° 8′ 46″

PARALLAX

106. Geocentric Place.—The positions of all heavenly bodies are referred to the center of the earth. The **geocentric place** of a heavenly body is the position on the celestial sphere that it appears to occupy when viewed from the center of the earth. The right ascensions and declinations of the geocentric places of the sun and moon are published in the American Ephemeris.

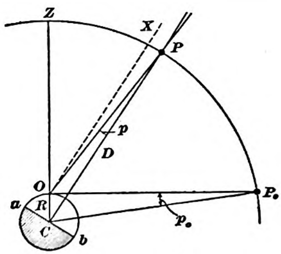

FIG. 15

107. The **parallax** of a heavenly body is the difference in direction of the body as actually observed and the direction that it would have if seen from the earth's center. Thus, in Fig. 15, where the observer is supposed to be at *O*.

the position of P in the sky (as seen from O) would be marked by the point where OP produced would pierce the celestial sphere. Its position as seen from C would be determined by producing CP. The angle POX formed by OP and a line parallel to CP is the parallax of P for an observer at O.

Since the angle POX equals the angle OPC, the parallax may also be defined as the angular distance between the observer's station and the center of the earth, as seen from the body observed.

108. Correction of Altitude and Zenith Distance for Parallax.—Let A, Fig. 16, be the position of the observer; C, the center of the earth; AH, the horizon; CH', a parallel to AH; Z, the zenith; and M, the position of a heavenly body. Then the angle z_a is the apparent or observed zenith distance of M, while z is its geocentric zenith distance. Likewise, h_a and h are, respectively, the apparent and the geocentric altitude of the body, and p is its parallax. The triangle CAM gives

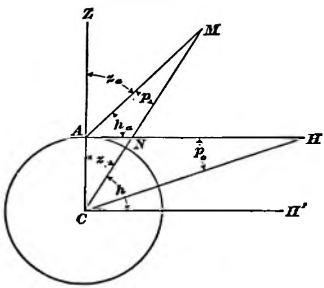

FIG. 16

$$z = z_a - p \qquad (1)$$

That is, the geocentric zenith distance is equal to the observed zenith distance minus the parallax.

The angle MNH, external to the triangle MNA, is equal to h, since NH is parallel to CH'. Therefore,

$$h = h_a + p \qquad (2)$$

That is, the geocentric altitude is equal to the observed altitude plus the parallax.

In the case of the stars, no correction of any kind need be applied for parallax, as the geocentric parallax of the nearest star is less than one-millionth of a second.

109. Formula for Parallax.—Referring to Fig. 16, let the distance CM of the body from the center of the earth be denoted by d, and the radius CA by r. In the triangle CAM, the sides CA and CM, or r and d, are to each other as the sines of the opposite angles p and CAM; but instead of sin CAM, the sine of its supplement z_a may be used. Therefore, $\frac{r}{d} = \frac{\sin p}{\sin z_a}$, and

$$\sin p = \frac{r}{d} \sin z_a \qquad (1)$$

Since $z_a = 90° - h_a$, we have, also, $\sin z_a = \cos h_a$, and, therefore,

$$\sin p = \frac{r}{d} \cos h_a \qquad (2)$$

110. The **horizontal parallax** of a body is the geocentric parallax of the body when the latter is in the horizon. Thus, in Fig. 16, the angle AHC, or p_o, is the horizontal parallax of the body H. The horizontal parallax of the sun may be found tabulated for every tenth day of the year, in the American Ephemeris.

111. Formula for Horizontal Parallax.—When a body is in the horizon, its apparent zenith distance z_a is 90°, and therefore $\sin z_a = 1$. Writing in formula **1**, Art. **109**, p_o for p and 1 for $\sin z_a$, the following formula for horizontal parallax is obtained:

$$\sin p_o = \frac{r}{d}$$

112. Parallax of the Sun.—Since $\frac{r}{d} = \sin p_o$, formula **2**, Art. **109**, may be written

$$\sin p = \sin p_o \cos h_a \qquad (1)$$

In the case of the sun, p and p_o are so small that the angles themselves may be used instead of their sines. By making this substitution, formula **1** becomes

$$p = p_o \cos h_a \qquad (2)$$

Table VIII, at the end of this Section, makes the application of formula **2** unnecessary.

EXAMPLE.—The altitude of the sun on May 14, 1903, was observed to be 24° 18′ 20″. It is required to correct this altitude for parallax.

SOLUTION.—(1) *By the formula.* From the Ephemeris, the horizontal parallax on May 14 is found to be +8.70″. Hence,

Parallax = 8.70 cos 24° 18′ = 8.70″ × 0.912 .	+7.93″
Observed altitude	24 18 20.00
Altitude corrected for parallax	24° 18′ 27.93″

(2) *By the table.*

Observed altitude	24° 18′ 20″
Correction for parallax from Table VIII	+8
Corrected altitude	24° 18′ 28″

The table will always be sufficiently accurate to correct altitudes that are measured with the transit or sextant.

EXAMPLES FOR PRACTICE

Correct the following measured altitudes of the sun for parallax:

(*a*) 25° 10′ 0″.	Ans.	(*a*) 25° 10′ 08″
(*b*) 60° 09′ 10″.		(*b*) 60° 09′ 14″
(*c*) 10° 10′ 5″.		(*c*) 10° 10′ 14″
(*d*) 5° 09′ 10″.		(*d*) 5° 09′ 19″

CORRECTION FOR SEMI-DIAMETER

113. Whenever the altitude of the sun or moon is observed with the sextant, it is either the highest or the lowest point of the disk of the body that is brought into coincidence with its reflection from the mercury. Similarly, when the observations are taken with the transit, the horizontal wire is placed tangent to the upper or lower edge of the apparent disk. Thus, as a result of the observation, the altitude of either the upper or the lower edge is obtained. The observed altitude must be corrected by adding to or subtracting from it the angular semi-diameter of the observed body, according as the lower or the upper edge has been observed (see Art. **94**).

CORRECTION (AT SEA) FOR THE DIP OF THE HORIZON

114. In observations of altitude at sea, where the measurement is made from the sea horizon, a correction is needed on account of the fact that this visible horizon does not coincide with the true astronomical horizon, but falls sensibly below it by an amount known as the **dip of the horizon.** The amount of this dip depends on the height of the observer's eye above the sea level.

In Fig. 17, C is the center of the earth, and O the observer, at an elevation h above the earth's (or ocean's) surface at A. The line OH is horizontal, while the tangent line OB corresponds to the sea horizon. The angle D is the *dip.*

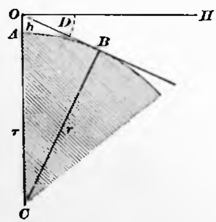

FIG. 17

Table IX, at the end of this Section, gives the correction for dip corresponding to various heights of the observer's eye above the surface of the ocean. These numbers include correction for refraction. The number taken from the table is to be subtracted from the measured altitude. If a table is not available, the correction for dip and refraction may be obtained from the formula

$$D'' = 59\sqrt{h}$$

where D'' = dip (refraction included), in seconds, and
h = height, in feet, of observer's eye above surface of sea.

EXAMPLE.—The altitude of Sirius above the visible sea horizon was observed to be 38° 40′ 10″. The measurement was made from the bridge of a steamer, the eye of the observer being approximately 34 feet above the water level. It is required to correct the altitude for dip.

SOLUTION.—

Measured altitude	38° 40′ 10″
From Table IX, correction for dip corresponding to 34 feet	−5 43
Corrected altitude	38° 34′ 27″

EXAMPLES FOR PRACTICE

1. The altitude of the lower edge of the sun was observed on January 5, 1903; the eye of the observer was 10 feet above the water line; the measured altitude was 20° 10′ 10″. Correct this altitude for semi-diameter and for dip. Ans. 20° 23′ 23″

2. Correct an altitude of 10° measured at a height of 10 feet, for dip. Ans. 9° 56′ 55″

3. Correct an altitude of 20°, measured at a height of 20 feet, for semi-diameter and for dip, the measurement being made on the upper edge of the sun on January 1, 1903. Ans. 19° 39′ 19″

ILLUSTRATIVE EXAMPLES OF THE DETERMINATION OF ALTITUDES

EXAMPLE 1.—At San Francisco, at 11 o'clock A. M. January 4, 1903, the altitude of the upper edge of the sun was observed with a sextant, using an artificial horizon; the circle reading was 46° 20′ 10″. At the same time, readings were taken for index error as follows:

MINUS READINGS	PLUS READINGS
31′ 0″	32′ 10″
31′ 20″	32′ 20″
31′ 10″	32′ 30″

What was the true altitude of the sun's center?

SOLUTION.—The first correction is that for index error.

Mean of minus readings	−31′ 10″
Mean of plus readings	+32 20
Algebraic sum	+1′ 10″
Index correction, ½ (algebraic sum)	+35
Circle reading = double altitude	46 20 10
Corrected double altitude	46° 20′ 45″
Altitude corrected for index error	23 10 22.5
Correction for refraction (Table VII)	−2 13
Correction for parallax (Table VIII)	+8
Final corrected altitude of sun's upper edge	23° 8′ 17.5″

There remains the correction for semi-diameter. We first find the semi-diameter (Art. 94).

Civil local time is January 4, 11 A. M.; astronomical time, Jan. 3	23h 0m 0.0s
Longitude of San Francisco	+3 1 27.1
Washington mean time, Jan. 4	2h 1m 27.1s
	or Jan. 4, .0843 da.

Hence, from Table IV, semi-diameter for January 4, .0843 da. = 16′ 17.8″.

Corrected altitude of sun's upper edge	23° 8′ 17.5″
Correction for semi-diameter	−16 17.8
Final corrected altitude of sun's center	22° 51′ 59.7″ Ans.

EXAMPLE 2.—The altitude of Vega was observed with a sextant and an artificial horizon; the circle reading was found to be +30° 28′ 40″. Immediately afterwards, readings were made on a star for index error as follows: Minus readings, 40″, 20″, 30″, 30″. To find the true altitude of Vega.

SOLUTION.—When a star is observed, no correction for semi-diameter or parallax need be applied; it is only necessary to correct the observation for index error and for refraction.

Observed double altitude	30° 28′ 40″
Correction for index error = mean of above four readings	−30
Corrected double altitude	30° 28′ 10″
Single altitude	15° 14′ 5″
Correction for refraction (Table VII)	−3 27
Final corrected altitude of star	15° 10′ 38″

EXAMPLES FOR PRACTICE

1. The altitude of a star was observed with a transit having an incomplete vertical circle; the correction for index error was −1′; the circle reading was 30° 10′ 30″. Find the true altitude.

Ans. 30° 7′ 52″

2. The altitude of a star was measured with a transit as explained in Art. 99; the measured altitude was 60° 12′ 10″. Find the true altitude. Ans. 60° 11′ 37″

TABLE I

CONVERSION OF ARC AND TIME

D.	H.M.	D.	H.M.	D.	H.M.	D.	H.M.	D.	H.M.	D.	H.M.
M.	M.S.	M.	M.S.	M.	M.S.	M.	M.S.	M.	M.S.	M.	M.S.
1	0 4	61	4 4	121	8 4	181	12 4	241	16 4	301	20 4
2	0 8	62	4 8	122	8 8	182	12 8	242	16 8	302	20 8
3	0 12	63	4 12	123	8 12	183	12 12	243	16 12	303	20 12
4	0 16	64	4 16	124	8 16	184	12 16	244	16 16	304	20 16
5	0 20	65	4 20	125	8 20	185	12 20	245	16 20	305	20 20
6	0 24	66	4 24	126	8 24	186	12 24	246	16 24	306	20 24
7	0 28	67	4 28	127	8 28	187	12 28	247	16 28	307	20 28
8	0 32	68	4 32	128	8 32	188	12 32	248	16 32	308	20 32
9	0 36	69	4 36	129	8 36	189	12 36	249	16 36	309	20 36
10	0 40	70	4 40	130	8 40	190	12 40	250	16 40	310	20 40
11	0 44	71	4 44	131	8 44	191	12 44	251	16 44	311	20 44
12	0 48	72	4 48	132	8 48	192	12 48	252	16 48	312	20 48
13	0 52	73	4 52	133	8 52	193	12 52	253	16 52	313	20 52
14	0 56	74	4 56	134	8 56	194	12 56	254	16 56	314	20 56
15	1 0	75	5 0	135	9 0	195	13 0	255	17 0	315	21 0
16	1 4	76	5 4	136	9 4	196	13 4	256	17 4	316	21 4
17	1 8	77	5 8	137	9 8	197	13 8	257	17 8	317	21 8
18	1 12	78	5 12	138	9 12	198	13 12	258	17 12	318	21 12
19	1 16	79	5 16	139	9 16	199	13 16	259	17 16	319	21 16
20	1 20	80	5 20	140	9 20	200	13 20	260	17 20	320	21 20
21	1 24	81	5 24	141	9 24	201	13 24	261	17 24	321	21 24
22	1 28	82	5 28	142	9 28	202	13 28	262	17 28	322	21 28
23	1 32	83	5 32	143	9 32	203	13 32	263	17 32	323	21 32
24	1 36	84	5 36	144	9 36	204	13 36	264	17 36	324	21 36
25	1 40	85	5 40	145	9 40	205	13 40	265	17 40	325	21 40
26	1 44	86	5 44	146	9 44	206	13 44	266	17 44	326	21 44
27	1 48	87	5 48	147	9 48	207	13 48	267	17 48	327	21 48
28	1 52	88	5 52	148	9 52	208	13 52	268	17 52	328	21 52
29	1 56	89	5 56	149	9 56	209	13 56	269	17 56	329	21 56
30	2 0	90	6 0	150	10 0	210	14 0	270	18 0	330	22 0
31	2 4	91	6 4	151	10 4	211	14 4	271	18 4	331	22 4
32	2 8	92	6 8	152	10 8	212	14 8	272	18 8	332	22 8
33	2 12	93	6 12	153	10 12	213	14 12	273	18 12	333	22 12
34	2 16	94	6 16	154	10 16	214	14 16	274	18 16	334	22 16
35	2 20	95	6 20	155	10 20	215	14 20	275	18 20	335	22 20
36	2 24	96	6 24	156	10 24	216	14 24	276	18 24	336	22 24
37	2 28	97	6 28	157	10 28	217	14 28	277	18 28	337	22 28
38	2 32	98	6 32	158	10 32	218	14 32	278	18 32	338	22 32
39	2 36	99	6 36	159	10 36	219	14 36	279	18 36	339	22 36
40	2 40	100	6 40	160	10 40	220	14 40	280	18 40	340	22 40
41	2 44	101	6 44	161	10 44	221	14 44	281	18 44	341	22 44
42	2 48	102	6 48	162	10 48	222	14 48	282	18 48	342	22 48
43	2 52	103	6 52	163	10 52	223	14 52	283	18 52	343	22 52
44	2 56	104	6 56	164	10 56	224	14 56	284	18 56	344	22 56
45	3 0	105	7 0	165	11 0	225	15 0	285	19 0	345	23 0
46	3 4	106	7 4	166	11 4	226	15 4	286	19 4	346	23 4
47	3 8	107	7 8	167	11 8	227	15 8	287	19 8	347	23 8
48	3 12	108	7 12	168	11 12	228	15 12	288	19 12	348	23 12
49	3 16	109	7 16	169	11 16	229	15 16	289	19 16	349	23 16
50	3 20	110	7 20	170	11 20	230	15 20	290	19 20	350	23 20
51	3 24	111	7 24	171	11 24	231	15 24	291	19 24	351	23 24
52	3 28	112	7 28	172	11 28	232	15 28	292	19 28	352	23 28
53	3 32	113	7 32	173	11 32	233	15 32	293	19 32	353	23 32
54	3 36	114	7 36	174	11 36	234	15 36	294	19 36	354	23 36
55	3 40	115	7 40	175	11 40	235	15 40	295	19 40	355	23 40
56	3 44	116	7 44	176	11 44	236	15 44	296	19 44	356	23 44
57	3 48	117	7 48	177	11 48	237	15 48	297	19 48	357	23 48
58	3 52	118	7 52	178	11 52	238	15 52	298	19 52	358	23 52
59	3 56	119	7 56	179	11 56	239	15 56	299	19 56	359	23 56
60	4 0	120	8 0	180	12 0	240	16 0	300	20 0	360	24 0

TABLE II

PART OF TABLE III OF THE AMERICAN NAUTICAL ALMANAC, FOR CHANGING MEAN SOLAR INTO SIDEREAL TIME

(To be added to mean solar interval)

Mean Solar	0^h	1^h	2^h	For Seconds	
m	m s	m s	m s	s	s
0	0 0.000	0 9.856	0 19.713	0	0.000
1	0 0.164	0 10.021	0 19.877	1	0.003
2	0 0.329	0 10.185	0 20.041	2	0.005
3	0 0.493	0 10.349	0 20.206	3	0.008
4	0 0.657	0 10.514	0 20.370	4	0.011
5	0 0.821	0 10.678	0 20.534	5	0.014
6	0 0.986	0 10.842	0 20.699	6	0.016
7	0 1.150	0 11.006	0 20.863	7	0.019
8	0 1.314	0 11.171	0 21.027	8	0.022
9	0 1.478	0 11.335	0 21.191	9	0.025
10	0 1.643	0 11.499	0 21.356	10	0.027

TABLE III

PART OF TABLE II OF THE AMERICAN NAUTICAL ALMANAC, FOR CHANGING SIDEREAL INTO MEAN SOLAR TIME

(To be subtracted from sideral interval)

Side-real	0^h	1^h	2^h	For Seconds	
m	m s	m s	m s	s	s
0	0 0.000	0 9.830	0 19.659	0	0.000
1	0 0.164	0 9.993	0 19.823	1	0.003
2	0 0.328	0 10.157	0 19.987	2	0.005
3	0 0.491	0 10.321	0 20.151	3	0.008
4	0 0.655	0 10.485	0 20.314	4	0.011
5	0 0.819	0 10.649	0 20.478	5	0.014
6	0 0.983	0 10.813	0 20.642	6	0.016
7	0 1.147	0 10.976	0 20.806	7	0.019
8	0 1.311	0 11.140	0 20.970	8	0.022
9	0 1.474	0 11.304	0 21.134	9	0.025
10	0 1.638	0 11.468	0 21.297	10	0.027

TABLE IV

THE FIRST FEW LINES OF THE SOLAR EPHEMERIS TABLES, WHICH COMPRISE PAGES 400 TO 407 OF THE AMERICAN NAUTICAL ALMANAC

SOLAR EPHEMERIS, 1903. FOR WASHINGTON MEAN AND APPARENT NOON

Date	Apparent Right Ascension		Apparent Declination		Hourly Motion		Equation of Time for Apparent Noon	Semi-Diameter at Apparent Noon	Sidereal Time of Semi-Diameter Passing Meridian	Sidereal Time of Mean Noon
	Mean Noon	App. Noon	Mean Noon	App. Noon	Right Ascension	Declination				
	h m s	s	° ′ ″	″	s	″	m s	′ ″	m s	h m s
Jan. 1	18 44 05.48	6.11	−23 03 44.2	43.6	11.051	+11.64	+3 23.98	16 17.81	1 11.06	18 40 41.57
2	18 48 30.52	31.23	22 58 51.0	50.2	11.037	12.78	3 52.48	16 17.83	1 11.02	18 44 38.12
3	18 52 55.22	56.02	22 53 30.6	29.5	11.022	13.92	4 20.63	16 17.83	1 10.98	18 48 34.68
4	18 57 19.54	20.42	22 47 42.8	41.6	11.005	15.06	4 48.39	16 17.82	1 10.93	18 52 31.24
5	19 01 43.45	44.41	22 41 27.9	26.5	10.987	16.18	5 15.75	16 17.81	1 10.87	18 56 27.80
6	19 06 06.91	7.95	−22 34 46.0	44.4	10.968	+17.30	+5 42.67	16 17.81	1 10.81	19 00 24.35
7	19 10 29.90	31.02	22 27 37.4	35.5	10.948	18.41	6 09.11	16 17.80	1 10.75	19 04 20.91
8	19 14 52.39	53.59	22 20 02.2	0.0	10.926	19.51	6 35.05	16 17.79	1 10.69	19 08 17.47
9	19 19 14.36	15.63	22 11 60.7	58.3	10.903	20.60	7 00.46	16 17.76	1 10.62	19 12 14.02
10	19 23 35.77	37.12	22 03 33.1	30.5	10.880	21.68	7 25.32	16 17.72	1 10.54	19 16 10.58

TABLE V

PART OF THE TABLE OF 383 FIXED STARS, WHICH COMPRISES PAGES 304 TO 311 OF THE AMERICAN NAUTICAL ALMANAC

FIXED STARS, 1903. MEAN PLACES FOR 1903.0 (January 0.826^d, Washington)

Names of Star	Magnitude	Right Ascension	Annual Variation	Declination	Annual Variation
		h m s	s	° ′ ″	″
33 Piscium	4.7	0 00 22.254	+3.0716	− 6 15 00.68	+20.137
δ Piscium	4.8	0 43 38.935	+3.1092	+ 7 03 26.11	+19.640
γ Cassiopeiæ	2.3	0 50 50.911	3.5893	+60 11 29.65	19.550
μ Andromedæ	4.0	0 51 21.986	3.3174	+37 58 23.86	19.575
43 Cephei (H.)	4.6	0 55 23.710	7.4355	+85 44 13.13	19.460
ε Piscium	4.3	0 57 54.479	3.1102	+ 7 22 04.76	19.436
β Andromedæ	2.2	1 04 17.872	+3.3473	+35 06 22.95	+19.146
κ Tucanæ	4.9	1 12 28.743	2.0417	−69 23 29.15	19.142
f Piscium	5.1	1 12 47.697	3.0917	+ 3 06 13.54	19.018
θ′ Ceti	3.6	1 19 10.477	2.9976	− 8 41 01.59	18.647
α Ursæ Minoris (*Polaris*)	2.2	1 23 49.770	25.8359	+88 47 22.84	18.724
β Arietis	2.8	1 49 16.753	3.3061	+20 20 02.47	17.700
50 Cassiopeiæ	4.1	1 55 08.306	5.0381	+71 57 07.60	17.589
γ Andromedæ	2.2	1 57 56.494	+3.6665	+41 51 52.13	+17.400
α Arietis	2.1	2 01 42.178	3.3736	+23 00 14.26	17.142
β Trianguli	3.1	2 03 46.126	3.5578	+34 31 43.16	17.149
ξ′ Ceti	4.5	2 07 51.441	3.1755	+ 8 23 30.46	16.991
γ Trianguli	4.3	2 11 32.693	3.5549	+33 23 55.56	16.782
α Canis Majoris (*Sirius*)	−1.4	6 40 52.425	2.6435	−16 34 58.33	4.764
α Lyræ (*Vega*)	0.2	18 33 39.256	2.0313	+38 41 35.34	3.213

TABLE VI

PART OF THE TABLE GIVING THE EXACT POSITIONS OF 383 FIXED STARS, WHICH COMPRISES PAGES 312 TO 300 OF THE AMERICAN NAUTICAL ALMANAC

FIXED STARS, 1903. APPARENT PLACES FOR THE UPPER TRANSIT AT WASHINGTON

(*Constants of Struve and Peters*)

Mean Solar Date	α Ursæ Min. (*Polaris*)		Mean Solar Date	α Canis Majoris (*Sirius*)		Mean Solar Date	α Lyræ (*Vega*)		Mean Solar Date	λ Pegasi	
	Right Ascension	Declination *North*		Right Ascension	Declination *South*		Right Ascension	Declination *North*		Right Ascension	Declination *North*
	h m	° ′		h m	° ′		h m	° ′		h m	° ′
	1 24	+88 47		6 40	−16 34		18 33	+38 41		22 41	+23 03
Jan.	s	″	Jan.	s	″	Jan.	s	″	Jan.	s	″
0.3	34.40	42.0	0.5	54.19	68.0	1.0	37.86	43.5	1.2	51.39	29.6
				.06	2.3		.11	3.1		.10	1.5
1.3	33.32	42.1	10.5	54.25	70.3	11.0	37.97	40.4	11.1	51.29	28.1
				.01	2.2		.15	3.0		.08	1.6
2.3	32.20	42.2	20.5	54.26	72.5	21.0	38.12	37.4	21.1	51.21	26.5
				.04	2.0		.20	2.9		.06	1.7
3.3	31.10	42.3	30.4	54.22	74.5	30.9	38.32	34.5	31.1	51.15	24.8
			Feb.	.08	1.7	Feb.	.24	2.5	Feb.	.03	1.8
4.3	30.05	42.4	9.4	54.14	76.2	9.9	38.56	32.0	10.1	51.12	23.0
5.3	29.04	42.5					.26	2.1		.01	1.7

TABLE VII

MEAN REFRACTION TO BE APPLIED TO ALL MEASURED ALTITUDES

(*Subtractive from apparent altitude*)

App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction
° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″
0 0	33 0			6 40	7 40	10 0	5 15	16 40	3 8
						10 10	5 10	16 50	3 6
		3 30	13 6			10 20	5 5	17 0	3 4
						10 30	5 0	17 10	3 3
				7 0	7 20	10 40	4 56	17 20	3 1
						10 50	4 51	17 30	2 59
						11 0	4 47	17 40	2 57
						11 10	4 43	17 50	2 55
		4 0	11 51	7 20	7 2	11 20	4 39	18 0	2 54
						11 30	4 34	18 10	2 52
						11 40	4 31	18 20	2 51
						11 50	4 27	18 30	2 49
1 0	24 29			7 40	6 45	12 0	4 23	18 40	2 47
						12 10	4 20	18 50	2 46
		4 30	10 48			12 20	4 16	19 0	2 44
						12 30	4 13	19 10	2 43
				8 0	6 29	12 40	4 9	19 20	2 41
						12 50	4 6	19 30	2 40
				8 10	6 22	13 0	4 3	19 40	2 38
						13 10	4 0	19 50	2 37
		5 0	9 54	8 20	6 15	13 20	3 57	20 0	2 35
						13 30	3 54	20 10	2 34
				8 30	6 8	13 40	3 51	20 20	2 32
						13 50	3 48	20 30	2 31
2 0	18 35	5 20	9 23	8 40	6 1	14 0	3 45	20 40	2 29
						14 10	3 43	20 50	2 28
				8 50	5 55	14 20	3 40	21 0	2 27
						14 30	3 38	21 10	2 26
		5 40	8 54	9 0	5 48	14 40	3 35	21 20	2 25
						14 50	3 33	21 30	2 24
				9 10	5 42	15 0	3 30	21 40	2 23
						15 10	3 28	21 50	2 21
		6 0	8 28	9 20	5 36	15 20	3 26	22 0	2 20
						15 30	3 24	22 10	2 19
				9 30	5 31	15 40	3 21	22 20	2 18
						15 50	3 19	22 30	2 17
3 0	14 36	6 20	8 3	9 40	5 25	16 0	3 17	22 40	2 16
						16 10	3 15	22 50	2 15
				9 50	5 20	16 20	3 12	23 0	2 14
						16 30	3 10	23 10	2 13

TABLE VII—*Continued*

App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction
° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″
23 20	2 12	26 40	1 53	34 0	1 24	48 0	0 51	68 0	0 23
23 30	2 11	26 50	1 52	34 30	1 23	49 0	0 49	69 0	0 22
23 40	2 10	27 0	1 51	35 0	1 21	50 0	0 48	70 0	0 21
23 50	2 9	27 15	1 50	35 30	1 20	51 0	0 46	71 0	0 19
24 0	2 8	27 30	1 49	36 0	1 18	52 0	0 44	72 0	0 18
24 10	2 7	27 45	1 48	36 30	1 17	53 0	0 43	73 0	0 17
24 20	2 6	28 0	1 47	37 0	1 16	54 0	0 41	74 0	0 16
24 30	2 5	28 15	1 46	37 30	1 14	55 0	0 40	75 0	0 15
24 40	2 4	28 30	1 45	38 0	1 13	56 0	0 38	76 0	0 14
24 50	2 3	28 45	1 44	38 30	1 11	57 0	0 37	77 0	0 13
25 0	2 2	29 0	1 42	39 0	1 10	58 0	0 35	78 0	0 12
25 10	2 1	29 30	1 40	39 30	1 9	59 0	0 34	79 0	0 11
25 20	2 0	30 0	1 38	40 0	1 8	60 0	0 33	80 0	0 10
25 30	1 59	30 30	1 37	41 0	1 5	61 0	0 32	81 0	0 9
25 40	1 58	31 0	1 35	42 0	1 3	62 0	0 30	82 0	0 8
25 50	1 57	31 30	1 33	43 0	1 1	63 0	0 29	83 0	0 7
26 0	1 56	32 0	1 31	44 0	0 59	64 0	0 28	84 0	0 6
26 10	1 55	32 30	1 30	45 0	0 57	65 0	0 26	86 0	0 4
26 20	1 55	33 0	1 29	46 0	0 55	66 0	0 25	88 0	0 2
26 30	1 54	33 30	1 26	47 0	0 53	67 0	0 24	90 0	0 0

TABLE VIII

SUN'S PARALLAX IN ALTITUDE TO BE APPLIED TO ALL MEASURED ALTITUDES OF THE SUN

(*Additive to observed altitude*)

Altitude Degrees	Parallax Seconds	Altitude Degrees	Parallax Seconds
0	9	54	5
6	9	57	5
12	9	60	4
16	8	63	4
20	8	66	3
25	8	69	3
30	8	72	3
34	7	75	2
36	7	78	2
40	7	81	1
45	6	84	1
48	6	87	0
51	5	90	0

TABLE IX

DIP OF THE HORIZON, TO BE APPLIED TO ALL ALTITUDES MEASURED AT SEA

(*Subtractive from observed altitude*)

Height Feet	Dip	Height Feet	Dip	Height Feet	Dip
	′ ″		′ ″		′ ″
1	0 59	13	3 32	26	5 0
2	1 23	14	3 40	28	5 11
3	1 42	15	3 48	30	5 22
4	1 58	16	3 55	35	5 48
5	2 11	17	4 2	40	6 12
6	2 24	18	4 9	45	6 34
7	2 36	19	4 16	50	6 56
8	2 46	20	4 23	60	7 35
9	2 56	21	4 29	70	8 12
10	3 5	22	4 36	80	8 46
11	3 15	23	4 42	90	9 18
12	3 24	24	4 48	100	9 48

PRACTICAL ASTRONOMY

(PART 2)

DETERMINATION OF LATITUDE

GENERAL CONSIDERATIONS

1. Latitude can be most readily determined in the field by measuring the altitude of a heavenly body with either the sextant or the transit. There are three methods, as follows: (1) by observing the altitude of one or more stars when on the meridian; (2) by observing the altitude of the sun when on the meridian; (3) by observing the north star, Polaris, at any time.

When the observer has acquired some experience, he will find that observations on the stars will give more accurate results than can be obtained from the sun. This is because the image of a star in the telescope is a small, well-defined point that permits a more exact setting of the horizontal wire (with the transit), or a more accurate coincidence of the images (with a sextant), than is possible when the sun is observed. Besides, stars suitable for observation can be employed at any hour of the night, while the sun can only be observed for latitude at or near the instant of apparent noon.

By suitably selecting the stars to be observed, the errors of the instrument can be nearly eliminated. This is a very important fact, since the errors of adjustment, especially in the transit instrument, may be so large as to render an accurate determination of the latitude impossible if only a single heavenly body is observed. For these reasons, the

first and the third method should generally be employed in preference to the second.

2. General Formulas for the Determination of Latitude.—From a measured altitude of a heavenly body at the instant of its meridian passage, the latitude of a place is found as follows:

Let Fig. 1 represent a section of the celestial sphere made by the plane of the meridian. The earth is at O; HZH' is the meridian; HH', the horizon; Z, the zenith; EO, the plane of the equator; and P, the north pole. Hence, the arc PH = the arc ZE = the latitude (*Practical Astronomy*, Part 1).

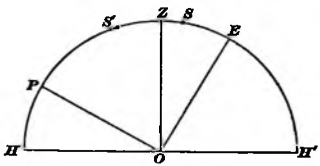

FIG. 1

If the observer is assumed to be in the northern hemisphere, and S is any heavenly body on the meridian, south of the zenith, we have

$$ZE = ZS + SE$$

Or, latitude = declination + zenith distance (for a south star).

If the body is on the meridian north of the zenith, as at S', we have

$$ZE = S'E - S'Z$$

Or, latitude = declination − zenith distance (for a north star).

In applying these formulas, the zenith distance is determined by observing the altitude, correcting it for refraction and index error (and for semi-diameter and parallax, if the sun is observed, and for dip, if the observation is made at sea), and subtracting the result from 90°.

EXAMPLE.—The altitude of the star Sirius was observed at Philadelphia on January 10, 1903; the circle reading was 66° 56′ 50″, the index error was +22″. To find the latitude of Philadelphia.

SOLUTION.—

Observed double altitude	66° 56′ 50″
Correction for index error	+22
Corrected double altitude	66° 57′ 12″

Apparent altitude	33° 28′ 36″
Refraction (Table VII, *Practical Astronomy*, Part 1)	−1 26
True altitude	33° 27′ 10″
Zenith distance = 90° − true altitude	56° 32′ 50″
Declination of Sirius from Ephemeris (Table V, *Practical Astronomy*, Part 1)	−16 34 58
Latitude	39° 57′ 52″ Ans.

FIRST METHOD OF DETERMINING LATITUDE: BY OBSERVING A STAR ON THE MERIDIAN

3. Selection of Stars.—Before going into the field, the observer should select at least two stars for observation, one of which passes north and the other south of the zenith. The altitudes of the two stars when on the meridian should be as nearly equal as possible. If three or four such pairs of stars are observed, and the mean of the resulting six or eight values of the latitude is formed, the value obtained will be much more accurate than could have been derived from a single pair. The disagreement of the individual values of the latitude will serve to indicate to the observer what the accuracy of his observations is.

The observer, after deciding at what time in the evening he will begin his observations, should first change this mean time into sidereal time. Since the right ascension of a star is equal to the sidereal time of its transit (*Practical Astronomy*, Part 1), he should then find, from the Ephemeris (Table V, *Practical Astronomy*, Part 1), a star whose right ascension is, as nearly as possible, equal to the sidereal time chosen. If in the northern hemisphere, the declination of the star is greater than the latitude [which usually is known within a few degrees, or can be approximately found, by observing the altitude of Polaris (Art. 2)], the star will pass north of the zenith; but if the declination is less, the star will pass south of the zenith (Art. 2). After selecting one star, another having approximately the same altitude when on the meridian may be found as follows:

Let l' be the approximate latitude of the place, d_n and z_n the declination and zenith distance, respectively, of a north star; d_s and z_s similar quantities for a south star. Then (Art. 2),

$$d_n = z_n + l', \; d_s = l' - z_s$$

If a north star is chosen as the first star, the second, or south, star must be so chosen that z_s will be approximately equal to z_n, and that, therefore, its declination will be approximately equal to $l' - z_n$. Similarly, if a south star is chosen as the first star, the second, or north, star must be so chosen that its zenith distance z_n will be approximately equal to z_s and that, therefore, its declination will be approximately equal to $z_s + l'$.

The stars selected should be bright (say of the first, second, or third magnitude). After selecting the first, the second may be selected by looking down the list of stars (Ephemeris, or Table V, *Practical Astronomy*, Part 1) until a bright star is found having approximately the required declination, as determined above.

EXAMPLE.—To select a pair of stars for observation at Philadelphia on January 5, 1903, the longitude being $-7^m\ 37^s$ from Washington and $5^h\ 0^m\ 38.5^s$ from Greenwich, and the latitude about 40°. The observer possesses an ordinary watch that keeps 75th-meridian time (that is, the local time of places whose longitude from Greenwich is 75°, or 5^h), and is 20 seconds fast.

SOLUTION.—Since Philadelphia is $5^h\ 0^m\ 38.5^s$ west of Greenwich, a watch keeping standard 75th-meridian time is 38.5^s fast on local time. Hence, the watch used is actually $38.5^s + 20.0^s = 58.5^s$ fast on local mean time.

Suppose that it is desired to begin observations about 6 P. M. From the complete table of the Ephemeris, of which a part is given as Table IV in *Practical Astronomy*, Part 1, we find, sidereal time of mean noon at Washington = $18^h\ 56^m$. Sidereal time corresponding to 6^h after mean noon is, roughly, $24^h\ 56^m$, that is, $0^h\ 56^m$. Looking now at the list of stars in the Ephemeris, or Table V, *Practical Astronomy*, Part 1, it is found that the star γ Cassiopeia is well situated for observation at about this time; its magnitude is but 2.3, so that it is quite bright, and as its declination is +60°, it is not too near the horizon (*Practical Astronomy*, Part 1). The latitude of Philadelphia is about 40°; hence, γ Cassiopeia is a north star (Art. 3). To find the approximate zenith distance when on the meridian, the latitude 40° is subtracted from the

approximate declination (say 60° 11′), and the zenith distance of the first star is found to be 20° 11′. The zenith distance of the second star must also be about 20°, and hence its declination must be about $40° - 20° = +20°$. Looking on down the list, the stars β Arietis and α Arietis, both of which fulfil this condition, are found. The latter is the brighter of the two, and is therefore selected.

A small table to be taken into the field is then prepared as follows:

Name of Star	Right Ascension	Declination	Magnitude	Watch Time of Transit	Altitude of Transit
γ Cassiopeia	$0^h\ 50^m\ 51^s$	+60° 11′ 30″	2.3	$5^h\ 54^m\ 24.5^s$	69° 49′ N
α Arietis . .	2 1 42	+23 0 14	2.1	7 5 3.5	72 0 S

The first four columns are copied from the Ephemeris. To find the exact watch time of transit of γ Cassiopeia, the sidereal time $0^h\ 50^m\ 51^s$ is changed into local mean time (*Practical Astronomy*, Part 1), and to the result is added 58.5^s, the watch error. (This computation requires the complete tables of the Ephemeris; Table IV, *Practical Astronomy*, Part 1, is not sufficiently extended for the purpose.)

The last column is found by subtracting the latitude from the declination for the north star to obtain the zenith distance, and by subtracting the declination from the latitude for the south star, and finally subtracting the two zenith distances from 90° to obtain the altitudes.

METHODS OF OBSERVING

4. Observation With a Sextant.—The approximate value of the double altitude of the first star at the time of transit is set off on the limb of the instrument, and the telescope is directed to the image of the star in the mercury. The two images that will be seen in the field of the telescope are kept in coincidence by turning the tangent screw until the instant of meridian passage is shown by the watch. The reading is then taken; this is the desired (uncorrected) double altitude.

If the time is not known with accuracy, the method of procedure is as follows: The images are brought into coincidence, as before, a few minutes before the star reaches the meridian, and kept in coincidence by turning the tangent screw. Since the altitude of the star when on the meridian is its greatest altitude, the turning of the tangent screw is

stopped the instant that the readings cease to increase and begin to decrease. The resulting reading is, as before, the uncorrected double altitude of the star when on the meridian.

5. Observation With a Transit.—The adjustments of the transit having been carefully attended to, there are three ways of proceeding, according as the vertical circle is or is not complete, and as a mercury horizon is or is not used:

1. *If the vertical circle is not complete.*—The index error must be well determined according to directions given in *Hydrographic Surveying*. Then the horizontal wire is brought on the first star, and the vertical motion clamped. The horizontal wire is kept constantly bisecting the image of the star by turning the tangent screw until the star ceases to rise; the instant when the upward motion of the star ceases and the star begins to fall, the turning of the tangent screw is stopped. The reading of the vertical circle, corrected for index error and for refraction, is the desired meridian altitude.

2. *If the vertical circle is complete.*—The observation is begun 3 or 4 minutes before the time of meridian passage, and the altitude of the star is measured by the method given in *Practical Astronomy*, Part 1. The result should be corrected for refraction, but not for index error.

3. *If an artificial horizon is used.*—The method given in *Practical Astronomy*, Part 1, is employed. The readings are usually made in the following manner: The transit is directed to the star and the vertical circle read; two sightings are taken on the mercury image, and the corresponding angles read from the vertical circle; finally, one more reading on the star is taken. By this time the star should be about 5 minutes past the meridian. The mean of the direct plus the mean of the reflected readings will be the double altitude, which should be divided by two and corrected for refraction, but not for index error.

The artificial horizon must be moved to a new position before observing the second star, and therefore the two stars should be so selected that there will be an interval of

at least 10 or 15 minutes between the times at which they come to the meridian. In other words, their right ascensions should not differ by less than this amount.

EXAMPLES FOR PRACTICE

1. The altitude of Polaris was measured on January 2, 1903, with a transit having an incomplete vertical circle; the circle reading was 50° 12′ 0″; the correction due to index error was +2′. Find the latitude.
Ans. +49° 0′ 54″

2. The altitude of Polaris was measured on January 5, 1903, as described in *Practical Astronomy*, Part 1; the vertical circle readings were as follows: Telescope sighted at star, 36° 27′ 10″; telescope sighted at reflection in mercury, 323° 32′ 30″; telescope sighted at reflection in mercury, 323° 33′ 0″; telescope sighted at star, 36° 27′ 10″. Find the latitude.
Ans. +35° 13′ 38″

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6. Accuracy of the Preceding Methods of Observing.—In general, the latitude can be determined from a single altitude about as accurately as the circles can be read. The principal sources of error are refraction, index error, and want of adjustment of the instrument. If the star is not too low, the uncertainty of the refraction seldom amounts to more than 1 or 2 seconds, and if the second or third method is employed, the other errors are almost wholly eliminated.

If a very exact determination is required, three or four pairs of stars at varying altitudes should be observed, and the mean of the six or eight resulting values of the latitude should be taken. This result should be accurate within 5″ with the sextant and within 5″ to 20″ with the transit. The error to be expected depends largely on the excellence of the instrument employed.

MODIFICATION OF THE FIRST METHOD FOR VERY ACCURATE WORK

7. The modification consists in measuring several altitudes of a star when it is near the meridian, and applying a correction for the change of altitude that takes place during the observations. This correction is obtained from Table I

at the end of this Section. If this method is employed, the declination of the star must be less than $+24°$ or greater than $-24°$.

8. Directions for Observing.—Not more than 12 minutes before the star reaches the meridian, the observer should begin to measure repeatedly its altitude by the methods of Arts. **4** or **5**, and should record the exact time when each observation is taken. He should measure ten, twenty, or thirty altitudes in this manner, taking care not to prolong the observations for more than 12 minutes after the time of meridian passage.

9. Reduction of the Observations.—The altitude of the star is continually increasing until the time of meridian passage; after this it begins to decrease. If the observations are pursued without interruption, the time at which the greatest altitude is recorded is the time of meridian passage. If the observations are interrupted at this time, or if all the altitudes are measured before or after the star passes the meridian (as is sometimes done), the watch time of meridian passage must be computed (Art. **3**). The difference between each recorded time and the time of meridian passage is then determined, and the squares of the *time intervals* so obtained are to be taken from Table II at the end of this Section.

10. Change in Altitude.—The amount by which the altitude of the star changes in 1 minute is then determined. This quantity is found in Table I, and its value corresponding to the given declination and the approximate latitude may be taken out directly. The change in 1 minute is then multiplied by each square interval, and the products are added to the corresponding measured altitudes; the resulting sums will be the corrected meridian altitudes. The mean of these sums is next formed and corrected for index error and refraction. The result is the meridian altitude as obtained from the ten, twenty, or thirty measurements. The latitude is then found in the usual manner.

EXAMPLE.—The following observations were made on δ Aquilæ by Lewis Boss in connection with the northern boundary survey of the United States:

Number	Circle Reading	½ Circle Reading	Watch Time
1	99° 5′ 35″	49° 32′ 47.5″	20^h 1^m 35^s
2	6 10	33 5	2 37
3	7 5	33 32.5	3 57
4	7 55	33 57.5	5 5
5	8 10	34 5	6 41
6	8 0	34 0	7 52
7	7 50	33 55	8 51
8	7 40	33 50	9 47
9	7 5	33 32.5	10 41
10	99 6 55	49 33 27.5	20 12 0

A sextant was used of which the index error was −3′ 43″. The approximate latitude was +49°, and the declination of δ Aquilæ, +8° 32′ 11.5″. To find the true latitude.

SOLUTION.—The time at which the star had its maximum altitude was 20^h 6^m 41^s, which is therefore the time of meridian passage. The differences between this time and the recorded times are first formed and placed in the second column of the following table. The squares of the intervals found in column three are taken directly from Table II at the end of this Section.

Number	Interval From Time of Meridian Passage	Square of Interval	Product	Meridian Altitudes
1	5^m 6^s	26.0	52.0	49° 33′ 39.5″
2	4 4	16.5	33.0	33 38.0
3	2 44	7.5	15.0	33 41.5
4	1 36	2.6	5.2	34 2.7
5	0 0	0.0	0.0	34 5.0
6	1 11	1.4	2.8	34 2.8
7	2 10	4.7	9.4	34 4.4
8	3 6	9.6	19.2	34 9.2
9	4 0	16.0	32.0	34 4.5
10	5 19	28.3	56.6	49 34 24.1

The numbers in the third column are then multiplied by the number corresponding to latitude 49° and declination +8.5° from Table I. This number is found to be 2.0; the products are put in the

fourth column. Finally, the products of column four are added to the corresponding single altitudes; the sums are found in column five, and these are the final meridian altitudes. The mean of these ten values is determined next, this being the meridian altitude as determined from all the measurements.

Final observed meridian altitude	49° 33′ 59.8″
Index correction = $\frac{1}{2} \times (-3' \, 43'')$	−1 51.5
Refraction (Table VII, *Practical Astronomy*, Part 1)	−48.5
True altitude	49° 31′ 19.8″
Zenith distance = 90° − true altitude	40° 28′ 40.2″
Declination of δ Aquilæ	+8 32 11.5
Latitude	49° 0′ 51.7″ Ans.

SECOND METHOD OF DETERMINING LATITUDE: BY OBSERVING THE SUN ON THE MERIDIAN

11. Since the sun is in motion among the stars, its declination is constantly either increasing or decreasing, and hence the sun does not attain its maximum altitude exactly at noon. From this cause, a slight error enters into the method now to be explained, and when this method is used, the errors of the sextant or transit cannot be completely eliminated, as in the methods explained in Arts. **3** and **4**. Nevertheless, when the latitude is not required with an accuracy greater than 20″ to 30″, observations on the sun are often the most convenient for determining the latitude. Two cases will be described, namely: (1) observation of a single altitude when the sun is on the meridian; and (2) observations of two or more altitudes when the sun is near the meridian.

12. Case I.—Some 10 or 15 minutes before noon, the observer should begin to observe the lower edge of the sun with either the sextant or the transit. He should repeat this observation every minute or two. At first, the altitude will be increasing, but immediately after noon it will begin to decrease. The maximum altitude obtained is the meridian altitude, and when this is corrected for index error, refraction, parallax, and semi-diameter, the true altitude of the sun's center will be obtained.

When the local mean time is known within a minute or two, the approximate time of the sun's meridian passage may be obtained by subtracting from the mean time the equation of time taken from the Ephemeris (*Practical Astronomy*, Part 1). The observation may then be begun 1 or 2 minutes before apparent noon.

13. To Find the Watch Time of Apparent Noon When the Watch is Running on Standard Time and Has a Known Error.—The local apparent time of noon is $12^h\,0^m\,0^s$. Adding to this the equation of time, the local mean time of the sun's meridian passage is obtained. To this sum must be added the difference in longitude between the place of observation and the standard meridian, if the place of observation is west of the standard meridian; otherwise, that difference should be subtracted. The result is the standard time of apparent noon. The known error of the watch is then added or subtracted, as the case may be, and the result is the desired watch time.

EXAMPLE.—To find the time of apparent noon at Philadelphia, January 10, 1903, as shown by a watch that is 52^s fast on standard 75th-meridian time, Philadelphia being $5^h\,0^m\,39^s$ west of Greenwich.

SOLUTION.—

Apparent solar time	$12^h\,0^m\,0^s$
Equation of time from Ephemeris, or from Table IV, *Practical Astronomy*, Part 1	$+7\ \ 25$
Local mean time of apparent noon	$12^h\,7^m\,25^s$
Philadelphia is west of principal meridian	$+39$
Standard time of apparent noon	$12^h\,8^m\,4^s$
Error of watch	$+52$
Watch time of apparent noon	$12^h\,8^m\,56^s$ Ans.

EXAMPLES FOR PRACTICE

1. Find the standard time of apparent noon at Washington on January 10, 1903, the longitude of Washington being $5^h\,8^m\,16^s$ from Greenwich. Ans. $12^h\,15^m\,41^s$

2. Find the watch time of apparent noon at Philadelphia on January 10, 1903, the watch being $3^m\,10^s$ slow on standard time, and the longitude of Philadelphia being $+5^h\,0^m\,39^s$. Ans. $12^h\,4^m\,54^s$

14. Case II.—Either the sextant or the transit may be used, and the watch time of each observation must be recorded. If the sextant is employed, the observations should begin, if possible, 5 or 10 minutes before apparent noon, and be continued as long as may be necessary to make the sun's meridian passage occur about the middle of the series. If the transit is employed, very good results may be obtained by the methods of observing explained in *Practical Astronomy*, Part 1. Two or four such series should be taken, but in general not more. In any case, one-half the altitudes should be measured on the upper edge of the sun and one-half on the lower, in order to avoid the correction for semi-diameter. The measurements are reduced by the method of Art. **9**.

The following example will fully illustrate the method of recording the observations and of computing the latitude:

EXAMPLE.—The observations tabulated below were made by the method given in *Practical Astronomy*, Part 1. The instruments were an artificial horizon, a transit instrument having a complete vertical circle, and a mean-time watch. The watch time of transit was $11^h\ 54^m\ 17.5^s$ (Art. **14**); the approximate latitude was $+48°\ 2'$; and the declination of the sun (*Practical Astronomy*, Part 1) was $-14°\ 7'\ 18.1''$. To find the true latitude.

RECORD OF OBSERVATIONS

Edge of Sun	Pointing	Reading of Vertical Circle	Watch Time
Upper	Direct	28° 14′ 20″	11h 50m 44s
	Mercury	331 46 0	51 42
	Mercury	331 46 20	52 28
	Direct	28 15 20	53 50
Lower	Direct	207 41 50	56 32
	Mercury	152 18 40	58 24
	Mercury	152 18 40	59 56
	Direct	207 41 30	12 1 14

NOTE.—The readings of the circle are here given exactly as they were recorded. It will be noticed that the graduation with this transit was from 0° to 360°. The instrument was turned 180° in azimuth and the telescope inverted between the two series of measurements, in order to completely eliminate all errors of adjustment.

SOLUTION.—

	FIRST SERIES	SECOND SERIES
Mean of readings on the sun direct	28° 14′ 50″	207° 41′ 40″
Mean of readings on sun reflected from mercury	331 46 10	152 18 40
Double altitude of edge of sun . .	56° 28′ 40″	55° 23′ 0″
Single altitude of edge of sun . . .	28° 14′ 20″	27° 41′ 30″

	Measured Altitudes	Time	Interval	Square	Product	Meridian Altitude
Upper edge	28° 14′ 20″	$11^h\,52^m\,11.0^s$	2^m 6.5^s	4.4	6.2″	28° 14′ 26.2″
Lower edge	27 41 30	11 59 1.5	4 44.0	22.4	31.4	27 42 1.4

The column headed Time contains the mean of the four times recorded while observing the upper and lower edges, respectively. The differences between these two means and the watch time of apparent noon ($11^h\,54^m\,17.5^s$) is the Interval; the square of this interval is taken from Table II. From Table I is next taken the number corresponding to a latitude of 48° 2′ and a declination of 14° 7′ of a "different name" (that is, different sign) from the latitude. This is found to be 1.4, and it is multiplied by the squares to give the desired corrections, which are written in the column headed Product. These products are finally added to the Measured Altitudes of the second column to form the Meridian Altitudes. The solution is then completed as follows: Mean of the meridian altitudes is equal to the altitude of the sun's center, which need not be corrected for semi-diameter, nor for index error (*Practical Astronomy*, Part 1) 27° 58′ 13.8″

Refraction (Table VII, *Practical Astronomy*, Part 1)	−1 47.0
Parallax (Table VIII, *Practical Astronomy*, Part 1)	+8.0
Corrected altitude of sun's center	27° 56′ 34.8″
Zenith distance = 90° − altitude	62° 3′ 25.2″
Declination of sun	−14 7 18.1
Latitude	47° 56′ 7.1″

Ans.

THIRD METHOD OF DETERMINING LATITUDE: BY OBSERVING POLARIS

15. As the north star is near the pole of the heavens and is easily identified in the sky, even by an inexperienced observer, it is frequently recommended in textbooks on astronomy that this star be used for the determination of latitude. The observations may be made by one of two methods: (1) star on the meridian; (2) star in any position.

16. The star may be observed on the meridian exactly as described in Arts. **1** to **6**. For determining the time of meridian passage, the right ascension, which may be taken from the Ephemeris, is necessary, and for finding the resulting latitude, the declination must be known. A simple method of finding the time of the meridian passage of Polaris without recourse to its right ascension is given in *Transit Surveying*, Part 2. The declination must, however, be taken from the Ephemeris.

17. If the observer possesses an Ephemeris, he may determine his latitude by observing the altitude of Polaris at any time, and using the tables of that work entitled "For Finding the Latitude by Polaris." The simple directions for using them are given in full. Even the tables of the Ephemeris are, however, liable to an error of 30″ or more in the resulting latitude. If a higher degree of accuracy is required, the following rule should be applied:

Rule.—*Convert the mean time of the observation into sidereal time, and from the result, subtract the right ascension of Polaris; the remainder is the hour angle of the star. Reduce the hour angle to arc measure.*

Subtract the declination from 90°; the result is the polar distance.

Correct the measured altitude for refraction and for index error, if the latter has not been eliminated in making the observations. From the corrected altitude, subtract the product of the polar distance by the cosine of the hour angle, and to the

remainder, add the second correction taken from Table III at the end of this Section; the result is the desired latitude.

EXAMPLE.—The altitude of Polaris was measured ten times with a transit having an incomplete vertical circle, by the method of Art. **16.** The mean of the circle readings was 39° 33′ 50″; the mean of the corresponding recorded times, reduced to sidereal time, was $10^h\ 45^m\ 8.9^s$. The index error was +57.4″. The right ascension and declination of Polaris, taken from the Ephemeris, were $1^h\ 15^m\ 6.0^s$ and 88° 41′ 6.2″, respectively. To find the latitude.

SOLUTION.—

Measured single altitude	+39° 33′ 50.0″
Correction for refraction (Table VII, *Practical Astronomy*, Part 1)	−1 8.6
Index error	+57.4
Corrected true altitude	39° 33′ 38.8″

Polar distance = 90° − declination = 1° 18′ 53.8″ = 4,733.8″.

Hour angle = sidereal time − right ascension = $9^h\ 30^m\ 2.9^s$ = 142° 30′ 43.5″.

Polar distance × cosine of hour angle = 4,733.8 × −.79348	−1° 2′ 36.2″
True altitude	39 33 38.8
Altitude-product	40° 36′ 15.0″

The second correction, from Table III, corresponding to $10^h\ 45^m$ and 39° 30′, is 15.2″. Adding this correction to the altitude-product, the resulting latitude is found to be 40° 36′ 30.2″. Ans.

DETERMINATION OF TIME

18. The problem of the determination of time in astronomy always consists in determining the *error* of a chronometer or clock. The **error** of a clock is the number of hours, minutes, and seconds that must be subtracted from the time shown by the clock to obtain the true time. If the clock indicates a time later than the true time, it is said to be fast; if it indicates a time earlier than the true time, it is said to be slow. When a clock is fast, its error is considered positive; and when it is slow, its error is negative. Thus, if at the instant of mean local noon a watch indicates $12^h\ 5^m\ 10^s$, the watch is fast $5^m\ 10^s$, and its error on local mean time is $+5^m\ 10^s$. To obtain the true time from that of such watch at any instant, $+5^m\ 10^s$ must be subtracted.

In general, if

T = true time at any instant;

T_c = time shown by clock at same instant;

e = clock error;

we have, $e = T_c - T$, $T = T_c - e$, $T_c = T + e$

These equations apply whether the clock is keeping local, standard, or sidereal time.

EXAMPLE 1.—The error of a watch on local mean time is $+2^m\ 4^s$. What is the true time when the watch reads 9 A. M.?

SOLUTION.—

$$\begin{aligned} T_c &= 9^h\ 0^m\ 0^s \text{ A. M.} \\ e &= \underline{\quad +2\quad 4} \\ T &= 8^h\ 57^m\ 56^s \text{ A. M.} \quad \text{Ans.} \end{aligned}$$

EXAMPLE 2.—The error of a watch is -40^s. What is the watch time of true local mean noon?

SOLUTION.—

$$\begin{aligned} T &= 12^h\ 0^m\ 0^s \\ e &= \underline{\qquad\quad -40} \\ T_c &= 11^h\ 59^m\ 20^s \quad \text{Ans.} \end{aligned}$$

EXAMPLES FOR PRACTICE

1. A clock is 20 seconds fast. What is the true time when the clock indicates 6^h? Ans. $5^h\ 59^m\ 40^s$

2. The right ascension of a star is $11^h\ 2^m\ 8.04^s$; the error of a sidereal clock is -11.08^s. Find the clock time of the star's transit. Ans. $11^h\ 1^m\ 56.96^s$

3. A watch keeping mean time has an error of $-1^m\ 10^s$; the equation of time is $-16^m\ 4.01^s$. Find the watch time when the sun is on the meridian. Ans. $11^h\ 42^m\ 45.99^s$

4. A sidereal clock has an error of $+16.02^s$; when a star was on the meridian the clock time was $20^h\ 8^m\ 19.22^s$. What was the right ascension of the star? Ans. $20^h\ 8^m\ 3.20^s$

19. In *Practical Astronomy*, Part 1, it was shown that the apparent solar time at any instant is simply the hour angle of the center of the sun at that instant, and that the sidereal time may be found by adding the observed hour angle of any star to its right ascension. Hence, the mean time at any instant may be found in one of two ways: (1) by determining

the hour angle of the sun's center and adding to this the equation of time; (2) by determining the hour angle of any star, adding to it the star's right ascension, and finally changing the resulting sidereal time into mean time. The hour angle of any heavenly body is found by measuring the altitude of the body and applying a trigonometric formula, as will be explained farther on.

FIRST METHOD OF DETERMINING TIME: FROM OBSERVATIONS ON THE SUN

20. Method of Making the Observations.—The observations should never be made later than 10 A. M. nor earlier than 2 P. M., and in general the less the altitude of the sun the better; though the altitude should not be less than 15°, since the correction for refraction then becomes uncertain.

If the observer uses a sextant, he first brings the edges of the direct and reflected images into contact. He next moves the index of the vernier forwards on the limb if the sun is rising, backwards on the limb if the sun is setting, 10′, 20′, or 30′, until it stands at an even reading. He then directs the telescope again to the mercury, when he will see two images either slightly separated or slightly overlapping, according as he is observing the upper or the lower edge of the sun. In the first case, the images will appear to be coming together, as at (*a*), Fig. 2; in the second case, to be slowly separating, as at (*b*). The instant they are seen to become tangent to each other, as at (*c*), the observer notes the exact time and the circle setting. The vernier is then moved forwards or backwards, as the case may be, 10′, 20′, or 30′, and a second observation is taken. Five such measurements of altitude usually constitute a series. If two series are taken, one on the upper and one on the lower edge of the sun, no correction for semi-diameter is necessary.

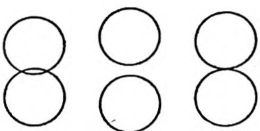

(a) (b) (c)

FIG. 2

21. If a transit is used, the telescope is first directed to the sun, and the horizontal wire placed tangent to the upper or to the lower edge. The index of the vernier is then moved forwards on the vertical circle 20′ or 30′, if the sun is rising, backwards if the sun is setting, until it stands at an even reading.

If the lower edge is observed, the appearance in the telescope will be either that of Fig. 3 (*a*), if the sun is rising, or that of (*c*), if it is setting; while if the upper edge is observed, the appearance will be as at (*b*) in the afternoon, and as at (*d*) in the forenoon. In (*c*) and (*d*), the sun is approaching the cross-wire; in (*a*) and (*b*), it is moving off the cross-wire. The instant the cross-wire is seen to be tangent to the disk, as in (*e*) or (*f*), the observer notes and records

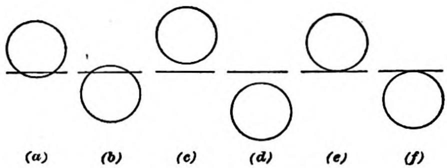


FIG. 3

the time, and also the reading of the vernier on the circle. The vernier is then moved forwards or backwards 10′, 20′, or 30′, and the observation is repeated. Five such observations usually constitute a series.

If one series is taken with the telescope direct and a second series with the telescope reversed, the index error and the errors of adjustment will be eliminated. If, in addition, one-half of the observations are made on the upper edge of the sun and the other half on the lower edge, no correction for semi-diameter need be applied. Finally, the mercury horizon may be used, and each altitude measured, as explained in *Practical Astronomy*, Part 1.

22. Method of Observing the Time.—The observer should place the clock or chronometer in such a position

that its beats can be heard distinctly. The instant the images are in contact, Fig. 2 (*c*), or the disk tangent to the wire, Fig. 3 (*e*) and (*f*), he should begin to count the clock beats and continue to count them until he can take the clock reading. The interval of time corresponding to the number of ticks counted until the time was read from the clock is to be subtracted from the clock reading to give the instant of observation.

23. To Find the True Altitude.—The mean of the measured altitudes should first be corrected for index error, if the sextant has been used, and also if the transit has been used without either reversing or using a mercury horizon. If the observations have been taken with the transit, half with the telescope direct and half with the telescope inverted, or with the transit in connection with a mercury horizon, no correction for index error is required. The resulting altitude should be further corrected for refraction and parallax, and, if the observations are made on a single edge of the sun, for semi-diameter.

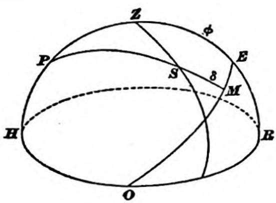


FIG. 4

24. Formula for Finding the Sun's Hour Angle From the True Altitude.—Let Fig. 4 represent a portion of the celestial sphere; let Z be the zenith of the observer; P, the celestial pole; HOR, the horizon; and OME, the equator. If S is the sun, the angle ZPS is the required hour angle; SM, the declination of the sun; PS, its polar distance; and ZS, its zenith distance.

If φ is the latitude of the observer, then $PZ = 90° - \varphi$ (*Practical Astronomy*, Part 1).

Let z = sun's zenith distance = ZS;
t = sun's hour angle = apparent solar time = ZPS;
δ = sun's declination = SM;
$m = \varphi - \delta$ = latitude − declination.

Then, the hour angle t is determined by means of the following formula:*

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\sin \frac{1}{2}(z+m) \sin \frac{1}{2}(z-m)}{\cos \varphi \cos \delta}}$$

The value of t is $-$ in the morning and $+$ in the afternoon.

To find the mean time and the error of the watch, the equation of time is added to the apparent solar time, the result being the local mean time of the observation. The difference between this result and the mean of the recorded times is the error of the watch on local time.

EXAMPLE 1.—The altitude of the lower edge of the sun was measured at Philadelphia on the afternoon of January 10, 1903. The instrument used was a sextant of which the index error was $+2' 40''$. The circle reading was $38° 10' 0''$; the corresponding watch time was $2^h 30^m 56^s$. The latitude of Philadelphia being $+39° 58'$ ($= \varphi$), the equation of time $+7^m 27.7^s$, and the declination of the sun $-22° 1' 39''$ ($= \delta$), to find the error of the watch.

SOLUTION.—To find the true altitude:

Observed double altitude	38° 10′ 00″
Index error	+2 40
Double altitude	38° 12′ 40″
Single altitude	19° 6′ 20″
Refraction (Table VII, *Practical Astronomy*, Part 1)	−2 43
Parallax (Table VIII, *Practical Astronomy*, Part 1)	+8
Semi-diameter (Table IV, *Practical Astronomy*, Part 1)	+16 18
True altitude of sun's center	19° 20′ 3″

To find the hour angle:

$$z = 90° - 19° 20' 3'' = 70° 39' 57''$$

$$m = 39° 58' - (-22° 1' 38.9'') = 61° 59' 39''$$

$$\tfrac{1}{2}(z+m) = \frac{70° 39' 57'' + 61° 59' 39''}{2} = 66° 19' 48''$$

$$\tfrac{1}{2}(z-m) = \frac{70° 39' 57'' - 61° 59' 30''}{2} = 4° 20' 9''$$

*This, and a few similar formulas used in this Section, are derived from the principles of spherical trigonometry, which the student may find in any work on the subject (Wood's "Plane and Spherical Trigonometry" is recommended for its conciseness). Spherical trigonometry is not included in this Course, as the applications of it that the Course contains are too few to justify a complete treatment of that subject, which is comparatively complicated.

Substituting the given values in the formula,

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\sin 66^\circ 19' 48'' \sin 4^\circ 20' 9''}{\cos 39^\circ 58' \cos 22^\circ 1' 39''}};\ \tfrac{1}{2} t = 18^\circ 11' 29'';\ t = 36^\circ 22' 58''$$

Dividing by 15 to reduce to time measure, we find $2^h\ 25^m\ 32^s$ P. M. as the apparent solar time.

To find the watch error:

Apparent solar time	$2^h\ 25^m\ 32.0^s$
Equation of time	+7 27.7
Local mean time	$2^h\ 32^m\ 59.7^s$
Watch time	2 30 56.0
Hence, the watch was slow	$2^m\ 3.7^s$

on local time, and the watch error was $-2^m\ 3.7^s$. Ans.

EXAMPLE 2.—For the purpose of illustrating a complete series, both on the upper and on the lower edge of the sun, we shall take the following data from a survey made in Colorado. The observations were taken with an engineer's transit, as explained in Art. 21. The index error was $-28''$; the latitude, $+38^\circ 04'$; the longitude, $+1^h\ 44^m\ 41^s$; the declination of the sun, $+18^\circ 42'\ 17''$; and the equation of time, $+6^m\ 13^s$. To find the error of the chronometer.

Edge of Sun	Circle Readings			Time		
Upper	44°	25′	00″	8^h	35^m	12.0^s
	44	30	0		35	39.5
	44	35	0		36	3.5
	44	40	0		36	30.5
	44	45	0		36	56.5
Lower	44	25	0	8	37	55.5
	44	30	0		38	22.0
	44	35	0		38	48.0
	44	40	0		39	14.5
	44	45	0		39	41.0
Means	44	35	0	8	37	26.3

SOLUTION.—

Mean circle reading	44° 35′ 0″
Correction for index error	−28
Measured altitude	44° 34′ 32″
Refraction (− 49″) and parallax (+ 6″)	−43
True altitude	44° 33′ 49″

No correction for semi-diameter.

To find the hour angle:

$$m = 38^\circ\ 4'\ 0'' - 18^\circ\ 42'\ 17'' = 19^\circ\ 21'\ 43''$$
$$z = 90^\circ - 44^\circ\ 33'\ 49'' = 45^\circ\ 26'\ 11''$$
$$\tfrac{1}{2}(z+m) = \frac{45^\circ\ 26'\ 11'' + 19^\circ\ 21'\ 43''}{2} = 32^\circ\ 23'\ 57''$$
$$\tfrac{1}{2}(z-m) = \frac{45^\circ\ 26'\ 11'' - 19^\circ\ 21'\ 43''}{2} = 13^\circ\ 2'\ 14''$$

Substituting the given values in the formula,

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\sin 32^\circ\ 23'\ 57'' \sin 13^\circ\ 2'\ 14''}{\cos 38^\circ\ 4' \cos 18^\circ\ 42'\ 17''}}$$

$\frac{1}{2} t = 23^\circ\ 44'\ 27''$; $t = 47^\circ\ 28'\ 54''$. Dividing by 15 to reduce to time measure, $t = 3^h\ 9^m\ 55.6^s$. The hour angle is minus in the morning. The apparent solar time is found by subtracting $3^h\ 9^m\ 56^s$ from 12 hr., which gives $8^h\ 50^m\ 4.4^s$ A. M. apparent solar time.

To find the chronometer error:

Apparent solar time	$8^h\ 50^m\ 4.4^s$
Equation of time	$+6\ \ 13.0$
Local mean time	$8^h\ 56^m\ 17.4^s$
Mean of the ten recorded times	$8\ \ 37\ \ 26.3$
Hence, the chronometer was slow	$18^m\ 51.1^s$

and its error was $-18^m\ 51.1^s$. Ans.

EXAMPLES FOR PRACTICE

1. Ten altitudes of the sun were measured with a transit as in Art. 21, one-half of them being on the lower and one-half on the upper edge of the sun. The transit had a complete vertical circle, and the index error was eliminated. The mean of the ten measured altitudes was $28^\circ\ 30'\ 44''$; the mean of the ten corresponding times was $2^h\ 5^m\ 45.5^s$ P. M. The declination of the sun being $-12^\circ\ 8'\ 16''$, the equation of time being $-15^m\ 50.5^s$, and the latitude being $+39^\circ\ 57'\ 6''$, find the error of the chronometer. Ans. $+33.5^s$

2. Ten double altitudes of the sun were measured with the sextant, one-half being on the upper and one-half on the lower edge. The mean circle reading (double altitude) was $77^\circ\ 40'$, and the mean of the corresponding times was $9^h\ 53^m\ 19.3^s$ A. M. The index correction being $-15'\ 50''$; the declination, $-3^\circ\ 2'\ 26''$; the latitude, $+40^\circ\ 36'\ 24''$; and the equation of time, $-10^m\ 11.5^s$, find the chronometer error.
Ans. $+1^m\ 41.8^s$

SECOND METHOD OF DETERMINING TIME: FROM THE ALTITUDE OF A STAR

25. The altitude of a bright star when 3 or 4 hours from the meridian is measured as described in Art. **20**; the resulting altitude is corrected for index error and refraction, and the corresponding hour angle is computed exactly as in Art. **24**. To this hour angle is added the right ascension of the star, and the sum, which is the sidereal time, is changed into mean time and compared with the time as shown by the chronometer. The following example will fully illustrate the process:

EXAMPLE.—The altitude of the star δ Coronæ Borealis was measured with a transit instrument having an incomplete circle, and with the aid of a mercury horizon, as explained in *Practical Astronomy*, Part 1. The observations were made as described in Art. **20**, the resulting measurements being as follows:

Telescope Circle		Time
Direct	$+47^\circ\ 45'\ 0''$	$10^h\ 2^m\ 56.0^s$
Reflected	$-47\ \ 44\ \ 40$	$4\ \ 4.8$
Reflected	$-47\ \ 44\ \ 25$	$4\ \ 26.5$
Direct	$+47\ \ 44\ \ 10$	$4\ \ 57.5$

The right ascension of the star was $15^h\ 29^m\ 34.1^s$; the declination, $+27^\circ\ 7'\ 32''$; the approximate latitude, $+38^\circ\ 4'$. The sidereal time of mean noon was computed and found to be $8^h\ 21^m\ 15.7^s$. To find the error of the chronometer.

SOLUTION.—To find the true altitude:

Mean of the direct readings	$+47^\circ\ 44'\ 35''$
Mean of the reflected readings	$-47\ \ 44\ \ 33$
Measured double altitude	$95^\circ\ 29'\ 08''$
Measured altitude	$47^\circ\ 44'\ 34''$
Refraction	-52
True altitude	$47^\circ\ 43'\ 42''$

No correction for index error.

To find the star's hour angle:

$$m = +\ 38^\circ\ 4' - 27^\circ\ 7'\ 32'' = 10^\circ\ 56'\ 28''$$
$$z = 90^\circ - 47^\circ\ 43'\ 42'' = 42^\circ\ 16'\ 18''$$

$$\tfrac{1}{2}(z+m) = \frac{42^\circ\ 16'\ 18'' + 10^\circ\ 56'\ 28''}{2} = 26^\circ\ 36'\ 23''$$

$$\tfrac{1}{2}(z-m) = \frac{42^\circ\ 16'\ 18'' - 10^\circ\ 56'\ 28''}{2} = 15^\circ\ 39'\ 55''$$

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\sin 26^\circ\ 36'\ 23'' \sin 15^\circ\ 39'\ 55''}{\cos 38^\circ\ 4' \cos 27^\circ\ 7'\ 32''}};\ \tfrac{1}{2} t = 24^\circ\ 32'\ 47''$$

$t = 49^\circ\ 5'\ 34''$. Dividing by 15 to reduce to time measure, $t = 3^h\ 16^m\ 22^s$.

To find the sidereal time, t is added to the right ascension $15^h\ 29^m\ 34.1^s$; therefore, the sidereal time $= 18^h\ 45^m\ 56.1^s$.

To find the mean time:

Sidereal time of mean noon	8^h	21^m	15.7^s
Sidereal interval past mean noon	10	24	40.4
Correction, Table II of Ephemeris		−1	42.3
Mean solar time	10^h	22^m	57.1^s
Chronometer time = mean of above four recorded times	10	4	6.2
Hence, the clock was slow		18^m	50.9^s

This observation was made in the evening of the same day as that of the second example of Art. 24, in order to verify the result found in the morning. When this is done, the observation in the afternoon or evening should be made on a body whose altitude is, as nearly as possible, equal to that of the body observed in the morning. Errors of the vertical circle readings and of adjustment are thus largely eliminated.

DETERMINATION OF LONGITUDE

26. By a Chronometer.—The difference in longitude between two points is simply the difference between their local times. Hence, when the error of the watch or chronometer from the local time of one station has been determined by one of the preceding methods, it is only necessary to compare it with the error from local time at a second station whose longitude is known, to find the difference in longitude between the two stations.

Between two fixed points of observation the method is as follows: One or more chronometers are compared with the standard clock at the first place of observation; they are then carried to the second place and compared with the standard clock there; finally, they are brought back to the starting place and a third comparison is made. If the errors of the two clocks on the local mean times are well determined, and

also the rate at which the first clock is gaining or losing time, the longitude can thus be determined with much accuracy.

Let the two points of observation be denoted by A and B. Let O_1 and C_1 be two corresponding times of the observatory clock and the chronometer, respectively, at A; that is, let C_1 be the time shown by the chronometer when the time indicated by the clock, after the clock error has been applied, is O_1. Let O_2 and C_2 be corresponding times at B, when the chronometer is taken to that station and compared with the clock. Finally, let O_3 and C_3 be corresponding times of the clock and chronometer at A, when the chronometer is brought back to A. Then, the difference d in longitude between the two stations is given by the formula

$$d = (C_2 - O_2) - (C_1 - O_1) - [(C_3 - O_3) - (C_1 - O_1)]\frac{C_2 - C_1}{O_3 - O_1}$$

In applying this formula, it should be borne in mind that $C_2 - C_1$ denotes the interval elapsed between the first and the second comparison, and $O_3 - O_1$ (for which $C_3 - C_1$ may be substituted without any serious error) denotes the time elapsed between the first and the third comparison.

EXAMPLE.—On the morning of May 28, a chronometer was compared with the observatory clock at Philadelphia. It was then carried to Washington and compared with the standard clock of the Naval Observatory, and on the morning of the next day was brought back and compared with the Philadelphia clock. From a long series of observations, it was known that the Philadelphia clock was at that time 38.42^s fast, and was gaining 1.12^s each 24 hours; the Washington clock was 8.60^s slow. To find the longitude of Philadelphia from Washington, the comparisons being as follows:

Place	Observatory Clock	Chronometer
Philadelphia	May 28, $11^h\ 14^m\ 20^s$	$11^h\ 36^m\ 24.22^s$
Washington	May 28, 21 17 00	21 47 11.64
Philadelphia	May 29, 9 58 00	10 20 4.85

SOLUTION.—First the clock times are corrected by applying the errors and rates given above.

First time	$11^h\ 14^m\ 20.00^s$
Error	-38.42
O_1	$11^h\ 13^m\ 41.58^s$

Second time	$21^h\ 17^m\ 00.0^s$
Error	$+8.6$
O_2	$21^h\ 17^m\ 8.6^s$
Third time	$9^h\ 58^m\ 00.0^s$
$-\left(38.42 + \frac{22.8}{24} \times 1.12^s\right)$	-39.48
O_3	$9^h\ 57^m\ 20.52^s$

The fraction $\frac{22.8}{24} \times 1.12^s$ is the gain of the clock between the first and third times, 22.8 being the number of hours elapsed between them. Thus, O_1, O_2, and O_3 are the absolutely correct times at the instants of comparison; that is, the times the clocks would have shown were they wholly free from error.

To apply the formula given in Art. **26**, we have

$C_2 - O_2 = 30^m\ 3.04^s$; $C_1 - O_1 = 22^m\ 42.64^s$

$C_3 - O_3 = 22^m\ 44.33^s$; $C_2 - C_1 = 10^h\ 10^m\ 47.42^s = 10.180^h$

$O_3 - O_1 = 22^h\ 43^m\ 38.94^s = 22.728^h$

Substituting these values in the formula,

$$d = 30^m 3.04^s - 22^m 42.64^s - (22^m 44.33^s - 22^m 42.64^s)\frac{10.180}{22.727} = 7^m\ 19.64^s$$

Ans.

27. Method of Comparing a Watch or Chronometer With a Standard Clock.—It is important that the student know how to find accurately to $\frac{1}{10}$ second, or closer, the difference between the times shown by his watch and by the clock with which he compares it. The method is very simple; it consists in noting the watch time of chosen clock beats, and may be thus stated:

Rule.—*Look at the clock and begin to count the beats; still counting each beat, look at the watch and note the watch time estimated to tenths of a second when a chosen clock second is beaten. Repeat the observation five or ten times, and find the mean of the difference between the corresponding times. If the work has been carefully done, this mean should not be uncertain by more than* $\frac{1}{10}$ *of a second.*

For example, if we wish to record the watch time corresponding to $9^h\ 34^m\ 30^s$, we look at the clock at about the twentieth second, and then, looking at the watch and counting each second as we hear the clock beat it, we note the exact watch instant when the thirtieth second is beaten. We

write down the seconds and tenths of a second from the watch, and add the hours and minutes immediately afterwards.

Thus, in comparing a watch with the observatory clock, the following five clock times were first written and then it was observed that when the clock beat $8^h\ 20^m\ 10^s$, the watch showed $8^h\ 20^m\ 16.4^s$, and similarly with the others:

Clock	Watch	Difference
$8^h\ 20^m\ 10^s$	$8^h\ 20^m\ 16.4^s$	6.4^s
8 20 30	8 20 36.5	6.5
8 20 50	8 20 56.5	6.5
8 21 10	8 21 16.4	6.4
8 21 30	8 21 36.4	6.4

The mean of the difference is 6.44^s. The clock on this date was 12.64^s fast, and therefore the watch was $6.44 + 12.64 = 19.1^s$ fast, which is probably correct within $\frac{1}{10}$ second.

28. At nearly all telegraph offices in the United States, the standard time is now received, either from Washington or from a nearer observatory, at exactly standard noon each day. At many seaports, time balls or other time signals also serve to show the navigator and others on each day the instant of standard noon. If the observer has but a single watch or chronometer, whose error on local mean time he determines by his observations, he must in some way find its *daily rate;* that is, the amount that the watch or chronometer gains or loses in 24 hours.

29. Daily Rate.—To find the daily rate, the error at two different times is determined either by observation or by comparison with a clock whose error is known. The difference between the two errors divided by the number of days and decimals of a day elapsed between the two determinations is the **daily rate.**

EXAMPLE.—At station *A*, on July 2, 9 A. M., a watch was found, by the method of Art. 22, to be $1^m\ 18.1^s$ fast. On July 4, 3 P. M., it was

found to be $1^m\ 34.2^s$ fast at station *A*, and on July 5, at 12 noon, comparing it with the standard 75th-meridian time at a telegraph office in Philadelphia, it was found to be 43.4^s fast. To find the longitude of station *A* from Washington.

SOLUTION.—The rate of the chronometer is first found. From July 2, 9 A. M., to July 4, 3 P. M., is 2 da. 6 hr. = 2.25 da. Since in 2.25 da. the watch gained 16.1 sec., the daily rate was $16.1^s \div 2.25 = 7.15^s$. (This is a very large rate.)

Next, the last chronometer time must be freed from the effect of rate. From July 2, 9 A. M., to July 5, 12 M., is 3 da. 3 hr. = 3.125 da. The gain during this time was $3.125 \times 7.15^s = 22.34^s$. Hence, the last reading freed from rate is $43.4^s - 22.3^s = 21.1^s$ fast.

Thus, the chronometer was $1^m\ 18.1^s$ fast on station *A* local time, and 21.1^s fast on 75th-meridian time. Hence, station *A* is $1^m\ 18.1^s - 21.1^s = 57.0^s$ west of the standard meridian; that is, $5^h\ 0^m\ 57^s$ west of Greenwich. Since Washington is $5^h\ 8^m\ 15.78^s$ west of Greenwich (Ephemeris), it follows that station *A* is $7^m\ 18.78^s$ east of Washington. Ans.

30. Use of Several Chronometers.—It has so far been assumed that the observer had only one accurate watch or chronometer. If a standard clock the error and rate of which are accurately known is not easily accessible, the observer must make observations at frequent intervals (preferably once each day) in order to keep informed regarding the rate of his chronometer. This assumes that he remains for several days at one station. When several stations are occupied successively and each for a short time, and also on sea voyages, it is necessary to carry three chronometers whose rates and whose errors on Greenwich or Washington mean time have been well determined before starting and which will be again examined immediately after the return. It is necessary to have three chronometers, since, if there were but two, there would be no way of telling which of the two was in error should one of them begin suddenly to gain or to lose.

DETERMINATION OF AZIMUTH

INTRODUCTION

31. The line in which the plane of the meridian intersects the plane of the horizon is called the **north-and-south line,** or the **true meridian.** It follows that the north-and-south line passes through the north and the south point of the horizon.

32. As explained in *Compass Surveying*, Part 1, the **true azimuth** of a line on the surface of the earth is the angle that the line forms with the true meridian. In astronomy, the term azimuth is always understood to mean true azimuth.

In what follows, azimuth will be reckoned from the north toward the east, unless otherwise stated. A negative azimuth is an azimuth reckoned in the opposite direction from that in which positive azimuths are reckoned. Thus, if positive azimuths are reckoned from the north toward the east, the azimuth of PG, Fig. 5, is either $+NESG$ or $-NAG$.

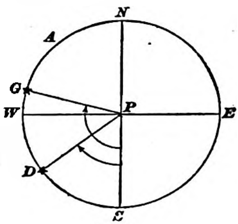


FIG. 5

33. Methods of Determining Azimuth.—The following are the most important methods of finding the azimuth of a line by astronomical observations with the engineers' transit: (1) by observing the sun directly at any hour angle; (2) by observing the sun with the aid of a solar transit; (3) by observing Polaris at eastern or western elongation; (4) by observing a south star at any hour angle; (5) by observing Polaris at any hour angle.

All these methods of determining azimuth are valuable, and when applied under proper conditions give good results; but the third, fourth, and fifth are the most satisfactory.

Observations of Polaris at eastern or western elongation afford at once the simplest and most accurate method of determining azimuth, but this method possesses the single disadvantage that it sometimes requires the observations to be made at an inconvenient hour. When this is the case, the fourth or fifth method can be substituted advantageously. The method by observing Polaris at culmination, described in *Transit Surveying*, Part 2, is rather unsatisfactory, as it requires that the observer's watch keep absolutely correct time, and that the observation be made at exactly the proper instant.

FIRST METHOD OF DETERMINING AZIMUTH: BY THE SUN AT ANY HOUR ANGLE

34. Outline of Method.—Let Fig. 6 represent the celestial sphere, $NESW$ being the horizon plane and NZS the plane of the meridian. Let the observer be at O, and let the straight line joining the observer with some fixed mark or reference point in the horizon plane have the direction OM. It is this line whose azimuth NOM is to be determined.

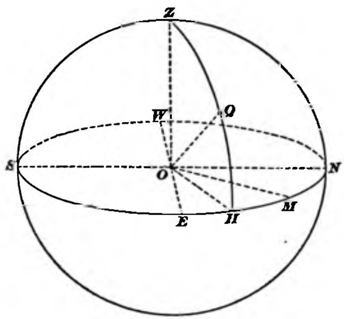


Fig. 6

Let Q be the position of the sun on the celestial sphere. Then NOH is the azimuth of the sun, and QOH its altitude. When the altitude QOH has been measured, the azimuth NOH, or NH, can be computed by trigonometry. The horizontal angle MOH, between the reference mark and the sun is measured, always turning the instrument toward the right, or in a clockwise direction. The azimuth of the line is determined by subtracting the horizontal angle MOH from the azimuth NOH of the sun.

35. Method of Making the Observations.—The transit must be provided with a vertical circle, and also with a colored-glass cap to protect the eye. The adjustments of the levels must be made with great care, especially those of the plate level parallel to the vertical circle; the index error must also be well determined, and both the vertical and horizontal wires must be in good adjustment.

The instrument is set up over the station *P*, Fig. 7, and the vernier of the horizontal circle set to read zero. The telescope is then directed to a flag at *M*, care being taken to make the final bisection by turning the lower tangent screw, so that the setting of the vernier will not be altered.

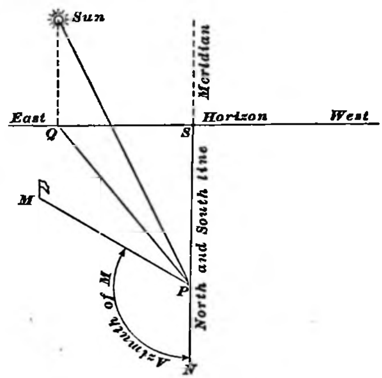


Fig. 7

The upper plate is now unclamped and the telescope is directed to the sun. After making this pointing as described in the next article, the readings of both the horizontal and vertical circle are recorded. This completes a single observation. It is customary to take five or six such sightings on the sun, the five or six corresponding readings of the horizontal and vertical circle being recorded. The lower plate is kept constantly clamped throughout the series of observations. A sighting is finally taken on the azimuth mark *M*; if the reading differs appreciably from zero, this shows that the plates have slipped, and the observations must be rejected. The two sightings on the mark with the five or six pointings on the sun constitute a series.

36. Sighting at the Sun.—The angular diameter of the sun is about 0° 32′. In the high-power telescopes now

generally used on transits, such an image will largely fill up the field of view. It is impossible to bisect such a large image with sufficient accuracy, and, consequently, when the telescope is pointed to the sun, the intersection of the cross-wires cannot be placed accurately at the center of the sun. It is necessary to place the horizontal and vertical wires tangent to the apparent disk, Fig. 8, and to apply to the readings on both the horizontal and the vertical circle a correction for semi-diameter. The novice will be surprised at the rapidity of the sun's motion when viewed through a high-power telescope. In the forenoon, the sun is moving upwards and to the right. For morning work, therefore, the telescope is set so that the upper edge of the sun is somewhat below the middle cross-wire and the right-hand edge is somewhat to the left of the vertical wire. If the telescope has stadia wires, care must be taken that neither the upper nor the lower stadia wire is mistaken for the middle wire. Then, with one hand on the tangent screw to the vertical arc and the other hand on the tangent screw to the horizontal plate, either or both screws are adjusted so as to make the sun reach both cross-wires at the same instant. Some practice will be required in order to do this accurately. In the afternoon, the sun is moving downwards and to the right, and the telescope should be so set that the sun will move into the upper left-hand corner.

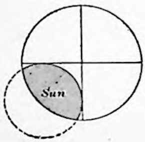


FIG. 8

37. Corrections.—The measured altitude of the sun's edge must be corrected for index error, refraction, and semi-diameter. The reading of the horizontal circle must be corrected for semi-diameter. The correction to the vertical circle reading is simply the angular semi-diameter of the sun, and this should be subtracted from the reading if the upper edge of the sun has been observed, but added to the reading if the wire has been placed tangent to the lower edge of the sun.

The correction to the horizontal reading is effected in the following manner: Let O, Fig. 9, be the center of the celestial

sphere, Z the zenith, and $W H K$ the horizon; let the telescope be at O and directed to the sun's edge at T, and let $S T$ be the radius of the sun.

When the intersection of the cross-wires is moved from T to S, the horizontal angle $K O H$ is turned off on the horizontal circle; it is desired to find this horizontal angle, or what is the same thing, the angular length of the arc $K H$, which is the correction for semi-diameter to be applied to the horizontal circle reading.

The angles $T R S$ and $K O H$ are equal, since each is equal to the spherical angle $K Z H$ (*Practical Astronomy*, Part 1). Hence, $\frac{\text{arc } S T}{\text{arc } H K} = \frac{R S}{O H} = \frac{R S}{O S}$; for, as shown in geometry, the circumferences of any two circles are to each other as their radii, and arcs subtending equal angles are to each other as their radii.

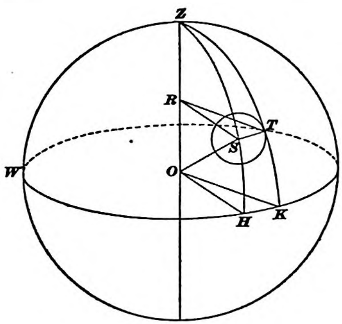


FIG. 9

But the angle $S O H$ is the sun's altitude, and, consequently, the equal angle $R S O$ also equals the sun's altitude. In the triangle $R S O$, $\frac{R S}{O S} = \cos R S O =$ cosine of sun's altitude.

Hence, cosine of sun's altitude $= \frac{\text{arc } S T}{\text{arc } H K}$, or arc $H K =$ arc $S T \div$ cosine of sun's altitude.

Hence, the correction to be applied to the reading of the horizontal circle is found by dividing the sun's semi-diameter by the cosine of its altitude. This correction is to be added to the reading of the horizontal circle if the wire is placed tangent to the left edge of the sun, but subtracted from the reading of the horizontal circle if the wire is placed tangent to the right edge of the sun.

The semi-diameter of the sun may be taken with sufficient accuracy from the following table, which is given in *Practical Astronomy*, Part 1, and is here repeated for convenience:

Time of Year (Approximately)	Semi-Diameter of Sun	Time of Year (Approximately)	Semi-Diameter of Sun
January 1 . .	16′ 18″	July 1	15′ 45″
April 1	16 2	October 1 . .	16 2

38. Formula for Finding the Sun's Azimuth When the True Altitude is Known.

Let z = zenith distance of the sun;
t = hour angle;
δ = declination;
φ = latitude of observer;
a = azimuth, counted from north toward east.

The value of a is computed by the following formula, which is derived by the principles of spherical trigonometry:

$$\sin \tfrac{1}{2} a = \sqrt{\frac{\cos \frac{1}{2}(z + \varphi + \delta) \sin \frac{1}{2}(z + \varphi - \delta)}{\sin z \cos \varphi}}$$

In applying this formula, it should be remembered that there are always two angles corresponding to a given sine. Thus, if $\sin A = .5$, the angle A may be either 30° or 150° (= 180° − 30°), since any two supplementary angles have the same sine. This is true in all cases, the formula $\sin (180° - A) = \sin A$ being perfectly general, whether A is greater or less than 180°. For example, $\sin 275° = \sin (180° - 275°) = \sin (-95°) = -\sin 95° = -\sin (180° - 95°) = -\sin 85°$.

As explained in Art. 32, a negative azimuth equal to $-a$ is the same as a positive azimuth equal to $360° - a$.

It will be observed that, in the forenoon, the sun is east of the meridian, and its azimuth is, therefore, less than 180°. In the afternoon, the sun is west of the meridian, and its azimuth is greater than 180°. Of the two values of $\frac{1}{2}a$ given by the formula, one is acute and the other obtuse. For morning observations, the acute angle should be used; for afternoon observations, the obtuse.

The zenith distance z is obtained by subtracting the corrected altitude from 90°. The value of φ must be known at least approximately.

The desired azimuth of the line is obtained by subtracting the corrected mean reading of the horizontal circle from the azimuth of the sun. If the minuend of this subtraction is greater than the subtrahend, it should be increased by 360°. Let the student verify this by drawing a sketch showing the object M, Fig. 7, on the right of PS.

It must be remembered that the horizontal angles are measured in a clockwise direction.

EXAMPLE.—The following observations were taken in the morning, in the manner described in Art. 35:

Vertical Circle	Horizontal Circle	Diagram of Field
22° 48.5′	237° 41.0′	
22 12.5	238 11.0	
21 44.5	238 34.0	
21 19.0	238 55.0	
20 49.5	239 19.5	
20 28.0	239 38.0	

The declination of the sun was +14° 45′ 40″, the approximate latitude was 39° 58′, and the semi-diameter of the sun was 15′ 54″; the index error was eliminated. To find the azimuth of the sun.

SOLUTION.—

Mean of the vertical circle readings	21° 33′ 40″
Refraction (Table VII, *Practical Astronomy*, Part 1)	−2 24
Parallax (Table VIII, *Practical Astronomy*, Part 1)	+8
Semi-diameter, which is to be subtracted (see field diagram)	−15 54
True altitude	21° 15′ 30″
Zenith distance = 90° − true altitude	68 44 30
Mean reading of horizontal circle	238° 43′ 5″
Correction for semi-diameter = 15′ 54″ ÷ cos 21° 15′ 30″ = 954″ ÷ 0.932, to be subtracted (see field diagram)	−17 4
True horizontal angle to the center of the sun .	238° 26′ 1″

To find the azimuth of the sun:

$$z = 68° \ 44' \ 30''; \ \varphi = 39° \ 58'' \ 0''; \ \delta = 14° \ 45' \ 40''$$

$$\tfrac{1}{2}(z + \varphi + \delta) = 61° \ 44' \ 5''; \ \tfrac{1}{2}(z + \varphi - \delta) = 46° \ 58' \ 25''$$

Substituting these values in the formula,

$$\sin \tfrac{1}{2} a = \sqrt{\frac{\cos 61° \ 44' \ 5'' \ \sin 46° \ 58' \ 25''}{\sin 68° \ 44' \ 30'' \ \cos 39° \ 58'}}$$

$$\tfrac{1}{2} a = 44° \ 7' \ 14''; \ a = 88° \ 14' \ 28''$$

The acute angle corresponding to $\sin \frac{1}{2} a$ is taken, because the observation was made in the forenoon. As the horizontal reading 238° 26′ 01″ is greater than the computed azimuth of the sun, 360° should be added to the latter, before subtracting the former. Then, the azimuth of the mark is

$$360° + 88° \ 14' \ 28'' - 238° \ 26' \ 1'' = 209° \ 48' \ 27''. \quad \text{Ans.}$$

EXAMPLE FOR PRACTICE

The following observations were taken in the manner described in Art. **35**:

Approximate Time A. M.	Vertical Circle	Horizontal Circle	Diagram of Field
8h 40m	43° 09′ 0″	64° 42′ 0″	
8 42	43 35 30	65 10 30	

The declination of the sun was +19° 43′ 10″; the latitude was +40° 36′ 27″; and the semi-diameter was 15′ 49″. Find the azimuth.

Ans. 36° 34′ 31″

---

39. In this method, each single observation may be reduced separately, if desired. When this is done, the agreement of the individual values of the azimuth will furnish a test of the accuracy with which the observer has made the observations, but not a test of the accuracy of the result, since the errors of adjustment, which always produce a comparatively large error in all determinations of azimuth, have not been eliminated. The additional labor of reducing each observation separately is considerable, and is not justified by the character of the work done. It would be much better to devote the additional time to taking another series; the two series may then each be reduced as in the preceding article.

If the vertical circle of the transit is not complete, the best determination of azimuth will be secured by taking several series of observations. If the vertical circle is complete, at least two series of observations should be taken, one with the telescope direct and one with it reversed, if this method is used at all. When the transit has a complete vertical circle, the following modification of the method is preferred:

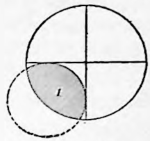


Fig. 10

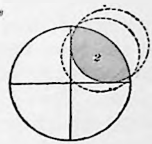


Fig. 11

40. Modification of the Method When the Vertical Circle is Complete.—In this modification, the errors of adjustment, the index error, and the corrections for semi-diameter are eliminated by the method of taking the observations. The computation is performed exactly as in Art. **38**, except that the corrections just referred to are omitted.

The instrument is set up as before, with the horizontal plate reading 0° when sighting at the azimuth mark. For forenoon work, the sun should be so sighted that it occupies position *1*, Fig. 10, with reference to the cross-wires. The time, vertical angle, and horizontal angle are noted as before. Then the upper plate is loosened, the instrument turned 180° in azimuth, the telescope inverted, and the sun sighted again, as in position *2*, Fig. 11. In position *1*, Fig. 10, the sun is moving toward both wires; in position *2*, Fig. 11, the telescope should be set approximately as shown by the dotted circle, so that the sun will clear both wires at the same instant. For afternoon work, the positions shown in Figs. 12 and 13 should be used. The observations are taken in *pairs;* if the second observation of a pair cannot be obtained promptly after the

Afternoon

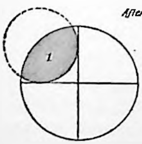


Fig. 12

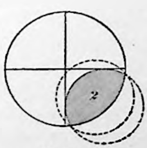


Fig. 13

first one (owing to a passing cloud, or some other cause), the first must be ignored and considered as useless.

41. It should be noted that the reversal of the transit between the observations eliminates the index error of the vertical circle, the error of level in the horizontal axis of the telescope, and the error of collimation of the telescope. By sighting in diagonal corners of the field of view and taking the mean of the observations, the corrections (both horizontal and vertical) due to the semi-diameter of the sun are eliminated. To simplify the notes, 180° should be added to (or subtracted from) the horizontal plate reading when the instrument is inverted.

EXAMPLE.—The following measurements were taken in the manner described in this article. The four means of the circle readings were formed in the field. The declination of the sun was −9° 30′ 5″, and the approximate latitude +39° 57′. To find the azimuth.

Telescope	Time P. M.	Vertical Circle	Horizontal Circle
Direct . . .	3:27	19° 39′ 00″	99° 52′ 00″
Inverted . .	3:29	19 52 00	99 49 00
Mean . . .	3:28	19 45 30	99 50 30
Direct . . .	3:32	18 46 00	100 55 30
Inverted . .	3:34	19 3 00	100 49 00
Mean . . .	3:33	18 54 30	100 52 15
Direct . . .	3:36	18 4 30	101 46 00
Inverted . .	3:38	18 23 30	101 35 00
Mean . . .	3:37	18 14 00	101 40 30
Direct . . .	3:40	17 26 30	102 29 30
Inverted . .	3:42	17 43 00	102 21 00
Mean . . .	3:41	17 34 45	102 25 15

SOLUTION.—

Mean of the four vertical circle readings	18° 37′ 11″
Refraction (Table VII, *Practical Astronomy*, Part 1)	−2 48
Parallax (Table VII, *Practical Astronomy*, Part 1)	+8
True altitude of center	18° 34′ 31″
Zenith distance = 90° − true altitude	71° 25′ 29″

To find the azimuth of the sun: $z = 71° 25' 29''$; $\varphi = 39° 57' 0''$; $\delta = -9° 30' 5''$; $\frac{1}{2}(z + \varphi + \delta) = 50° 56' 12''$; $\frac{1}{2}(z + \varphi - \delta) = 60° 26' 17''$.

Substituting these values in the formula of Art. **38**,

$$\sin \tfrac{1}{2} a = \sqrt{\frac{\cos 50° 56' 12'' \sin 60° 26' 17''}{\sin 71° 25' 29'' \cos 39° 57'}}$$

The two values of $\frac{1}{2} a$ are $60° 17' 15''$ and $119° 42' 45''$ ($= 180° - 60° 17' 15''$). As the observations were made in the afternoon, the obtuse angle should be used (see Art. **38**). This gives

$$a = 2 \times 119° 42' 45'' = 239° 25' 30''$$

The mean of the four horizontal readings is $101° 12' 8''$. Subtracting this from the azimuth of the sun, the azimuth of the line is found to be $239° 25' 30'' - 101° 12' 8''$, or $138° 13' 22''$. Ans.

SECOND METHOD OF DETERMINING AZIMUTH: BY THE SOLAR ATTACHMENT

42. Description.—The foregoing methods have the advantage of requiring no extra equipment to the transit, except an inexpensive colored-glass cap, but they require considerable numerical computation. Some transits are provided with a device known as the **solar attachment,** by means of which the true meridian can be located directly from observation. There are several forms of solar attachment and of solar transit, but they all have the condition that when the latitude and declination have been properly set off and the instrument so turned that an image of the sun is formed in a specified place, the main telescope of the transit will lie in the meridian. A detailed description is here given of one of the best and simplest forms of solar attachment, known as the **Saegmuller solar attachment.**

A view of the attachment alone is shown in Fig. 14. It consists of a small telescope EO, having a colored-glass cap over the object glass O, and a prismatic eyepiece E, which permits the observer to look into the side of the eyepiece instead of into its end, thus enabling him to make an observation with the telescope nearly vertical. A level bubble is on the top of the telescope. At the ends of the level bubble are two circular sights s and s', the latter being somewhat smaller than the former. When directing the telescope to the sun, the quickest method is so to turn it that the shadow of the

smaller sight s' will fall on the sight s, forming a ring of light around its edge. When this occurs, the image of the sun will be found within the field of view in the telescope. A clamp screw C secures the telescope axis to the standards S, and a tangent screw T permits slow motion. The standards and the telescope revolve about an axis, called the **polar axis,** hidden by the standard in Fig. 14. A clamp screw M and a slow-motion screw N permit the accurate adjustment of the

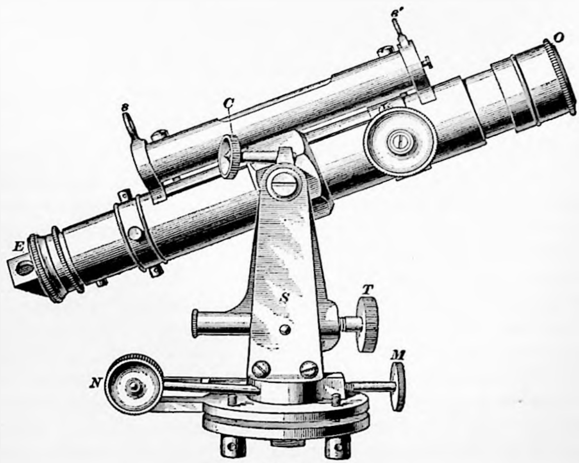


FIG. 14

instrument about its polar axis. By means of the screws in the base plate of the attachment, the polar axis is so adjusted as to be perpendicular to the line of collimation or optical axis, and also to the transverse axis of the transit telescope.

43. Preparation for Making the Observations. First, a table of declinations of the sun to be set off on the vertical circle is prepared. The declination of the sun for the beginning of each hour during the time in which the observations are to be made may be taken from the Ephemeris,

or from Table IV, *Practical Astronomy*, Part 1. These declinations are set off on the vertical circle of the transit in order to cause the solar telescope to point directly to the sun. As the apparent position of the sun in the sky is modified by refraction, a correction for refraction must be applied before setting off the declination on the vertical circle. If this were not done, the solar telescope would not point to the apparent position of the sun in the sky, but to its true position, and the sun would not be visible. This correction, which is to be added to the declination, may be taken directly from Table IV at the end of this Section, for any given latitude and hour angle.

EXAMPLE.—What declination should be set off on the vertical circle when the latitude is 39° 57′, the time 8:30 A. M., and the declination +3° 10′?

SOLUTION.—For 8:30 A. M. the hour angle is equal to $12 - 8.5 = 3.5$. Assume the latitude to be 40° (which is near enough); for declination 0° and hour angle 3, we have 1′ 9″, while for hour angle 4, we have 1′ 36″; hence, for hour angle 3.5 it should be the mean of these, or 1′ 22″. For declination +5°, we similarly compute 1′ 9″ for hour angle 3.5. The difference is $1' 22'' - 1' 9'' = 0' 13''$, and interpolating proportionally between the two values, for declination 3° 10′, we have $1' 22'' - \frac{3\frac{1}{6} \times 13}{5} = 1' 14''$. The declination to be set is, therefore, $3° 10' + 1' 14'' = 3° 11' 14''$. Ans.

44. Adjustment of the Instrument.—Adjust the eyepieces of both telescopes for parallax; focus the object glass of the solar for observing the sun; focus the object glass of the transit telescope for observing the azimuth mark. These adjustments should be made before observation is commenced, so as to avoid the jarring of the instrument that might occur if it were done later.

Level the instrument with extreme care. The plate bubble parallel to the telescope is almost invariably too sluggish for the accuracy required in this work. Therefore, clamp the telescope nearly horizontal; by repeated reversions of the whole instrument about its vertical axis, with corresponding adjustments of the leveling screws and the tangent screw to the vertical arc, the instrument may be so leveled that the

bubble of the telescope level will remain in the center of the tube for any position of the instrument. Under these conditions, the vertical axis is truly vertical and the reading of the vertical arc will be its index error, assuming that the line of collimation is parallel to the axis of the bubble tube. The exact error of the adjustment of the plate bubbles is then apparent and should be noted, so that any accidental change of level that may occur during the time of taking observations may be at once detected. The parallelism of the line of collimation and the axis of the telescope level can be tested and adjusted by the peg method described in *Leveling*.

45. Principle of the Instrument.—Set vernier A of the horizontal plates at 0° and point at the azimuth or reference mark. Loosen the upper plate and swing the telescope approximately into the meridian, with the objective end toward the south. If the declination is south, point the transit telescope upwards with a vertical angle equal to the value of the declination as modified by refraction. If the declination is north, point the telescope downwards with a vertical angle $K O C$, Fig. 15, equal to the modified declination. Then place the solar telescope in the same vertical plane as the transit telescope, make it horizontal by means of its level bubble, and clamp it securely, using screws C and T, Fig. 14.

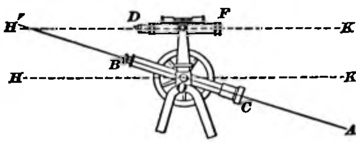


FIG. 15

Since the solar telescope is now horizontal, Fig. 15, and since the transit telescope is inclined to the horizon by an angle $K O A$ equal to the sun's declination, it follows that the optical axis of the solar telescope and that of the transit telescope are inclined to each other at an angle $F H' C$, which is equal to the sun's declination.

Now bring the transit telescope into the plane of the equator. As the angle $E O K$, Fig. 16, between the plane of the equator and the plane of the horizon is equal to 90° minus

the latitude, this may be done by turning the transit telescope upwards until it makes a vertical angle with the plane of the horizon equal to 90° minus the latitude. The polar axis of the solar is thus made approximately parallel to the earth's axis OP, since it is perpendicular to the transit telescope BC, which now lies in the plane of the equator OE. It will be exactly parallel when the transit telescope is exactly in the meridian. The solar telescope is pointing above (or below) the plane of the equator by an angle $RH'E$ equal to the sun's declination, since this is the angle between the axes of the two telescopes and the large telescope lies in the

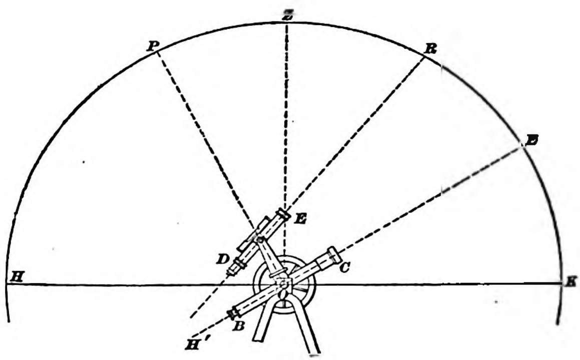


FIG. 16

equator plane. Hence, if we now turn the solar telescope about its polar axis, its line of collimation produced will trace out in the sky a small circle parallel to the equator. The declination of all points on this small circle will be equal to the sun's declination. Hence, the polar axis being in the plane of the meridian and parallel to the earth's axis, it follows that by turning the solar telescope about the polar axis it may be pointed directly to the sun. The converse is also true, that when the solar telescope is pointed directly to the sun, the polar axis will lie in the plane of the meridian. But since the polar axis and the optical axis of the transit

telescope are in the same vertical plane, it follows that when the polar axis lies in the plane of the meridian the optical axis of the transit telescope will lie in this plane also. Hence, when the solar telescope is pointed at the sun, the transit telescope will be in the plane of the meridian.

46. Method of Making the Observations.—Having adjusted the inclination of the two telescopes to the horizon, by setting off the proper angles on the vertical circle, as just explained, bring the transit telescope into the meridian as nearly as possible, and clamp the upper plate. Direct the solar telescope to the sun by the shadow of the sights on top of the solar telescope, and, with screw *M*, Fig. 14, clamp the motion about the polar axis. Then, with one hand on the slow-motion screw of the polar axis *N*, and the other hand on the tangent screw of the upper transit plate, point the solar telescope exactly at the center of the sun. When the instrument is in this position, the reading of the horizontal plate gives one value of the angle between the azimuth mark and the meridian. To obtain another value, which shall be independent of the previous determination, loosen the upper transit plate, the clamp screws *C* and *M* of the solar, and repeat the operation, making due allowance, if necessary, for any change of declination that may have taken place in the interval. With practice, several observations may be taken in a very few minutes, during which time no appreciable change of declination will take place, even when the motion of the sun in declination is most rapid. Watch the levels carefully for any indication of jarring or disturbance of the instrument. The lower plate should be kept clamped throughout, and the vernier should always read 0° when pointing at the azimuth mark. After completing a set of readings, check the reading on the azimuth mark to see if the plates have slipped. Observe the needle reading when the instrument is in the meridian; this will show the declination of the needle for that time and place. The mean of all the horizontal angles read is the desired azimuth.

EXAMPLE.—It is desired to make observations for azimuth with a solar transit at Philadelphia between 9 A. M. and 10 A. M. on

January 5, 1903. The latitude of Philadelphia is +39° 58′, and the longitude is $-7^m\ 37^s$. It is required to make out a table of corrected declination settings for use in the field.

SOLUTION.—First find the true declination of the sun for the hours of 9 A. M. and 10 A. M., Philadelphia mean time.

January 5, 9 A. M. civil time = January 4, 21^h A. M. astronomical time. Longitude of Philadelphia (Ephemeris) is $-7^m\ 37^s = .127^h$. Washington time corresponding to 9 A. M. is January 4, 20.873^h. The declination (Table IV, *Practical Astronomy*, Part 1) for noon of January 4 at Washington is	−22° 47′ 42.8″
Hourly motion in declination is +15.06″. Multiplying this hourly motion by the number of hours that have elapsed between Washington noon and 9 A. M., we obtain the increase in the sun's declination during this period = +15.06″ × 20.873	+5′ 14.3″
Declination at 9 A. M., to the nearest second	−22° 42′ 28.0″
Add to the declination at 9 A. M. the hourly motion	+15
Declination at 10 A. M.	−22° 42′ 13″

We now form the following table:

Time	Declination	Correction for Refraction	Vertical-Circle Setting
$9^h\ 0^m$	−22° 42′ 28″	+3′ 2″	−22° 39′ 26″
9 10	−22 42 25	+2 54	−22 39 31
9 20	−22 42 23	+2 46	−22 39 37
9 30	−22 42 20	+2 38	−22 39 42
9 40	−22 42 18	+2 30	−22 39 48
9 50	−22 42 15	+2 22	−22 39 53
10 0	−22 42 13	+2 14	−22 39 59

The second column is found by simply adding one-sixth of 15.06″ = 2.51″, successively, to the declination, beginning with that for 9 A. M. The third column is computed from Table IV. As explained in Art. **43**, corresponding to the latitude of 40° and an hour angle of 3 hours, we have

Correction for a declination of −20°	+2′ 36″
Correction for a declination of −23° 27′	+3 9
Difference	+33″

Hence, the correction corresponding to the given declination of $-22° 42'$ is found by interpolation to be

$$2' 36'' + \left(\frac{2° 42'}{3° 27'} \times 33''\right) = 2' 36'' + \left(\frac{270}{345} \times 33''\right) = 3' 2''.$$

Similarly, the correction for 10 A. M. is found to be $+2' 14''$, and the other numbers of the column are found by simply adding one-sixth of the difference between these two successively. Finally, by adding each correction to the corresponding number of the preceding column, we obtain the last column of settings for use in the field.

47. Cross-Wires in the Solar.—The Saegmuller solar telescope is provided with three horizontal and three vertical cross-wires, as shown in Fig. 17. The spacing of the extreme wires is purposely made very nearly equal to the apparent diameter of the sun, and it is therefore very easy to equalize the margin on all sides and thus sight the telescope directly at the center of the sun. The complication of allowing for the semi-diameter of the sun, as in previous methods, is thus avoided.

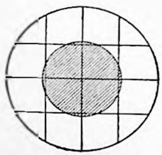


FIG. 17

THIRD METHOD OF DETERMINING AZIMUTH: BY OBSERVING POLARIS AT ELONGATION

48. General Considerations.—The most accurate method for determining azimuth is by observing Polaris at elongation. Since at that time the star is moving in a direction nearly parallel to the plane of the meridian and hence perpendicular to the plane of the horizon, its azimuth remains practically unchanged for several minutes. A method of determining azimuth by observing Polaris at elongation was given in *Transit Surveying*, Part 2. The one explained below is more accurate, although not so convenient.

49. General Formulas.—The following formulas are obtained from spherical trigonometry:

Let a = azimuth of Polaris;
δ = declination of Polaris;
t = hour angle of Polaris at elongation;
φ = observer's latitude.

Then, $$\cos t = \frac{\tan \varphi}{\tan \delta} \qquad (1)$$

$$\sin a = \frac{\cos \delta}{\cos \varphi} \qquad (2)$$

If the right ascension of Polaris is denoted by α, the sidereal time of elongation is (see *Practical Astronomy*, Part 1) $\alpha \pm t$, the plus sign applying to western elongation, and the minus sign to eastern elongation. By reducing this sidereal time to mean time, the mean time of elongation is obtained. By measuring at that time the angle between the star and the mark whose azimuth is required, the latter azimuth is readily determined by adding the azimuth of Polaris to or subtracting it from the angle between the star and the mark.

The quantities δ and α are taken from the Ephemeris.

50. Making the Observations.—The manner in which observations are made is different from that employed in any of the preceding methods. In every other case, the vernier of the horizontal circle was first set at zero, but in this method, which is susceptible of a high degree of accuracy, it is better to distribute the readings around the horizontal circle. The telescope being directed to the reference mark, the horizontal circle is read; then, without loosening the lower clamp, the telescope is pointed to the star, and the circle is read again. The position of the lower plate is next changed, the vernier of the horizontal circle turned 180°, the telescope inverted, and the observation repeated, taking one reading on the mark and one on the star. The process is continued in the same manner, inverting the telescope and changing the position of the vernier on the horizontal plate after each complete observation. The azimuth computed by formula 2, Art. 49, is added to the angle between the star and the mark, if Polaris is at eastern elongation; otherwise, it is subtracted. The result is the azimuth of the mark.

It is not necessary to record the time, but the observations must not be extended more than 10 or 12 minutes on each side of elongation.

FOURTH METHOD OF DETERMINING AZIMUTH: BY OBSERVING A SOUTH STAR AT ANY HOUR ANGLE

51. This method requires not even the roughest knowledge of the time; the observations may be made at any time, provided only that the star observed, which may be any south star that can be identified by the observer, is at least 2 hours from the meridian; and in order to secure increased accuracy, the observations may be continued as long as desired. It is probable that this method will be preferred by the student, but he should bear in mind that when azimuth is determined by observations on Polaris at elongation (third method), the results will be more accurate than can be obtained in any other way. In the preceding method, only the vertical wire was used; in the present method both vertical and horizontal wires must be in accurate adjustment, and the star brought exactly to their intersection.

52. Method of Observing.—The instrument having been carefully leveled, clamp the lower motion, sight at the mark, and read the horizontal circle. Unclamp the upper motion, bring the intersection of the cross-wires accurately on the star, and read both the horizontal and the vertical circle. This completes one observation.

Now unclamp both lower plates, change the position of the vernier on the horizontal circle by 20° or 30°, revolve the instrument in azimuth 180°, invert the telescope, and repeat the observation. If the transit has not a complete vertical circle, it will not be possible to invert it. In this case, if the highest accuracy is desired, a mercury horizon should be used and each alternate pointing be made at the star reflected in the mercury.

Six observations, at least, should be taken as above described, the position of the lower plate being changed after each observation, and the telescope used alternately direct and inverted, if possible.

53. Rule for Computing the Azimuth.—*Subtract each reading of the horizontal circle when pointed on the mark*

from the corresponding reading when pointed on the star, and form the mean of the resulting differences. Next, form the mean of the vertical circle readings and correct this for index error if the telescope was not inverted and if a mercury horizon was not used; otherwise, no correction for index error need be applied. Correct also for refraction: the result is the true altitude of the star. Subtract the true altitude from 90°, to obtain the zenith distance.

Take from the Ephemeris, or from Table VI, Practical Astronomy, Part 1, the value of the declination. Find the azimuth of the star by applying the formula of Art. 38. Subtract the corrected mean reading of the horizontal circle from the azimuth of the star, adding 360° to the latter azimuth, if necessary. The result is the desired azimuth of the line.

It should be constantly borne in mind that in turning the instrument from the mark to the star, it should be turned toward the right.

EXAMPLE.—On January 5, 1903, at Philadelphia, the following readings were taken on the star Sirius when the star was east of the meridian. The latitude of Philadelphia being 39° 58′, find the azimuth of the mark.

Telescope	Vertical Circle			Horizontal Circle: Pointing on Mark			Horizontal Circle: Pointing on Star		
Direct . . .	20°	17′	10″	300°	27′	10″	330°	29′	0″
Inverted . .	200	17	20	114	30	0	144	33	20
Inverted . .	200	18	10	175	35	10	205	38	50
Direct . . .	20	18	30	60	24	30	90	29	30
Direct . . .	20	19	20	108	10	0	138	15	50
Inverted . .	200	20	30	3	6	10	33	12	30

SOLUTION.—To find the true altitude:

Mean of vertical circle readings	20° 18′ 30″
Refraction (Table VII, *Practical Astronomy*, Part 1)	−2 33
True altitude	20° 15′ 57″
Zenith distance = 90° − true altitude	69° 44′ 3″

To find the azimuth of the star:

$z = 69° \ 44' \ 3''$, $\varphi = 39° \ 58'$, $\delta = -16° \ 35' \ 9''$ (Table VI, *Practical Astronomy*, Part 1); $\frac{1}{2}(z + \varphi + \delta) = 46° \ 33' \ 27''$; $\frac{1}{2}(z + \varphi - \delta)$

$= 63° 8' 36''$. Substituting these values in the formula of Art. **38**, we have,

$$\sin \tfrac{1}{2} a = \sqrt{\frac{\cos 46° 33' 27''; \sin 63° 8' 36''}{\sin 69° 44' 3'' \cos 39° 58'}}$$

$$\tfrac{1}{2} a = 67° 28'; \; a = 134° 56'$$

Since the star is east of the meridian, the acute angle corresponding to $\sin \frac{1}{2} a$ is used. The mean of the differences of the circle readings is $30° 4' 20''$. Subtracting this from the azimuth of the star, the azimuth of the mark is found to be $104° 51' 40''$. Ans.

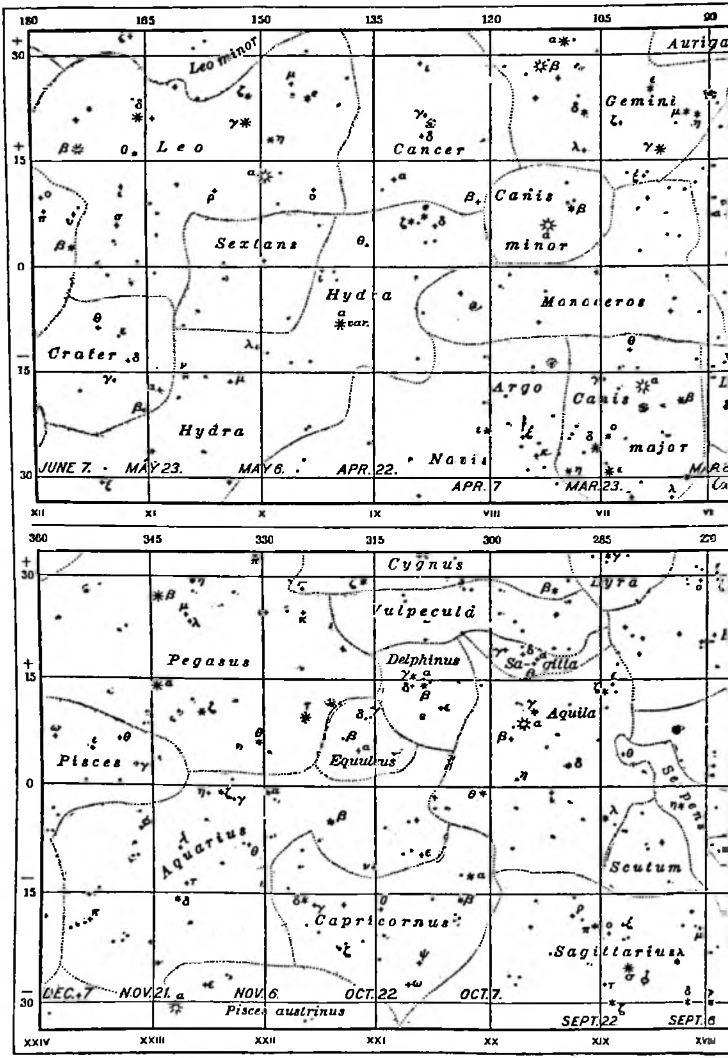
FIFTH METHOD OF DETERMINING AZIMUTH: BY OBSERVING POLARIS AT ANY HOUR ANGLE

54. Advantages of the Method.—The greatest advantage of the method about to be described is that the observations can be made at about the time of sunset. It is therefore unnecessary for the observer to place a light at the station that he is observing, or to illuminate the wires of his instrument, and this makes the method more convenient than any other in which a star is employed.

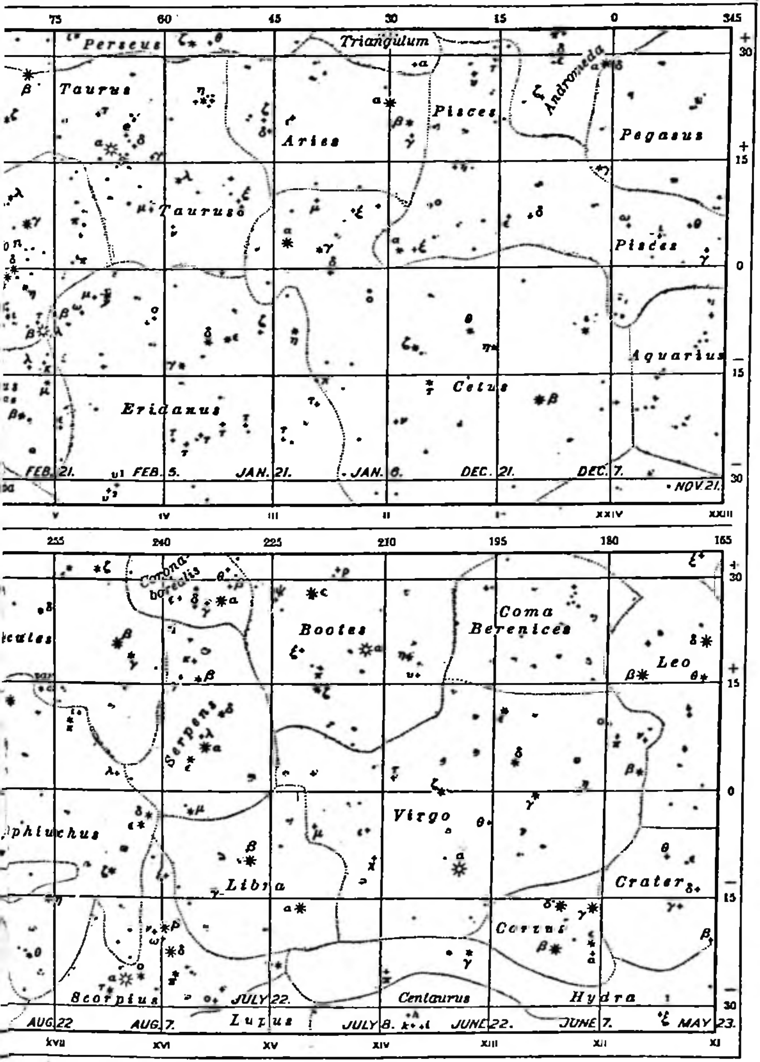
This method is more accurate than any of those in which the sun is observed, because the motion of Polaris is so slow that the error of the watch need only be known within a minute or two. This error can be quickly and easily found in the field, as described in Art. **55.**

It is strongly recommended that the student use either this method or that of observing Polaris at elongation, since these are far more accurate than observations on the sun can ever be. If, however, he cannot choose his time for observing, but is compelled to make his observations at various times during the day, he should employ the methods of Arts. **34** to **41.**

55. To Find the Sidereal Time and the Watch Error.—If the mean local time is known, it may be easily converted into sidereal time, as was explained in *Practical Astronomy*, Part 1. Otherwise, the sidereal time may be determined with sufficient accuracy as follows: Point the telescope at Polaris and carefully bisect the star with the vertical wire. Then revolve the telescope on the horizontal axis



Map of Stars between



…h and 30 South Declination.

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until it points toward the south. Since Polaris is always nearly in the meridian, the optical axis of the telescope, after being revolved, will point nearly toward the south.

Now point the telescope at such an elevation that one of the bright southern stars shown in the map of Fig. 18 will pass across the field. By sighting along the top of the tube, several such stars can always be found. These are, of course, all moving across the sky from east to west.

When the star chosen is seen to enter the field, clamp the vertical motion and bring the star near the horizontal wire. At the instant that it passes the vertical wire, note the watch time. Since the star is very nearly on the meridian, the true sidereal time at this instant is very nearly equal to the right ascension of the star; and hence the watch error is obtained by subtracting the time shown by the watch from the right ascension. This error, being added algebraically to any watch time, gives the corresponding sidereal time.

56. Explanation of the Map.—The map, Fig. 18, shows all the brighter south stars whose declinations lie between $-30°$ and $+30°$. The student should notice that the lower edge of the map is not parallel to the horizon, but is parallel to the celestial equator. The vertical lines are hour circles; the right ascension of each of these hour circles is written at the top of the map in degrees, and at the bottom in hours. At the bottom of the map is also written the time of the year when each of these hour circles is due south at 7 P. M. For example, on July 8, the circle whose right ascension is 14 hours is due south at 7 P. M.; that is, it then coincides with the meridian. The bright star α Bootis is then 10 minutes east of the meridian, and α Virginis is about 45 minutes west of the meridian. Either of these stars would be a good one to observe on this date.

The name of each star is found as follows: The dotted lines show the limits of the various constellations. Each star of a constellation is named by merely prefixing the Greek letter shown on the map to the name of the constellation. Thus, δ Virginis is the star δ of the constellation

Virgo. As the names of the constellations are in Latin, they are written in the genitive case when preceded by the letter of a star. Thus, α Virginis may be considered equivalent to α of Virgo.

EXAMPLE.—What star on the map has a right ascension of $11^h\ 44^m$, and a declination of $+15°\ 7'$?

SOLUTION.—Looking at the position indicated, we find a star marked β in the constellation Leo. Hence, the star is β Leonis. Ans.

EXAMPLE FOR PRACTICE

Tell the names of the stars in the following positions: (*a*) Right ascension $19^h\ 46^m$, declination $+8°\ 34'$; (*b*) right ascension $0^h\ 3^m$, declination $+28°\ 33'$; (*c*) right ascension $7^h\ 34^m$, declination $+5°\ 29'$. (*d*) What stars are well suited for observation soon after sunset on October 7?

Ans.
- (*a*) α Aquilæ
- (*b*) α Andromedæ
- (*c*) α Canis minoris
- (*d*) β and ε Cygni, β and γ Delphini, α, γ, δ, and θ Aquilæ, γ Sagittarii, and α and β Capricorni

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57. To Compute the Azimuth of Polaris, the Sidereal Time Being Known.

Let t = hour angle of Polaris;
α = its right ascension;
δ = its declination;
θ = sidereal time;
φ = latitude of observer;
K = quantity given in Table V;
a = azimuth of Polaris at time θ.

Then, $t = \theta - \alpha$, and a is approximately given by the formula

$$a = -\frac{\sin t}{\cos \varphi}\left[(90° - \delta) + K \tan \varphi\right]$$

EXAMPLE.—The right ascension of Polaris being $1^h\ 24^m\ 0^s$, its declination $+88°\ 47'\ 26''$, the sidereal time $9^h\ 27^m$, and the latitude of the observer $+39°\ 58'$, find the azimuth of Polaris.

SOLUTION.—In order to apply the formula, $\varphi = +39°\ 58'$, $\delta = +88°\ 47'\ 26''$, and $t = 9^h\ 27^m\ 0^s - 1^h\ 24^m\ 0^s = 8^h\ 3^m\ 0^s$, which is equal to

120° 45′. The value of K corresponding to 120.8° is found, from Table V, to be −46″. Substituting these values in the formula,

$$a = -\frac{\sin 120° 45'}{\cos 39° 58'}[(90° - 88° 47' 26'') - 46'' \times \tan 39° 58'] = -1° 20' 38''$$

Therefore, the azimuth of Polaris reckoned from the north point is −1° 20′ 38″. Ans.

EXAMPLE FOR PRACTICE

In the above example, find the azimuth of Polaris when the sidereal time is $9^h\ 59^m\ 30^s$. Ans. −1° 12′ 54″

NOTE.—By comparing this answer with that of the preceding example, it may be seen how slowly Polaris moves.

58. Method of Making the Observations.—Set the transit up over the instrument station and level very carefully. Clamp the lower motion and point on the reference mark M. Clamp the upper motion, and, using the tangent screw, carefully bisect the mark with the vertical wire. Read and record both verniers.

Unclamp the upper motion, point on Polaris, and carefully bisect the star with the vertical wire, using the slow-motion screw as before. At the instant the bisection is perfected, read and record the exact watch time in hours, minutes, and seconds, and then read and record both verniers.

Now unclamp the upper motion, again direct the telescope to the mark, and read both verniers. Then sight again to the star, and note both the reading of the verniers and the time. Proceed in the same manner until four or five readings have been obtained. The difference between the mean of the readings on the mark and that of the readings on the star gives the difference between the azimuth of the mark and that of the star. Notice that the lower motion remains securely clamped throughout all these observations.

To make the observation that gives the watch error, point again on Polaris and clamp both motions. Then, being careful not to disturb either the upper or the lower motion, plunge the telescope and point it at such an elevation that any bright south star will pass across the field. Find the name of the star from the map (Art. 56); without disturbing the instrument in any way, look through the telescope, and

at the instant the star crosses the vertical wire, record the watch time. This completes the observation.

59. Rule for Computing the Azimuth of the Line. *Find from the Ephemeris the right ascension and declination of Polaris, and also the right ascension of the south star observed. Compute the watch error (Art. 24). Form the average of the four vernier readings on the mark and also of the vernier readings on the star, and subtract the average of the readings on Polaris from the average of the readings on the mark. The remainder is the angle between the star and the mark. Form the average of the two watch times, and add to this the watch error. The sum is the sidereal time of the observation. Compute the azimuth of Polaris (Art. 58). Add, algebraically, the azimuth of Polaris to the angle between the star and the mark. The sum will be the desired azimuth of the mark.*

EXAMPLE.—Observations made as above described gave the following results: Average of readings on the station, 108° 17′ 30″; average of readings on the star, 30° 3′ 5″; mean of recorded times, $6^h\ 25^m\ 30^s$. The telescope was pointed on α Leonis (see Fig. 18), and the watch time of its transit across the vertical thread was taken; this time was $7^h\ 1^m\ 30^s$. The latitude being +39° 58′; the right ascension of Polaris, $1^h\ 24^m\ 0^s$; its declination, +88° 47′ 26″; and the right ascension of α Leonis, $10^h\ 3^m\ 0^s$, to find the azimuth of the line.

SOLUTION.—First compute the watch error and the sidereal time.

The sidereal time of the transit of α Leonis is equal to its right ascension	$10^h\ 3^m\ 0^s$
The corresponding watch time was	7 1 30
Hence, the watch error was	$3^h\ 1^m\ 30^s$
The average watch time of the pointings on Polaris was	6 25 30
Hence, the sidereal time of the observation was	$9^h\ 27^m\ 0^s$

Next, compute the azimuth of Polaris. (This computation is given in the example, Art. 58.) Azimuth of Polaris = −1° 20′ 38″. Finally, compute the azimuth of the line.

Reading on Station B	108° 17′ 30″
Reading on Polaris	30 3 5
Difference	78° 14′ 25″
Azimuth of Polaris	−1 20 38
Azimuth of line	76° 53′ 47″

Ans.

EXAMPLE FOR PRACTICE

Find the azimuth of the line observed in the following series of observations, the latitude of the observer being $+29° 26'$:

Observations in the field	Average of four readings on station . .	300° 17′ 10″
	Average of four readings on Polaris . .	306° 4′ 0″
	Average of the two watch times	$7^h\ 3^m\ 0^s$
	Watch time of transit of β Aquarii	$7^h\ 22^m\ 20^s$
Quantities found from the Ephemeris	Right ascension of Polaris	$1^h\ 4^m\ 4.7^s$
	Declination of Polaris	+88° 29′ 58″
	Right ascension of β Aquarii . . .	$21^h\ 26^m\ 20^s$

Ans. 355° 42′ 26″

TABLE I

VARIATION OF ALTITUDE IN 1^m FROM NOON

Latitude Degrees	Declination of Different Name From the Latitude											
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°
	"	"	"	"	"	"	"	"	"	"	"	"
0					28.1	22.4	18.7	16.0	14.0	12.4	11.1	10.1
1				28.1	22.4	18.7	16.0	14.0	12.4	11.2	10.1	9.3
2			28.1	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6
3		28.1	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0
4	28.1	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0	7.4
5	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0	7.4	7.0
6	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6
7	16.0	14.0	12.4	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6	6.2
8	14.0	12.4	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6	6.2	5.9
9	12.4	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6	6.2	5.9	5.6
10	11.1	10.1	9.3	8.6	8.0	7.4	7.0	6.6	6.2	5.9	5.6	5.3
11	10.1	9.3	8.6	8.0	7.4	7.0	6.6	6.2	5.9	5.6	5.3	5.1
12	9.2	8.5	7.9	7.4	7.0	6.5	6.2	5.9	5.6	5.3	5.0	4.8
13	8.5	7.9	7.4	6.9	6.5	6.2	5.8	5.6	5.3	5.0	4.8	4.6
14	7.9	7.4	6.9	6.5	6.2	5.8	5.5	5.3	5.0	4.8	4.6	4.4
15	7.3	6.9	6.5	6.1	5.8	5.5	5.3	5.0	4.8	4.6	4.4	4.2
16	6.8	6.5	6.1	5.8	5.5	5.2	5.0	4.8	4.6	4.4	4.2	4.1
17	6.4	6.1	5.8	5.5	5.2	5.0	4.8	4.6	4.4	4.2	4.1	3.9
18	6.0	5.7	5.5	5.2	5.0	4.8	4.6	4.4	4.2	4.1	3.9	3.8
19	5.7	5.4	5.2	4.9	4.7	4.5	4.4	4.2	4.0	3.9	3.8	3.6
20	5.4	5.1	4.9	4.7	4.5	4.3	4.2	4.0	3.9	3.8	3.6	3.5
21	5.1	4.9	4.7	4.5	4.3	4.2	4.0	3.9	3.7	3.6	3.5	3.4
22	4.9	4.7	4.5	4.3	4.1	4.0	3.9	3.7	3.6	3.5	3.4	3.3
23	4.6	4.4	4.3	4.1	4.0	3.8	3.7	3.6	3.5	3.4	3.3	3.2
24	4.4	4.2	4.1	3.9	3.8	3.7	3.6	3.5	3.4	3.3	3.2	3.1
25	4.2	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.1	3.0
26	4.0	3.9	3.8	3.6	3.5	3.4	3.3	3.2	3.1	3.0	3.0	2.9
27	3.9	3.7	3.6	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8
28	3.7	3.6	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7
29	3.5	3.4	3.3	3.2	3.1	3.1	3.0	2.9	2.8	2.8	2.7	2.6
30	3.4	3.3	3.2	3.1	3.0	3.0	2.9	2.8	2.7	2.7	2.6	2.5
31	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.6	2.5	2.5
32	3.1	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.5	2.4
33	3.0	2.9	2.9	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3
34	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.3
35	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2
36	2.7	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1
37	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.2	2.1	2.1
38	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.1	2.0
39	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.1	2.0	2.0	2.0
40	2.3	2.3	2.2	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9
41	2.3	2.2	2.2	2.1	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8
42	2.2	2.1	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8
43	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7
44	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7
45	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6
46	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6
47	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.6
48	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.6	1.5	1.5
49	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5
50	1.6	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.4
52	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.3
54	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3
56	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.2
58	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1
60	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0
62	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9
64	1.0	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
66	0.9	0.9	0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
68	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7
70	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7

TABLE I—*Continued*

Latitude Degrees	Declination of Different Name From the Latitude												
	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°
	″	″	″	″	″	″	″	″	″	″	″	″	″
0	9.2	8.5	7.9	7.3	6.8	6.4	6.0	5.7	5.4	5.1	4.9	4.6	4.4
1	8.5	7.9	7.4	6.9	6.5	6.1	5.7	5.4	5.1	4.9	4.7	4.4	4.2
2	7.9	7.4	6.9	6.5	6.1	5.8	5.5	5.2	4.9	4.7	4.5	4.3	4.1
3	7.4	6.9	6.5	6.1	5.8	5.5	5.2	4.9	4.7	4.5	4.3	4.1	3.9
4	7.0	6.5	6.2	5.8	5.5	5.2	5.0	4.7	4.5	4.3	4.1	4.0	3.8
5	6.5	6.2	5.8	5.5	5.2	5.0	4.8	4.5	4.3	4.2	4.0	3.8	3.7
6	6.2	5.8	5.5	5.3	5.0	4.8	4.6	4.4	4.2	4.0	3.9	3.7	3.6
7	5.9	5.6	5.3	5.0	4.8	4.6	4.4	4.2	4.0	3.9	3.7	3.6	3.5
8	5.6	5.3	5.0	4.8	4.6	4.4	4.2	4.0	3.9	3.7	3.6	3.5	3.4
9	5.3	5.0	4.8	4.6	4.4	4.2	4.1	3.9	3.8	3.6	3.5	3.4	3.3
10	5.0	4.8	4.6	4.4	4.2	4.1	3.9	3.8	3.6	3.5	3.4	3.3	3.2
11	4.8	4.6	4.4	4.2	4.1	3.9	3.8	3.6	3.5	3.4	3.3	3.2	3.1
12	4.6	4.4	4.3	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0
13	4.4	4.3	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9
14	4.2	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8
15	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8
16	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7
17	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7	2.6
18	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.5
19	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.6	2.5
20	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.6	2.5	2.4
21	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.4	2.4
22	3.2	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.4	2.4	2.3
23	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.4	2.4	2.3	2.3
24	3.0	2.9	2.8	2.8	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2
25	2.9	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2
26	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.1	2.1
27	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.2	2.2	2.1	2.1	2.1
28	2.6	2.6	2.5	2.5	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.1	2.0
29	2.6	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.0	2.0	2.0
30	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9
31	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9
32	2.3	2.3	2.2	2.2	2.2	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8
33	2.3	2.2	2.2	2.1	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8	1.8
34	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8
35	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.8	1.8	1.8	1.7	1.7
36	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7
37	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.6
38	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6
39	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.6	1.6
40	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5
41	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5
42	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5
43	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.4	1.4
44	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4
45	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4
46	1.6	1.6	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.3	1.3
47	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.3	1.4	1.3	1.3
48	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3
49	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2
50	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2
52	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.1	1.1
54	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1
56	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0
58	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
60	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9	0.9	0.9	0.9	0.9	0.9
62	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8
64	0.9	0.9	0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
66	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	
68	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7			
70	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6					

TABLE I—*Continued*

Latitude Degrees	Declination of Same Name as the Latitude											
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°
	"	"	"	"	"	"	"	"	"	"	"	"
0					28.1	22.4	18.7	16.0	14.0	12.4	11.1	10.1
1						28.0	22.4	18.6	16.0	13.9	12.4	11.1
2							28.0	22.3	18.6	15.9	13.9	12.3
3								27.9	22.3	18.5	15.8	13.8
4	28.1								27.8	22.2	18.5	15.8
5	22.4	28.0								27.7	22.1	18.4
6	18.7	22.4	28.0								27.6	22.0
7	16.0	18.6	22.3	27.9								27.4
8	14.0	16.0	18.6	22.3	27.8							
9	12.4	13.9	15.9	18.5	22.2	27.7						
10	11.1	12.4	13.9	15.8	18.5	22.1	27.6					
11	10.1	11.1	12.3	13.8	15.8	18.4	22.0	27.4				
12	9.2	10.1	11.1	12.3	13.8	15.7	18.3	21.9	27.3			
13	8.5	9.2	10.0	11.0	12.2	13.7	15.6	18.2	21.7	27.1		
14	7.9	8.5	9.2	10.0	10.9	12.1	13.6	15.5	18.0	21.6	26.9	
15	7.3	7.8	8.4	9.1	9.9	10.9	12.1	13.5	15.4	17.9	21.4	26.7
16	6.8	7.3	7.8	8.4	9.1	9.8	10.8	12.0	13.4	15.3	17.8	21.3
17	6.4	6.8	7.2	7.8	8.3	9.0	9.8	10.7	11.9	13.3	15.2	17.6
18	6.0	6.4	6.8	7.2	7.7	8.3	8.9	9.7	10.6	11.8	13.2	15.0
19	5.7	6.0	6.3	6.7	7.2	7.6	8.2	8.9	9.6	10.6	11.7	13.1
20	5.4	5.7	6.0	6.3	6.7	7.1	7.6	8.1	8.8	9.5	10.5	11.6
21	5.1	5.4	5.6	5.9	6.3	6.6	7.0	7.5	8.1	8.7	9.5	10.4
22	4.9	5.1	5.3	5.6	5.9	6.2	6.6	7.0	7.5	8.0	8.6	9.4
23	4.6	4.8	5.0	5.3	5.5	5.8	6.1	6.5	6.9	7.4	7.9	8.5
24	4.4	4.6	4.8	5.0	5.2	5.5	5.8	6.1	6.4	6.8	7.3	7.8
25	4.2	4.4	4.6	4.7	5.0	5.2	5.4	5.7	6.0	6.4	6.8	7.2
26	4.0	4.2	4.3	4.5	4.7	4.9	5.1	5.4	5.7	6.0	6.3	6.7
27	3.9	4.0	4.1	4.3	4.5	4.7	4.9	5.1	5.3	5.6	5.9	6.2
28	3.7	3.8	4.0	4.1	4.3	4.4	4.6	4.8	5.0	5.3	5.5	5.8
29	3.5	3.7	3.8	3.9	4.1	4.2	4.4	4.6	4.7	5.0	5.2	5.5
30	3.4	3.5	3.6	3.7	3.9	4.0	4.2	4.3	4.5	4.7	4.9	5.1
31	3.3	3.4	3.5	3.6	3.7	3.8	4.0	4.1	4.3	4.4	4.6	4.8
32	3.1	3.2	3.3	3.4	3.5	3.7	3.8	3.9	4.1	4.2	4.4	4.6
33	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.9	4.0	4.2	4.3
34	2.9	3.0	3.1	3.2	3.2	3.3	3.4	3.6	3.7	3.8	3.9	4.1
35	2.8	2.9	3.0	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.9
36	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7
37	2.6	2.7	2.7	2.8	2.9	2.9	3.0	3.1	3.2	3.3	3.4	3.5
38	2.5	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.0	3.2	3.2	3.3
39	2.4	2.5	2.5	2.6	2.7	2.7	2.8	2.9	2.9	3.0	3.1	3.2
40	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.7	2.8	2.9	3.0	3.0
41	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.6	2.7	2.8	2.8	2.9
42	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.6	2.7	2.8
43	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.7
44	2.0	2.1	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.5
45	2.0	2.0	2.0	2.1	2.1	2.2	2.2	2.2	2.3	2.3	2.4	2.4
46	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.2	2.2	2.2	2.3	2.3
47	1.8	1.9	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.1	2.2	2.2
48	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.1
49	1.7	1.7	1.8	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.1
50	1.6	1.7	1.7	1.7	1.8	1.8	1.8	1.8	1.9	1.9	1.9	2.0
52	1.5	1.6	1.6	1.6	1.6	1.6	1.7	1.7	1.7	1.8	1.8	1.8
54	1.4	1.4	1.5	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6	1.7
56	1.3	1.3	1.4	1.4	1.4	1.4	1.4	1.4	1.5	1.5	1.5	1.5
58	1.2	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.4	1.4	1.4
60	1.1	1.1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.3	1.3
62	1.0	1.0	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.2	1.2
64	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.1
66	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	1.0
68	0.8	0.8	0.8	0.8	0.3	0.8	0.8	0.8	0.8	0.8	0.9	0.9
70	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.8	0.8	0.8	0.8

TABLE I—*Continued*

Latitude Degrees	Declination of Same Name as the Latitude												
	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°
	″	″	″	″	″	″	″	″	″	″	″	″	″
0	9.2	8.5	7.9	7.3	6.8	6.4	6.0	5.7	5.4	5.1	4.9	4.6	4.4
1	10.1	9.2	8.5	7.8	7.3	6.8	6.4	6.0	5.7	5.4	5.1	4.8	4.6
2	11.1	10.0	9.2	8.4	7.8	7.2	6.8	6.3	6.0	5.6	5.3	5.0	4.8
3	12.3	11.0	10.0	9.1	8.4	7.8	7.2	6.7	6.3	5.9	5.6	5.3	5.0
4	13.8	12.2	10.9	9.9	9.1	8.3	7.7	7.2	6.7	6.3	5.9	5.5	5.2
5	15.7	13.7	12.1	10.9	9.8	9.0	8.3	7.6	7.1	6.6	6.2	5.8	5.5
6	18.3	15.6	13.6	12.1	10.8	9.8	8.9	8.2	7.6	7.0	6.6	6.1	5.8
7	21.9	18.2	15.5	13.5	12.0	10.7	9.7	8.9	8.1	7.5	7.0	6.5	6.1
8	27.3	21.7	18.0	15.4	13.4	11.9	10.6	9.6	8.8	8.1	7.5	6.9	6.4
9		27.1	21.6	17.9	15.3	13.3	11.8	10.6	9.5	8.7	8.0	7.4	6.8
10			26.9	21.4	17.8	15.2	13.2	11.7	10.5	9.5	8.6	7.9	7.3
11				26.7	21.3	17.6	15.0	13.1	11.6	10.4	9.4	8.5	7.8
12					26.5	21.1	17.5	14.9	13.0	11.5	10.3	9.3	8.4
13						26.2	20.9	17.3	14.8	12.8	11.3	10.1	9.2
14							26.0	20.7	17.1	14.6	12.7	11.2	10.0
15								25.7	20.4	16.9	14.4	12.5	11.1
16	26.5								25.4	20.2	16.7	14.3	12.4
17	21.1	26.2								25.1	20.0	16.5	14.1
18	17.5	20.9	26.0								24.8	19.7	16.3
19	14.9	17.3	20.7	25.7								24.5	19.5
20	13.0	14.8	17.1	20.4	25.4								24.2
21	11.5	12.8	14.6	16.9	20.2	25.1							
22	10.3	11.3	12.7	14.4	16.7	20.0	24.8						
23	9.3	10.1	11.2	12.5	14.3	16.5	19.7	24.5					
24	8.4	9.2	10.0	11.1	12.4	14.1	16.3	19.5	24.2				
25	7.7	8.3	9.0	9.9	10.9	12.2	13.9	16.1	19.2	23.8			
26	7.1	7.6	8.2	8.9	9.8	10.8	12.1	13.7	15.9	18.9	23.5		
27	6.6	7.0	7.5	8.1	8.8	9.6	10.6	11.9	13.5	15.6	18.6	23.1	
28	6.2	6.5	7.0	7.4	8.0	8.7	9.5	10.5	11.7	13.3	15.4	18.3	22.7
29	5.7	6.1	6.4	6.9	7.3	7.9	8.6	9.4	10.3	11.5	13.1	15.1	18.0
30	5.4	5.7	6.0	6.4	6.8	7.2	7.8	8.4	9.2	10.1	11.3	12.8	14.9
31	5.1	5.3	5.6	5.9	6.3	6.7	7.1	7.7	8.3	9.0	10.0	11.1	12.6
32	4.8	5.0	5.2	5.5	5.8	6.2	6.5	7.0	7.5	8.1	8.9	9.8	10.9
33	4.5	4.7	4.9	5.1	5.4	5.7	6.1	6.4	6.9	7.4	8.0	8.7	9.6
34	4.3	4.4	4.6	4.8	5.1	5.3	5.6	5.9	6.3	6.8	7.3	7.8	8.6
35	4.0	4.2	4.4	4.5	4.7	5.0	5.2	5.5	5.8	6.2	6.6	7.1	7.7
36	3.8	4.0	4.1	4.3	4.5	4.7	4.9	5.1	5.4	5.7	6.1	6.5	7.0
37	3.6	3.8	3.9	4.0	4.2	4.4	4.6	4.8	5.0	5.3	5.6	6.0	6.4
38	3.4	3.6	3.7	3.8	4.0	4.1	4.3	4.5	4.7	4.9	5.2	5.5	5.8
39	3.3	3.4	3.5	3.6	3.8	3.9	4.0	4.2	4.4	4.6	4.8	5.1	5.4
40	3.1	3.2	3.3	3.4	3.6	3.7	3.8	4.0	4.1	4.3	4.5	4.7	5.0
41	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.9	4.0	4.2	4.4	4.6
42	2.9	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.7	3.8	4.0	4.1	4.3
43	2.7	2.8	2.9	3.0	3.0	3.1	3.2	3.3	3.5	3.6	3.7	3.9	4.0
44	2.6	2.7	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.8
45	2.5	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5
46	2.4	2.4	2.5	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3
47	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.8	2.9	2.0	3.0	3.1
48	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.8	2.9	3.0
49	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.8
50	2.0	2.0	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6
52	1.8	1.9	1.9	1.9	2.0	2.9	2.1	2.1	2.1	2.2	2.2	2.3	2.4
54	1.7	1.7	1.7	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.1	2.1
56	1.5	1.6	1.6	1.6	1.6	1.7	1.7	1.7	1.8	1.8	1.8	1.9	1.9
58	1.4	1.4	1.5	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6	1.7	1.7
60	1.3	1.3	1.3	1.3	1.4	1.4	1.4	1.4	1.4	1.5	1.5	1.5	1.5
62	1.2	1.2	1.2	1.2	1.2	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.4
64	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.2	1.2
66	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.1
68	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	1.0	1.0
70	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.9

TABLE II

SQUARES OF INTERVAL

Seconds	Time From Meridian Passage												
	0^m	1^m	2^m	3^m	4^m	5^m	6^m	7^m	8^m	9^m	10^m	11^m	12^m
0	0.0	1.0	4.0	9.0	16.0	25.0	36.0	49.0	64.0	81.0	100.0	121.0	144.0
1	0.0	1.0	4.1	9.1	16.1	25.2	36.2	49.2	64.3	81.3	100.3	121.4	144.4
2	0.0	1.1	4.1	9.2	16.3	25.3	36.4	49.5	64.5	81.6	100.7	121.7	144.8
3	0.0	1.1	4.2	9.3	16.4	25.5	36.6	49.7	64.8	81.9	101.0	122.1	145.2
4	0.0	1.1	4.3	9.4	16.5	25.7	36.8	49.9	65.1	82.2	101.3	122.5	145.6
5	0.0	1.2	4.3	9.5	16.7	25.8	37.0	50.2	65.3	82.5	101.7	122.9	146.0
6	0.0	1.2	4.4	9.6	16.8	26.0	37.2	50.4	65.6	82.8	102.0	123.2	146.4
7	0.0	1.2	4.5	9.7	16.9	26.2	37.4	50.6	65.9	83.1	102.3	123.6	146.8
8	0.0	1.3	4.6	9.8	17.1	26.4	37.6	50.9	66.1	83.4	102.7	124.0	147.2
9	0.0	1.3	4.6	9.9	17.2	26.5	37.8	51.1	66.4	83.7	103.0	124.3	147.6
10	0.0	1.4	4.7	10.0	17.4	26.7	38.0	51.4	66.7	84.0	103.4	124.7	148.0
11	0.0	1.4	4.8	10.1	17.5	26.9	38.2	51.6	67.0	84.3	103.7	125.1	148.4
12	0.0	1.4	4.8	10.2	17.6	27.0	38.4	51.8	67.2	84.6	104.0	125.4	148.8
13	0.0	1.5	4.9	10.3	17.8	27.2	38.6	52.1	67.5	84.9	104.4	125.8	149.2
14	0.1	1.5	5.0	10.5	17.9	27.4	38.9	52.3	67.8	85.3	104.7	126.2	149.7
15	0.1	1.6	5.1	10.6	18.1	27.6	39.1	52.6	68.1	85.6	105.1	126.6	150.1
16	0.1	1.6	5.1	10.7	18.2	27.7	39.3	52.8	68.3	85.9	105.4	126.9	150.5
17	0.1	1.6	5.2	10.8	18.3	27.9	39.5	53.0	68.6	86.2	105.7	127.3	150.9
18	0.1	1.7	5.3	10.9	18.5	28.1	39.7	53.3	68.8	86.5	106.1	127.7	151.3
19	0.1	1.7	5.4	11.0	18.6	28.3	39.9	53.5	69.2	86.8	106.4	128.1	151.7
20	0.1	1.8	5.4	11.1	18.8	28.4	40.1	53.8	69.4	87.1	106.8	128.4	152.1
21	0.1	1.8	5.5	11.2	18.9	28.6	40.3	54.0	69.7	87.4	107.1	128.8	152.5
22	0.1	1.9	5.6	11.3	19.1	28.8	40.5	54.3	70.0	87.7	107.5	129.2	152.9
23	0.1	1.9	5.7	11.4	19.2	29.0	40.7	54.5	70.3	88.0	107.8	129.6	153.3
24	0.2	2.0	5.8	11.6	19.4	29.2	41.0	54.8	70.6	88.4	108.2	130.0	153.8
25	0.2	2.0	5.8	11.7	19.5	29.3	41.2	55.0	70.8	88.7	108.5	130.3	154.2
26	0.2	2.1	5.9	11.8	19.7	29.5	41.4	55.3	71.1	89.0	108.9	130.7	154.6
27	0.2	2.1	6.0	11.9	19.8	29.7	41.6	55.5	71.4	89.3	109.2	131.1	155.0
28	0.2	2.2	6.1	12.0	20.0	29.9	41.8	55.8	71.7	89.6	109.6	131.5	155.4
29	0.2	2.2	6.2	12.1	20.1	30.1	42.0	56.0	72.0	89.9	109.9	131.9	155.8
30	0.2	2.2	6.2	12.2	20.2	30.2	42.2	56.2	72.2	90.2	110.2	132.2	156.2
31	0.3	2.3	6.3	12.4	20.4	30.4	42.5	56.5	72.5	90.6	110.6	132.6	156.7
32	0.3	2.4	6.4	12.5	20.6	30.6	42.7	56.8	72.8	90.9	111.0	133.0	157.1
33	0.3	2.4	6.5	12.6	20.7	30.8	42.9	57.0	73.1	91.2	111.3	133.4	157.5
34	0.3	2.5	6.6	12.7	20.9	31.0	43.1	57.3	73.4	91.5	111.7	133.8	157.9
35	0.3	2.5	6.7	12.8	21.0	31.2	43.3	57.5	73.7	91.8	112.0	134.2	158.3
36	0.4	2.6	6.8	13.0	21.2	31.4	43.6	57.8	74.0	92.2	112.4	134.6	158.8
37	0.4	2.6	6.8	13.1	21.3	31.5	43.8	58.0	74.3	92.5	112.7	134.9	159.2
38	0.4	2.7	6.9	13.2	21.5	31.7	44.0	58.3	74.5	92.8	113.1	135.3	159.6
39	0.4	2.7	7.0	13.3	21.6	31.9	44.2	58.5	74.8	93.1	113.4	135.7	160.0
40	0.4	2.8	7.1	13.4	21.8	32.1	44.4	58.8	75.1	93.4	113.8	136.1	160.4
41	0.5	2.8	7.2	13.6	21.9	32.3	44.7	59.0	75.4	93.8	114.1	136.5	160.9
42	0.5	2.9	7.3	13.7	22.1	32.5	44.9	59.3	75.7	94.1	114.5	136.9	161.3
43	0.5	2.9	7.4	13.8	22.2	32.7	45.1	59.5	76.0	94.4	114.8	137.3	161.7
44	0.5	3.0	7.5	13.9	22.4	32.9	45.3	59.8	76.3	94.7	115.2	137.7	162.1
45	0.6	3.1	7.6	14.1	22.6	33.1	45.6	60.1	76.6	95.1	115.6	138.1	162.6
46	0.6	3.1	7.7	14.2	22.7	33.3	45.8	60.3	76.9	95.4	115.9	138.5	163.0
47	0.6	3.2	7.7	14.3	22.9	33.4	46.0	60.6	77.1	95.7	116.3	138.8	163.4
48	0.6	3.2	7.8	14.4	23.0	33.6	46.2	60.8	77.4	96.0	116.6	139.2	163.8
49	0.7	3.3	7.9	14.6	23.2	33.8	46.5	61.1	77.7	96.4	117.0	139.6	164.3
50	0.7	3.4	8.0	14.7	23.4	34.0	46.7	61.4	78.0	96.7	117.4	140.0	164.7
51	0.7	3.4	8.1	14.8	23.5	34.2	46.9	61.6	78.3	97.0	117.7	140.4	165.1
52	0.8	3.5	8.2	15.0	23.7	34.4	47.2	61.9	78.6	97.4	118.1	140.8	165.6
53	0.8	3.5	8.3	15.1	23.8	34.6	47.4	62.1	78.9	97.7	118.4	141.2	166.0
54	0.8	3.6	8.4	15.2	24.0	34.8	47.6	62.4	79.2	98.0	118.8	141.6	166.4
55	0.8	3.7	8.5	15.3	24.2	35.0	47.8	62.7	79.5	98.3	119.2	142.0	166.8
56	0.9	3.7	8.6	15.5	24.3	35.2	48.1	62.9	79.8	98.7	119.5	142.4	167.3
57	0.9	3.8	8.7	15.6	24.5	35.4	48.3	63.2	80.1	99.0	119.9	142.8	167.7
58	0.9	3.9	8.8	15.7	24.7	35.6	48.5	63.5	80.4	99.3	120.3	143.2	168.1
59	1.0	3.9	8.9	15.9	24.8	35.8	48.8	63.7	80.7	99.7	120.6	143.6	168.6

TABLE III

THE SECOND CORRECTION TO BE APPLIED WHEN DETERMINING LATITUDE FROM POLARIS*

Arguments: Sidereal Time and Altitude (Additive)

Sidereal Time		Altitude														Sidereal Time	
		10°		20°		30°		40°		50°		60°		70°			
h	m	′	″	′	″	′	″	′	″	′	″	′	″	′	″	h	m
0	0	0	1	0	2	0	3	0	5	0	7	0	10	0	16	12	0
	30	0	0	0	1	0	1	0	2	0	3	0	4	0	6		30
1	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	13	0
	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0		30
2	0	0	0	0	0	0	1	0	1	0	2	0	2	0	3	14	0
	30	0	1	0	1	0	2	0	3	0	5	0	7	0	11		30
3	0	0	1	0	3	0	5	0	7	0	10	0	14	0	22	15	0
	30	0	2	0	5	0	8	0	11	0	16	0	23	0	36		30
4	0	0	3	0	7	0	11	0	16	0	22	0	33	0	52	16	0
	30	0	4	0	9	0	14	0	21	0	30	0	43	1	8		30
5	0	0	5	0	11	0	18	0	26	0	37	0	53	1	25	17	0
	30	0	6	0	13	0	21	0	30	0	43	1	3	1	40		30
6	0	0	7	0	15	0	24	0	34	0	49	1	10	1	52	18	0
	30	0	8	0	16	0	25	0	37	0	52	1	16	2	1		30
7	0	0	8	0	17	0	27	0	39	0	55	1	20	2	7	19	0
	30	0	8	0	17	0	27	0	39	0	55	1	20	2	8		30
8	0	0	8	0	16	0	26	0	38	0	54	1	18	2	4	20	0
	30	0	7	0	15	0	25	0	36	0	51	1	14	1	57		30
9	0	0	7	0	14	0	22	0	32	0	46	1	7	1	46	21	0
	30	0	6	0	12	0	19	0	28	0	40	0	58	1	32		30
10	0	0	5	0	10	0	16	0	23	0	33	0	48	1	16	22	0
	30	0	4	0	8	0	12	0	18	0	26	0	37	0	59		30
11	0	0	3	0	6	0	9	0	13	0	19	0	27	0	43	23	0
	30	0	2	0	4	0	6	0	9	0	12	0	18	0	28		30
12	0	0	1	0	2	0	3	0	5	0	7	0	10	0	16	24	0

*For use of table, see Art. 17.

TABLE IV

EFFECT OF REFRACTION ON DECLINATION, FOR USE IN DETERMINING AZIMUTH BY THE SOLAR TRANSIT*

Latitude	Hour Angle	Declination										
		+ 23° 27′	+ 20°	+ 15°	+ 10°	+ 5°	0°	− 5°	− 10°	− 15°	− 20°	− 23° 27′
		′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
+20°	0	0 4	0 0	0 5	0 10	0 15	0 21	0 27	0 33	0 40	0 48	0 55
	1	0 3	0 0	0 6	0 11	0 16	0 22	0 28	0 34	0 41	0 49	0 56
	2	0 1	0 3	0 8	0 13	0 18	0 24	0 30	0 37	0 45	0 54	1 00
	3	0 4	0 7	0 12	0 18	0 23	0 30	0 36	0 44	0 52	1 02	1 11
	4	0 13	0 17	0 22	0 28	0 35	0 42	0 50	1 00	1 11	1 25	1 36
	5	0 34	0 39	0 47	0 57	1 07	1 19	1 36	1 57	2 28	3 14	4 01
	6	2 06	2 27	3 12	4 24	6 43	11 56					
+30°	0	0 07	0 10	0 15	0 21	0 27	0 33	0 40	0 48	0 58	1 09	1 18
	1	0 08	0 11	0 16	0 22	0 28	0 35	0 42	0 50	0 59	1 11	1 20
	2	0 10	0 14	0 19	0 25	0 32	0 39	0 46	0 55	1 06	1 18	1 27
	3	0 16	0 20	0 26	0 32	0 40	0 47	0 56	1 06	1 19	1 35	1 51
	4	0 28	0 32	0 39	0 47	0 56	1 06	1 19	1 36	1 58	2 28	2 59
	5	0 53	0 50	1 10	1 24	1 43	2 06	2 41	3 36	5 18	8 54	16 42
	6	2 11	2 33	3 23	4 51	8 00	17 27					
+40°	0	0 17	0 21	0 27	0 33	0 40	0 48	0 58	1 09	1 22	1 40	1 55
	1	0 18	0 22	0 28	0 34	0 42	0 50	1 00	1 11	1 25	1 43	2 00
	2	0 22	0 26	0 32	0 39	0 47	0 56	1 07	1 19	1 36	1 58	2 18
	3	0 29	0 33	0 41	0 48	0 58	1 09	1 22	1 39	2 02	2 36	3 09
	4	0 41	0 47	0 56	1 07	1 20	1 36	1 58	2 30	3 19	4 47	6 45
	5	1 07	1 16	1 31	1 51	2 20	3 02	4 15	6 47	14 18		
	6	2 11	2 35	3 27	5 03	8 42	22 26					
+50°	0	0 29	0 33	0 40	0 48	0 58	1 09	1 22	1 40	2 03	2 37	3 12
	1	0 30	0 34	0 42	0 50	1 00	1 11	1 25	1 44	2 09	2 46	3 24
	2	0 34	0 39	0 47	0 56	1 07	1 19	1 36	1 58	2 29	3 18	4 13
	3	0 42	0 48	0 56	1 07	1 21	1 37	1 59	2 32	3 22	4 55	7 02
	4	0 55	1 02	1 15	1 29	1 49	2 17	2 58	4 07	6 32	13 37	
	5	1 20	1 31	1 51	2 20	3 02	4 16	6 52	15 08			
	6	2 11	2 37	3 30	5 09	9 12	26 44					
+60°	0	0 43	0 48	0 58	1 09	1 22	1 40	2 03	2 37	3 32	5 16	7 46
	1	0 44	0 50	0 59	1 11	1 25	1 43	2 08	2 44	3 46	5 44	8 45
	2	0 48	0 54	1 05	1 18	1 34	1 55	2 26	3 11	4 34	7 44	13 41
	3	0 56	1 03	1 16	1 31	1 51	2 21	3 04	4 19	7 05	16 40	
	4	1 10	1 19	1 35	1 57	2 28	3 17	4 44	8 08	22 10		
	5	1 32	1 46	2 12	2 50	3 54	6 21	12 17				
	6	2 12	2 37	3 31	5 12	9 30	30 13					
Latitude	Hour Angle	+ 23° 27′	+ 20°	+ 15°	+ 10°	+ 5°	0°	− 5°	− 10°	− 15°	− 20°	− 23° 27′
		Declination										

*For use of table, see Art. 43.

TABLE V

FOR COMPUTING AZIMUTH BY POLARIS WHEN OBSERVED AT ANY HOUR ANGLE

Hour Angle Degrees	*K* Seconds	Hour Angle Degrees	Hour Angle Degrees	*K* Seconds	Hour Angle Degrees
0	+90	360	90	0	270
5	+90	355	95	− 8	265
10	+89	350	100	−16	260
15	+87	345	105	−23	255
20	+85	340	110	−31	250
25	+82	335	115	−38	245
30	+78	330	120	−45	240
35	+74	325	125	−52	235
40	+69	320	130	−58	230
45	+64	315	135	−64	225
50	+58	310	140	−69	220
55	+52	305	145	−74	215
60	+45	300	150	−78	210
65	+38	295	155	−82	205
70	+31	290	160	−85	200
75	+23	285	165	−87	195
80	+16	280	170	−89	190
85	+ 8	275	175	−90	185
90	0	270	180	−90	180

A SERIES OF QUESTIONS

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the questions contained in the following pages are divided into sections corresponding to the sections of the text of the preceding pages, so that each section has a headline that is the same as the headline of the section to which the questions refer. No attempt should be made to answer any of the questions until the corresponding part of the text has been carefully studied.

LEVELING

EXAMINATION QUESTIONS

(1) A leveling rod is held 360 feet from the instrument; the reading is 6.817 feet. After causing the level bubble to move over two divisions of the scale, the reading is 6.879 feet. What is the angular value in seconds of one division of the bubble scale? Ans. 17.8 sec.

(2) If the diameter of the clear aperture of the object glass of a telescope is 1.20 inches, and the diameter of the small disk of light at the eye end is .04 inch, what is the magnifying power of the telescope? Ans. 30

(3) What is meant by (*a*) the magnifying power and (*b*) the definition of a telescope?

(4) (*a*) What are turning points? (*b*) What is a bench mark? (*c*) What is a reference base or datum?

(5) (*a*) The elevation of a point where a backsight is taken is 61.84 feet, the rod reading on this point is 11.81 feet, and the foresight reading on a turning point is .49 foot; (*a*) what is the elevation of the turning point? When the instrument is moved forward and set up in a new position, the backsight reading on the same turning point is 9.57 feet, and the foresight reading on a station is 4.3 feet; what is (*b*) the new height of instrument, and (*c*) the elevation of the station?

Ans. $\begin{cases} (a) & 73.16 \text{ ft.} \\ (b) & 82.73 \text{ ft.} \\ (c) & 78.4 \text{ ft.} \end{cases}$

(6) (*a*) What are the principal sources of error in taking levels? (*b*) What would be the permissible error of closure for leveling in a preliminary railroad survey, the length of the circuit being 25 miles, and (*c*) in the United States Lake Survey, the length of the circuit being 50 miles?

Ans. $\begin{cases} (b) & .5 \text{ ft.} \\ (c) & .297 \text{ ft.} \end{cases}$

(7) (*a*) What is a profile? (*b*) What is the rate of grade? (*c*) What is understood by a 1-per-cent. grade?

(8) In a certain survey the elevation of the grade at Station 66 is 126.50; between Stations 66 and 100 there is an ascending grade of 1.25 per cent.; what is the elevation of the grade (*a*) at Station 93, (*b*) at Station 95 + 64, and (*c*) at Station 99 + 32?

Ans. $\begin{cases} (a) & 160.25 \text{ ft.} \\ (b) & 163.55 \text{ ft.} \\ (c) & 168.15 \text{ ft.} \end{cases}$

(9) An instrument is stationed at *A*, Fig. I, 100 feet

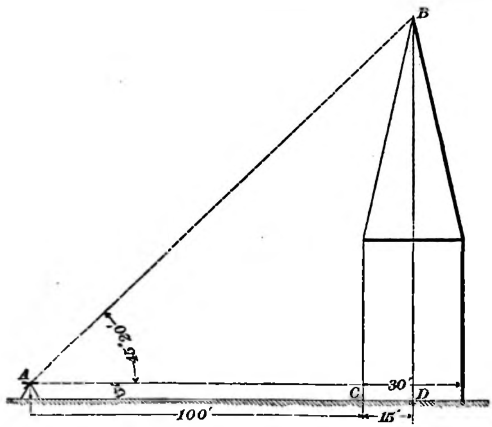

FIG. I

from the base of a church spire *B C*. The horizontal line of sight *A C* from instrument is 5 feet above the base of the

spire, which at that point is 30 feet in diameter. The angle CAB is 45° 20′; required the height of the spire.

Ans. 121.35 ft.

(10) (*a*) Complete the following level notes by working out the elevations, and check the elevation of the last turning point. (*b*) Plat the notes in a profile and, assuming the elevation of the grade at Station 40 to be 162.0 feet, draw on the profile a grade line descending at the rate of 80 feet to the mile. (*c*) Calculate the elevation of the grade at each station, and write it opposite the station in the column headed *Grade*.

Station	*Rod Reading*	*Height of Instrument*	*Elevation*	*Grade*	*Remarks*
B. M.	+ 5.53		161.42		*B. M. on poplar tree*
40	− 6.4			162.0	*70 ft. to left of Sta. 40*
41	− 7.2				
41 + 60	−10.9				
42	− 8.6				
43	− 8.8				
T. P.	− 8.66				
	+ 2.22				
44	− 4.8				
45	− 6.3				
46	− 8.8				
47	− 9.9				
48	−11.1				
T. P.	−11.24				
	+ 3.30				
49	− 4.7				
50	− 7.1				
51	− 8.7				
52	− 9.8				
53	−10.9				
T. P.	−11.62				

(11) The reading of barometer h_1 at the lower station is 29.40 inches and the temperature $t_1 = 74°$; the reading of

barometer h' at the higher station is 26.95 inches and the temperature $t' = 58°$; what is the difference in elevation?

Ans. 2,454 ft.

(12) It is desired to determine the height of an inaccessible tower situated on the bank of a river. A transit is set up on the opposite bank and leveled up carefully. The line of sight is then directed to the top of the tower and the vernier on the vertical circle is found to read 18° 10′. The instrument is then moved to a point 200 feet farther from the tower, in the same vertical plane with the first line of sight, and is set up at the same height as before. When leveled up carefully and the line of sight is directed to the top of the tower, the vernier on the vertical circle is found to read 13° 15′. Determine (*a*) the height of the tower above the instrument and (*b*) the horizontal distance from the first, or nearer, instrument point to the tower.

Ans. $\begin{cases} (a) & 166.76 \text{ ft.} \\ (b) & 508.20 \text{ ft.} \end{cases}$

(13) In order to determine the height of a distant summit, a transit is set up at some convenient point A, and the vertical angle that the line of sight makes with a horizontal line when the telescope is directed to the summit is measured and found to be 20° 18′. The transit is then moved to a point C nearer the summit, at a distance of 500 feet horizontally from the first instrument point A, and in the same vertical plane with this point and the summit. By sighting again on the distant summit, the angle between the line of sight and a horizontal line is measured and found to be 40° 55′. If, when at the point C the instrument is 3.0 feet lower than when at the point A, what is the elevation of the summit above the instrument at the point C?

Ans. 327.90 ft.

(14) If, in the preceding example, the instrument, when at the point C, is 3 feet higher than when at the point A, all other values remaining the same, what would be the elevation of the summit above the instrument at the point A?

Ans. 320.43 ft.

(15) Taking the radius of the earth as 20,890,590 feet and the coefficient of refraction as .0719, compute the error due to the combined effect of curvature and refraction in a sight of (*a*) 450 feet, (*b*) 850 feet, and (*c*) 1,250 feet.

Ans. $\begin{cases} (a) & .004 \text{ ft.} \\ (b) & .015 \text{ ft.} \\ (c) & .032 \text{ ft.} \end{cases}$

(16) The height of instrument is 126.37 feet and the foresight on a turning point is 5.36 feet. After the instrument has been moved forward and set up in a new position, the backsight on the turning point is 6.27 feet. What are the elevations to the nearest tenth of a foot of the three succeeding stations on which the rod readings are (*a*) 4.6 feet, (*b*) 9.3 feet, and (*c*) 11.8 feet, respectively?

Ans. $\begin{cases} (a) & 122.7 \text{ ft.} \\ (b) & 118.0 \text{ ft.} \\ (c) & 115.5 \text{ ft.} \end{cases}$

(17) A horizontal base line *A B*, 500 feet long, is measured on the plain at the foot of a mountain, the transit set up at *A*, and the telescope directed to *B*. When the instrument is turned in azimuth until a point at the top of the mountain is sighted, the angle turned on the horizontal circle is 77° 36′, and that turned on the vertical circle is 28° 16′. The transit is then set up at *B* and the telescope directed to *A*. When the instrument is turned in azimuth until the same point at the top of the mountain is sighted, the angle turned on the horizontal circle is 67° 45′. Determine the height of the mountain. Ans. 437.65 ft.

(18) (*a*) Complete the following level notes by working out the elevations, and check the elevation of the last turning point. (*b*) Plat the notes in a profile and, assuming the elevation of grade at Station 40 to be 112.0 feet, draw on the profile a grade line having a gradient of — 1.5 per cent. between Stations 40 and 53, and a gradient of — .8 per cent. between Stations 53 and 63.

Station	Backsight	Height of Instrument	Foresight	Elevation of Surface	Elevation of Grade	Cut or Fill	Remarks
B. M.	5.52			111.42			On poplar tree 60 ft. to left of Sta. 40
40			6.6		112.0		
41			10.5				
T. P.	7.83		11.28				
42			8.5				
43			0.9				
44			4.1				
45			5.8				
T. P.	0.49		10.87				
46			3.1				
47			6.2				
48			7.0				
49			10.1				
50			8.2				
51			5.7				
52			6.6				
T. P.	0.56		11.54				
53			2.0				
53 + 60			0.6				
54			4.5				
T. P.	3.24		10.65				
54 + 75			10.9				Mill Creek
55			4.7				
55 + 40			1.8				
56			1.2				
57			2.7				
58			1.8				
T. P.	11.37		0.98				
59			8.9				
60			1.0				
61			5.5				
62			6.0				
63			11.4				
T. P.			11.72				

(19) From the notes as completed in compliance with the preceding example, (*a*) calculate the elevation of the grade at each station and write it in the column headed *Elevation of Grade;* (*b*) also calculate the amount of cut or fill for each station and write it in the column headed *Cut or Fill.*

CIRCULAR CURVES

EXAMINATION QUESTIONS

(1) The angle of intersection between two tangents is 35° 10′, the degree of curve is 6° 15′; what is the tangent distance? Ans. 290.66 ft., nearly

(2) The angle of intersection is 30° 45′, the degree of curve, 5° 15′; what is the length of the curve?

Ans. 585.71 ft.

(3) The angle of intersection is 33° 06′, the station of the point of intersection, 20 + 37.8, the degree of curve, 5°; what is (*a*) the station of the P. C., (*b*) the length of curve, and (*c*) the station of the P. T.?

Ans. (*a*) 16 + 97.17; (*b*) 662 ft.; (*c*) 23 + 59.17

(4) The angle of intersection is 20° 10′, the tangent distance, 291.16 feet; required (*a*) the radius and (*b*) the degree of curve.

Ans. (*a*) 1,637.3 ft.; (*b*) 3° 30′

(5) The deflection angle for a chord of 48.2 feet is 1° 41.22′. What is the radius of the curve? Ans. 819.02 ft.

(6) The degree of curve is 6° 15′; what is the deflection angle for a chord of 72.7 feet? Ans. 2° 16.3′

(7) The degree of curve is 5° 30′; what is the tangent deflection for a chord of 50 feet? Ans. 1.199 ft.

(8) In a 5° curve, what is the chord deflection for a chord of 47.8 feet following one of 100 feet? Ans. 3.083 ft.

(9) The length of a chord is 60 feet, and the middle ordinate .63 foot. Required the degree of curve. Ans. 8.02° (The original curve probably 8°.)

(10) Determine the middle ordinate to a chord 60 feet long in a 5° curve. Ans. .393 ft.

(11) Suppose the required curve $ABCD$, Fig. I, is a 6° curve and that the offset distance AE, $= DH$, necessary

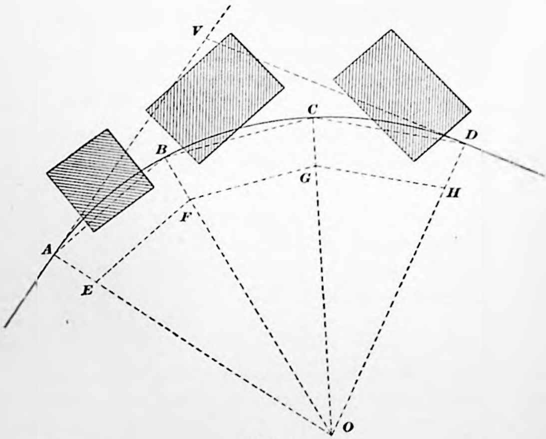

FIG. I

to avoid the obstacles is 80 feet. What is the length of the chords EF, FG, and GH, in order that the corresponding chords of the required curve will each be 100 feet in length? Ans. 91.63 ft.

(12) Suppose that in running the curve $ABCDEF$, Fig. II, it is found that there is a building between Stations 3 and 4, and also one between Stations 4 and 5. The points for Stations 4, 5, and 6 are to be located from the point B, which is Station 3, by means of the long chord BE. Assuming the curve to be a 7° curve, and the distance

between stations to be 100 feet, determine (*a*) the deflection angle $V'BE$ for the point E, and (*b*) the length of the long chord BE, locating Station 6 on the curve.

Ans. { (*a*) 10° 30′
(*b*) 298.52 ft.

(13) Referring to the preceding question, determine (*a*) the distances MC' and MD' from the center M of the chord

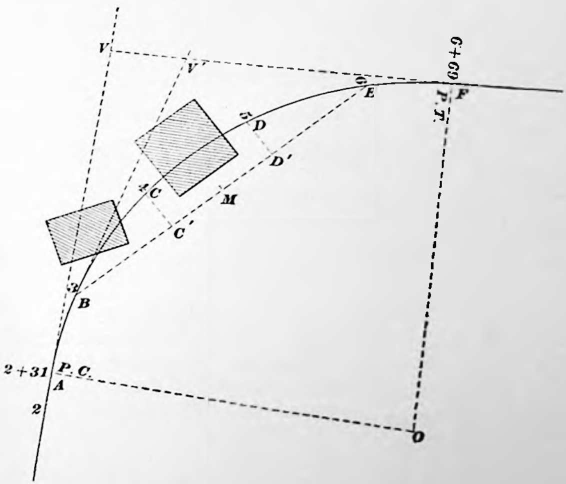

FIG. 11

to the ordinates $C'C$ and $D'D$, respectively; (*b*) the distance BC'; (*c*) the distance BD' from the beginning B of the chord to the same respective ordinates, as measured along the chord; and (*d*) the ordinates $C'C$ and $D'D$, respectively, thus locating Stations 4 and 5 on the curve.

Ans. { (*a*) 50.00 ft. each
(*b*) 99.26 ft.
(*c*) 199.26 ft.
(*d*) 12.19 ft. each

STADIA AND PLANE-TABLE SURVEYING

EXAMINATION QUESTIONS

NOTE.—The answers to those examples in which distances or elevations are required are in each case given to the nearest tenth of a foot. It is assumed that the vertical angle, as recorded in each case, is taken to a point on the stadia rod whose height is equal to the height of instrument. The examples in which the Stadia Reduction Table is not mentioned are to be solved by the formulas.

(1) The length intercepted by the stadia wires on the stadia rod when held at right angles to the horizontal line of sight is 3.47 feet; if the instrument constant is 1 foot and the stadia constant 100, what is the horizontal distance of the stadia rod from the center of the instrument?

Ans. 348.0 ft.

(2) The vertical angle that the line of sight makes with the horizontal is $+20° 45'$, and the length intercepted on the stadia rod is 6.48 feet; if the instrument constant is 1.25 feet and the stadia constant 102, determine: (*a*) the horizontal distance of the rod from the center of the instrument; (*b*) the elevation of the stadia point above the instrument point.

Ans. $\begin{cases} (a)\ 579.2 \text{ ft.} \\ (b)\ 219.4 \text{ ft.} \end{cases}$

(3) Determine the stadia constant *s* and the instrument constant *i* from the following data, using the method explained in Art. 10:

Distance Measured, Feet	Rod Reading, Feet
50	.480
100	.971
200	1.951
250	2.441
350	3.422

Ans. $\begin{cases} s = 101.941 \\ i = 1.068 \end{cases}$

(4) The length intercepted on the stadia rod is 6 feet, the vertical angle +8° 26′, the instrument constant .75, and the stadia constant 100; if the elevation of the instrument point is 506.3 feet, what is the elevation of the stadia point?

Ans. 593.5 ft.

(5) The distance intercepted on the rod is 5 feet, the vertical angle +15° 24′, the instrument constant 1, and the stadia constant 100. Calculate, by means of the Stadia Reduction Table: (*a*) the horizontal distance of the rod from the center of the instrument; (*b*) the difference in elevation between the stadia point and the instrument point. (*c*) If the elevation of the instrument point is 125.4 feet, what is the elevation of the stadia point?

Ans. $\begin{cases} (a)\ 465.7 \text{ ft.} \\ (b)\ 128.3 \text{ ft.} \\ (c)\ 253.7 \text{ ft.} \end{cases}$

(6) The distance on a measured base line from the instrument point to Station 3 is 300 feet, the instrument constant being 1 foot. With the line of sight horizontal, the length intercepted by the stadia wires on a rod held on Station 3 is 3.03 feet. (*a*) What is the stadia constant as determined by this reading? Assuming that the constant so determined is correct, determine the respective lengths that will be intercepted by the stadia wires when the rod is held at distances of: (*b*) 101 feet; (*c*) 201 feet; (*d*) 401 feet; and (*e*) 501 feet from the instrument point.

Ans. $\begin{cases} (a)\ 98.68 \\ (b)\ 1.01 \text{ ft.} \\ (c)\ 2.03 \text{ ft.} \\ (d)\ 4.05 \text{ ft.} \\ (e)\ 5.07 \text{ ft.} \end{cases}$

(7) The distance between the place where the upper stadia wire appears to cut the rod and the mark on the rod

to which the center cross-wire is sighted, which indicates the height of instrument, is 4.9 feet, the instrument constant is .92 foot, and the stadia constant is 101; determine the horizontal distance: (*a*) when the vertical angle is zero; (*b*) when the vertical angle is 10° 36′. Ans. $\begin{cases} (a)\ 990.7 \text{ ft.} \\ (b)\ 957.2 \text{ ft.} \end{cases}$

(8) The length on the stadia rod intercepted by the stadia wires is 7.3 feet, the instrument constant is 1 foot, and the stadia constant is 101; what are the vertical distances: (*a*) when the vertical angle is 10° 36′? (*b*) when the vertical angle is 21° 12′? Ans. $\begin{cases} (a)\ 133.5 \text{ ft.} \\ (b)\ 248.9 \text{ ft.} \end{cases}$

(9) Solve example 8 using the Stadia Reduction Table. Ans. $\begin{cases} (a)\ 133.5 \text{ ft.} \\ (b)\ 249.0 \text{ ft.} \end{cases}$

(10) Determine, by means of the Stadia Reduction Table, the elevation of the stadia point above datum, if the vertical angle between the horizontal and inclined line of sight is −20° 19′, the length intercepted on the rod 6.42 feet, the instrument constant 1.25 feet, and the stadia constant 100, the elevation of the instrument point above datum being 275.00 feet. Ans. 65.5 ft.

(11) What must be the length of each main division of a regraduated stadia rod: (*a*) for a stadia constant of 98.75? (*b*) for a stadia constant of 101.38? Ans. $\begin{cases} (a)\ 1.01 \\ (b)\ .99 \end{cases}$

(12) Assume the azimuths, vertical angles, and stadia readings from a station to four points, and write the notes for the observations, filling in, by means of the Stadia Reduction Table, the columns for horizontal distances and elevations. Assume the stadia and instrument constants, and also the elevation of the station occupied.

(13) Assume the length of a base line on the ground from which other points are to be located by the plane table. Explain how the platted position of any point is found by intersection from the extremities of the base line.

TOPOGRAPHIC SURVEYING

EXAMINATION QUESTIONS

(1) Define, in your own words, topographic surveying, and give a brief outline of the principal methods used.

(2) If the slope of an embankment is $1\frac{1}{2}:1$, and the distance from the top to the bottom of the embankment, measured along the slope, is 50 feet, what is the horizontal distance?

(3) The data being the same as in the preceding question, determine: (*a*) the difference between the elevation of the top and that of the bottom of the embankment; (*b*) the slope angle of the embankment.

(4) (*a*) What do you consider to be the advantages and disadvantages of the hand level, as compared with the Y level? (*b*) In general, for work of what character would you use one of these instruments in preference to the other?

(5) Give a brief description, in your own words, of the clinometer, and explain how it is used.

(6) Draw a diagram showing the right slope at a railroad (surveying) station. Assume the form of the slope, and explain the necessary operations to determine, by the hand level, the right-hand portion of the cross-section at that station. The cross-section is supposed to extend 250 feet on each side of the line.

(7) What would you understand by the notation $-\frac{13.7}{97}$ for 275, used in connection with cross-sectioning?

(8) Make a sketch of a small rectangular field, and explain how its topography may be determined by dividing it into squares. Assume the necessary dimensions and other conditions, and explain how the field is staked out and how the field operations are conducted. Write the elevations on the sketch.

(9) State, in a general manner, without going into details, what field operations are gone through in making a topographical map of a large area by using the plane table.

(10) Explain how to locate contours directly by means of the plane table and Y level. Make a sketch, assuming all necessary conditions, and explain how to locate a few points on two consecutive contours, say contours *65* and *70*.

(11) If the contour interval is 25 feet, could points on more than one contour be located by the method described in Art. 28, using the plane table and Y level? If not, why?

HYDROGRAPHIC SURVEYING

EXAMINATION QUESTIONS

(1) In order to make a hydrographic survey of a river, a base line *A B*, Fig. I, was measured, stations *C*, *D*, *E*, etc. were selected on the two sides, and all the angles in the triangles *1*, *2*, *3*, *4*, *5* carefully measured. The values of the

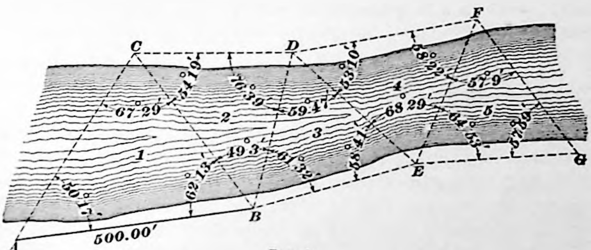

Fig. I

angles shown in the figure are those that were found by actual measurements: (*a*) Correct the angles (see Art. 6). (*b*) Having corrected the angles, compute the distances *A C*, *C B*, *C D*, *D B*, *B E*, *D E*, *D F*, *F E*, *E G*, *F G*.

Ans.

Corrected Angles			Lengths of Lines
Triangle 1	*Triangle 2*	*Triangle 3*	AC = 478.87 ft.
67° 29′ 20″	54° 18′ 40″	59° 47′	CB = 416.36 ft.
62° 13′ 20″	76° 38′ 40″	58° 41′	CD = 323.18 ft.
50° 17′ 20″	49° 2′ 40″	61° 32′	DB = 347.57 ft.
			BE = 351.57 ft.
Triangle 4	*Triangle 5*		DE = 357.66 ft.
			DF = 390.81 ft.
53° 9′ 40″	57° 8′ 40″		FE = 336.22 ft.
58° 21′ 40″	57° 58′ 40″		EG = 333.12 ft.
68° 28′ 40″	64° 52′ 40″		FG = 359.04 ft.

(2) A base line having been measured along the bank of a river, and a buoy having been placed near the middle of the river, it is required to determine the distance of the buoy from one end of the base line. Describe the operations necessary for the solution of the problem, assume the measurements, and work out the result.

(3) Give an example of the location of one sounding by two angles measured simultaneously from shore. Assume the necessary measurements, make the necessary calculations, and explain how the sounding is located or plotted on a map.

(4) What is the most accurate method of locating soundings: (*a*) in a large body of water? (*b*) in a narrow channel or creek?

(5) In very accurate work, would you use the method of locating soundings by time intervals? If not, why?

(6) A boat is on a range making an angle of 60° with the base line, and intersecting the latter line at a distance of 275 feet from the station occupied by the transit. If the angle between the line of sight and the base line is 49° 12′ at the moment a sounding is taken, find the distance of the sounding: (*a*) from the station occupied by the instrument; (*b*) from the intersection of the range with the base line.

Ans. $\begin{cases} (a) & 252.2 \text{ ft.} \\ (b) & 220.4 \text{ ft.} \end{cases}$

(7) At the moment of making a sounding, two angles were measured with the sextant, directing the line of sight from the boat to three points, *A*, *B*, and *C*, on shore. The angle between the lines *A B* and *B C* was 159° 30′, and the lengths of *A B* and *B C* were, respectively, 300 and 500 feet. The lines of sight from the boat to *A* and *B* made an angle of 39° 13′, and those to *B* and *C*, an angle of 23° 8′. Make a sketch and determine the distances of the sounding from *A*, *B*, and *C*, respectively.

Ans. $\begin{cases} 174.9 \text{ ft.} \\ 414.4 \text{ ft.} \\ 853.8 \text{ ft.} \end{cases}$

(8) Fig. II represents a contour map of a reservoir; the contour interval is 5 feet, as shown, the water-line being taken as zero depth. The areas included by the five contours are, respectively, A_0 = 11,500 square yards, A_1 = 8,740

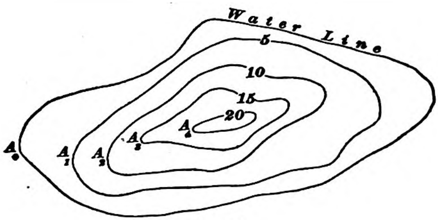

FIG. II

square yards, A_2 = 5,170 square yards, A_3 = 2,650 square yards, and A_4 = 615 square yards; find the volume of the reservoir, in cubic yards, by the end-area method.

Ans. 37,696 cu. yd.

(9) Compute the capacity of the reservoir in the preceding question by using the prismoidal formula.

Ans. 37,786 cu. yd.

(10) In Fig. III is shown the plane of a pond. Soundings were taken along the parallel ranges $A_1 A_1$, $A_2 A_2$, etc.,

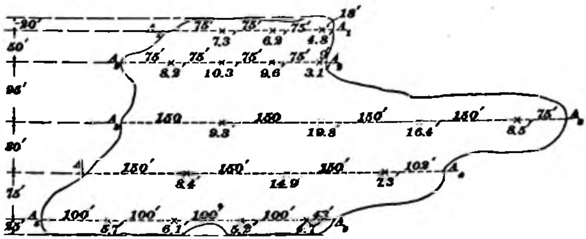

FIG. III

the latter being spaced as shown. The number under each cross-mark indicates the depth at that point; and that on the horizontal line between two cross-marks indicates the

horizontal distance between the corresponding points. Compute the areas of the cross-sections A_1A_1, A_2A_2, etc.

$$\text{Ans.}\left\{\begin{aligned} A_1A_1 &= 1{,}235.7 \text{ sq. ft.}\\ A_2A_2 &= 2{,}237.7 \text{ sq. ft.}\\ A_3A_3 &= 7{,}781.3 \text{ sq. ft.}\\ A_4A_4 &= 4{,}414.8 \text{ sq. ft.}\\ A_5A_5 &= 2{,}036.1 \text{ sq. ft.}\end{aligned}\right.$$

(11) From the data of the preceding example, compute the capacity of the pond, in cubic yards, using the method of end areas. Ans. 49,270 cu. yd.

(12) Still referring to Fig. III, make a careful sketch showing the cross-sections A_3A_3 and A_4A_4 to such a scale that the section A_3A_3 will be about $6\frac{1}{2}$ inches long and about 3 inches deep. Use the same scale for both sections, and state what horizontal and what vertical scale you have used.

UNITED STATES LAND SURVEYS

(PART 1)

EXAMINATION QUESTIONS

Note.—The student should answer all questions in his own language, and not copy from the Instruction Paper.

(1) (*a*) What are the two principal classes of land surveys? (*b*) What is the object of an original survey? (*c*) What is the object of a resurvey?

(2) (*a*) Into what divisions are the public lands of the United States divided by the government survey? (*b*) What is a township? (*c*) What is a section? (*d*) How are townships numbered?

(3) Indicate, by a diagram, the location: (*a*) of T. 6 N, R. 5 E; (*b*) of T. 2 N, R. 4 W. (*c*) Show, by a diagram, how the sections of a township are numbered. (*d*) How are the sections numbered in a township made fractional by a lake so that it contains less than 36 full sections?

(4) What is the effect of running the range lines to conform to true meridians?

(5) (*a*) When there is an excess or deficiency of measurement in subdividing a township into sections, what is done with it? (*b*) What sections are fractional because of this?

(6) What is: (*a*) a base line? (*b*) a principal meridian? (*c*) What are guide meridians and where are they located?

(7) (*a*) In what order are the township exteriors surveyed in a block of land 24 miles square bounded by

standard lines? (*b*) What lines have precedence in the order of survey? (*c*) How are the east-and-west lines run?

(8) In latitude 37° north, the bearing of the range line forming the east boundary of a township is due north; what should be the bearing of: (*a*) the first meridional section line west of this range line? (*b*) the second? (*c*) the third? (*d*) the fourth? and (*e*) the fifth?

Ans. (*a*) N 0° 1′ W
(*b*) N 0° 1′ W
(*c*) N 0° 2′ W
(*d*) N 0° 3′ W
(*e*) N 0° 3′ W

(9) (*a*) What streams and lakes are meandered in the public-land surveys and how is the meandering done? (*b*) How many kinds of meander corners are there and on what lines is each kind located? Illustrate by a diagram.

(10) What are the limits of closing for random lines?

(11) The south boundary of a township is on a correction line in latitude 43° north. (*a*) Calculate the theoretical length of its north boundary. (*b*) Calculate the angle of convergency between its eastern and western boundaries if on true meridians. (*c*) What are the least and greatest lengths permitted for the random line in running this boundary? (*d*) What is the theoretical length of the south line of Section 18 in this township? (*e*) What are the least and greatest lengths permitted for the random of this line?

Ans. (*a*) 479.32 ch.
(*b*) 4′ 51″
(*c*) 476.32 and 482.32 ch.
(*d*) 79.66 ch.
(*e*) 79.16 and 80.16 ch.

(12) (*a*) How are the lines of the government land surveys marked between corners? (*b*) What corners are established on these surveys and how are they defined or marked?

(13) Give a complete description of each of the following corners, with diagrams to illustrate them: (*a*) A standard

township corner; (*b*) a corner common to 4 townships; (*c*) a standard section corner; (*d*) a corner common to 4 sections; and (*e*) a quarter-section corner.

(14) (*a*) What are bearing trees? (*b*) How many bearing trees are marked for a corner? (*c*) How is the direction of a bearing tree given?

UNITED STATES LAND SURVEYS

(PART 2)

EXAMINATION QUESTIONS

(1) Describe and show by a diagram how Section 16 should be subdivided into half-quarters.

(2) Describe how the subdivision of Section 4 differs from that of a regular section, illustrating same by a diagram.

(3) Describe how the subdivision of Section 30 differs from that of a regular section, illustrating by a diagram.

(4) Describe and show by a diagram how Section 6 should be divided into half-quarters.

(5) Describe and show by a diagram how Section 25 should be subdivided when the southeast corner and south quarter post cannot be established because of being in a river.

(6) If the returned length of the north boundary of Section 6 is 77.32 chains, and this distance, as remeasured, is found to be 81.22 chains, what should be the remeasured length of that part of the line extending from the section corner between Sections 5 and 6 to the north quarter-section corner for Section 6? Ans. 42.02 ch.

(7) (*a*) What is the true legal length of each line of the public-land survey? (*b*) What is meant by original measure? (*c*) How is it used in practice?

(8) (*a*) What are accretions? (*b*) How are they apportioned between the adjoining land owners as the water recedes?

(9) What is the difference in principle between an original survey and a resurvey?

(10) Give an illustration of how you would proceed to find a lost corner in a particular case, real or imaginary, stating the kind of evidence by which the corner was identified when found.

(11) (*a*) In locating a deed on the ground, if the description is capable of more than one meaning, which should be taken? (*b*) In what light are descriptions in deeds to be interpreted? (*c*) If the calls in a description are inconsistent with each other, which is to prevail?

(12) (*a*) Where a conveyance is by metes and bounds and the contents are inconsistent with the bounds, which controls? (*b*) When a reference is made in the conveyance to a map or plat, what effect or value has the reference?

(13) (*a*) When land is bounded by a highway, how far does it extend? (*b*) When land is bounded by navigable waters, how far does it extend? (*c*) What force or effect has a meander line of the United States survey as a boundary line?

(14) If the variation is 4° 00′ west, what is the corrected bearing of a line whose original bearing was N 48° 52′ E?

Ans. N 52° 52′ E

(15) A certain line of a survey is found to have a bearing N 4° 45′ W, and its bearing, as recorded in a former survey, was N 1° 30′ E; what should be the present bearings of certain other lines of the same survey whose original bearings, as recorded in the notes of the former survey, were: (*a*) N 88° 30′ E, (*b*) S 87° 15′ E, (*c*) S 3° 45′ W, and (*d*) N 46° 45′ W?

Ans. (*a*) N 82° 15′ E
(*b*) N 86° 30′ E
(*c*) S 2° 30′ E
(*d*) N 53° 00′ W

MAPPING

(PART 1)

EXAMINATION QUESTIONS

(1) A map is made to a scale of 250 feet to the inch; what is the scale, expressed as a ratio? (See Art. 2.)

Ans. $\frac{1}{3000}$

(2) The scale of a map is $\frac{1}{3960}$; how many chains to the inch does this scale represent? Ans. 5

(3) Suppose that you have a line $1\frac{1}{2}$ inches long, and that you wish to lay off an angle of 35° 10′ from one of the extremities of the line, by means of a semicircular protractor 4 inches in diameter, and graduated to half degrees; explain fully, by means of a sketch, how you would proceed.

(4) Take a line and lay off at one end of it an angle of 37° 16′: (*a*) by the chord method; (*b*) by the tangent method. Make a sketch, and explain in detail all the operations.

(5) Explain how the line to which the accompanying notes refer would be platted by tangents.

Station	Angle
3 + 92	
4 + 39	R. 17° 54′
5 + 97	L. 49° 32′
7 + 81	

Make a sketch and explain fully all the operations.

(6) Draw a line to represent a meridian, take any point on it, and explain how a line having an azimuth, counted from the north, of 296° 10′, would be drawn from that point, by means of a semicircular protractor graduated to half degrees.

MAPPING

(PART 2)

EXAMINATION QUESTIONS

(1) Define a contour map, and give a brief outline of the field operations that are necessary for the construction of a map of that kind.

(2) Are field sketches preferable to written descriptions, and if so, why?

(3) In making a profile drawing, why is the vertical scale usually made larger than the horizontal?

(4) Make a sketch showing a profile along *AB*, Fig. I, the numbers on this line being station numbers. Assume any convenient scales, and explain the details of the work.

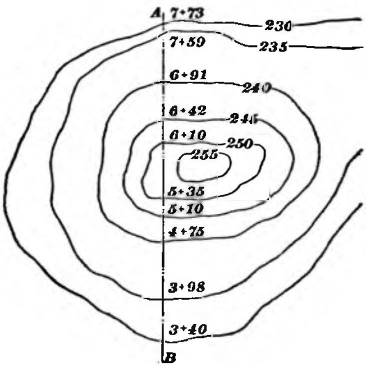

FIG. I

(5) In preparing the notes for a map of a stadia survey, what work must be done in the field, and what work can be more advantageously done in the office?

(6) On the margin are given the notes for a traverse line.

Line	Azimuth	Length Feet
A B	50° 12′	325.3
B C	118° 37′	487.6
C D	44° 13′	846.9
D E	321° 4′	203.7
E F	31° 49′	576.1

If this line is to be platted on a sheet 10 inches by 18 inches inside the border line, and the meridian is to be taken parallel to one of the side border lines, what scale would you use, and where would you begin the drawing, that the latter might be as large as possible, without, however, running outside of or too close (say less than about ½ inch) to the border lines? The azimuths are reckoned from the north.

HINT.—Compute first the latitudes and longitudes of the different corners, referred to a meridian through *A*.

PRACTICAL ASTRONOMY

(PART 1)

EXAMINATION QUESTIONS

(1) (*a*) What circles on the earth are secondaries to the equator? (*b*) What points are the poles of the equator? (*c*) What other circles on the earth have these same points for their poles? (*d*) What points are the poles of the meridian?

(2) (*a*) How is the angle between two great circles measured? What is the value of the angle between: (*b*) the meridian and the equator? (*c*) the prime vertical and the meridian?

(3) (*a*) What is the polar distance of the vernal equinox? (*b*) What is the declination of the south pole? (*c*) What is the hour angle of the zenith? (*d*) What is the azimuth of the north pole?

(4) (*a*) Through what points does the prime vertical pass? (*b*) The meridian? (*c*) In what direction and from what points is the azimuth measured? (*d*) Hour angles? (*e*) Right ascensions?

(5) (*a*) The sidereal time is 22 hours; what is the hour angle of the equinoctial colure? (*b*) The right ascension of a star is $18^h\ 10^m\ 20^s$; at what sidereal time will it be on the meridian?

(6) (*a*) Describe what is meant by standard time. (*b*) Find the standard time at which the hour angle of the sun is $-3^h\ 30^m$, the equation of time being $-3^m\ 12.4^s$, and the

longitude from Greenwich being $+6^h\ 44^m\ 10.7^s$. (*c*) Change August 17, 10 A. M., from civil date to astronomical date.

Ans. $\begin{cases} (b)\ 8^h\ 10^m\ 58.3^s \\ (c)\ \text{Aug. } 16,\ 22^h \end{cases}$

(7) The right ascension of a star is $13^h\ 2^m\ 40.1^s$; at what local mean solar time will this star's hour angle be 6 hours at a station whose longitude is $+1^h\ 7^m\ 3.5^s$, on January 5, 1903? Ans. $0^h\ 6^m\ 0.29^s$

(8) For the time 11 A. M., January 2, 1903, at a station whose longitude is $+3^h\ 58^m\ 55.4^s$, find: (*a*) the right ascension of the sun; (*b*) the declination; (*c*) the right ascension of Vega; (*d*) the declination; (*e*) the equation of time; (*f*) the semi-diameter of the sun.

Ans. $\begin{cases} (a)\ 18^h\ 49^m\ 3.43^s \\ (b)\ -22°\ 58'\ 12.9'' \\ (c)\ 18^h\ 33^m\ 37.87^s \\ (d)\ +38°\ 41'\ 43.2'' \\ (e)\ +3^m\ 55.97^s \\ (f)\ 16'\ 17.83'' \end{cases}$

(9) The altitude of the lower edge of the sun was observed at noon on January 3, 1903, with a transit and mercury horizon at a station whose longitude was $-3^h\ 1^m\ 10^s$, and found to be $20°\ 12'\ 10''$; what was the true altitude of the sun's center? Ans. $+20°\ 26'\ 2.2''$

(10) The wire of a transit was placed tangent to the upper edge of the sun, and the vertical circle was read; it was then placed tangent to the lower edge, and the vertical circle read again; the observations were then repeated on the image of the sun reflected from a mercury horizon. The four readings were: $30°\ 12'\ 10''$, $29°\ 40'\ 50''$, $329°\ 48'\ 0''$, $330°\ 19'\ 30''$. Find the true altitude of the sun's center.

Ans. $29°\ 54'\ 52.0''$

(11) The mean solar time at a place whose longitude is $+2^h\ 6^m$ was observed on January 4, 1903, to be $2^h\ 10^m\ 8^s$ P. M.; what was the sidereal time? Ans. $21^h\ 3^m\ 21.317^s$

(12) When a mean-time clock and a sidereal clock both indicate the same time, what is the right ascension of the mean sun?

(13) If the standard central time at a place whose longitude is 85° is $10^h\ 32^m$, what is the local time? Ans. $10^h\ 52^m$

(14) The altitude of a star was observed with a transit having an incomplete vertical circle; the correction for index error was $-1'\ 30''$; the circle reading was $60°\ 40'\ 30''$. Find the true altitude. Ans. $60°\ 38'\ 28''$

PRACTICAL ASTRONOMY

(PART 2)

EXAMINATION QUESTIONS

(1) The meridian altitude of Polaris was measured at Philadelphia on January 2, 1903, with a transit by the method of Art. 5. The following were the readings of the vertical circle:

Telescope direct	41° 11′ 25″
Telescope inverted	318 48 30
Telescope direct	41 11 55
Telescope inverted	318 48 30

Find the latitude. Ans. 39° 58′ 12.2″

(2) On January 25, 1903, the following series of altitudes were measured on the star Sirius when this star was passing the meridian. The index error was found to be +1′ 30″; the approximate latitude was 49°. Find the true latitude.

Number	Vertical Circle	Watch Time
1	24° 20′ 30″	$11^h\ 10^m\ 14^s$
2	24 20 30	11 12 30
3	24 21 00	11 14 30
4	24 21 30	11 16 50
5	24 20 50	11 18 50
6	24 20 30	11 20 54

Ans. 49° 4′ 14.4″

(3) Find the watch time of apparent noon on January 6, 1903, if the observer is in longitude $+2^h\ 3^m\ 18.1^s$ and his watch is $1^m\ 31.1^s$ slow on local mean time.

Ans. $12^h\ 4^m\ 13.8^s$

(4) At Washington, a watch is $9^m\ 11^s$ fast on standard eastern time; find the watch time of apparent noon on January 6, 1903, the longitude of Washington being $+5^h\ 8^m\ 15.75^s$ from Greenwich. Ans. $12^h\ 23^m\ 9.42^s$ P. M.

(5) Ten double altitudes of the sun were observed on the morning of January 7, 1903, a mercury horizon being used; one-half of the measurements were taken on the upper edge and one-half on the lower edge of the sun, and the watch time of each observation was recorded. The mean of the observed altitudes was 14° 10′ 58.2″; the mean of the recorded watch times was $9^h\ 9^m\ 13.2^s$ A. M. Find the true mean time and the watch error, the longitude being $-7^m\ 37^s$ and the latitude +39° 58′ 2″.

Ans. $\begin{cases} \text{Mean time, } 9^h\ 1^m\ 2.63^s \\ 8^m\ 10.6^s \text{ fast} \end{cases}$

(6) The following observations on the sun were made, as described in Art. **35**, for the purpose of determining the azimuth of a line by the method of Art. **38**. The latitude was +38° 53′ 18″, the declination of the sun was +13° 55′ 33″, the semi-diameter of the sun was 16′ 27″ and the correction due to index error of the vertical circle was −3′ 40″; find the azimuth.

Approximate Time P. M.	Vertical Circle	Horizontal Circle	Diagram of Field
$5^h\ 2^m$	29° 36′ 0″	25° 26′ 30″	
5 5	29 20 0	25 36 30	
5 6	29 2 0	25 46 0	
5 9	28 59 0	25 58 0	
5 10	28 45 0	26 2 0	
5 11	28 36 30	26 6 30	

Ans. 238° 45′ 50″

(7) The following observations for azimuth were made as described in Art. 40. The latitude being $+40° 36' 27''$ and the declination of the sun $+19° 54' 5''$, find the azimuth.

Telescope	Time A. M.	Vertical Circle	Horizontal Circle	Diagram of Field
Direct. . .	$8^h\ 40^m$	43° 09′ 00″	64° 48′ 00″	
Inverted. .	8 42	43 35 30	65 10 30	
Inverted. .	8 44	44 21 00	64 52 30	
Direct. . .	8 46	44 48 00	65 15 00	

Ans. 36° 44′ 18″

(8) Make a table of corrected declination settings for use in the field with a solar transit at San Francisco, between the hours of 3 and 4 P. M. on January 1, 1903, the longitude of San Francisco being $+3^h\ 1^m\ 27^s$ from Washington, and the latitude being $+36° 27'$.

(9) The following observations were made on Polaris when this star was passing eastern elongation at Washington on January 3, 1903. The method was that described in Art. 50. The declination of Polaris being 88° 47′ 42″, and the latitude of Washington 38° 53′ 20″, find the azimuth of the line.

Telescope	Pointing on Star	Pointing on Mark
Direct	30° 8′ 30″	130° 9′ 40″
Inverted	117 36 10	217 37 30
Direct	224 10 50	324 12 10
Inverted	340 8 00	80 9 30

Ans. 101° 34′ 13″

(10) On the morning of July 1, a chronometer was compared with the observatory clock at a certain station. It was then carried to Washington and compared with the

observatory clock there, and on the morning of July 2 was brought back and compared with the first clock. It was known that the clock at the first station was 10^s fast, and was gaining 1.2^s each 24 hours. The clock at Washington was 5.8^s slow. Find the longitude of the station from Washington, the comparison being as follows:

	Observatory Clock	Chronometer
Station, July 1 . . .	$8^h\ 48^m\ 2^s$	$8^h\ 56^m\ 24^s$
Washington, July 1 .	18 13 36	21 14 30
Station, July 2 . . .	9 56 24	10 4 46

Ans. $2^h\ 52^m\ 15.6^s$

(11) The error of a watch on local mean time is $-2^m\ 4.8^s$ and on standard 75th-meridian time it is $+8^m\ 42.8^s$. What is the longitude west from Greenwich?

Ans. $4^h\ 49^m\ 12.4^s$

A KEY

TO ALL THE QUESTIONS AND EXAMPLES
INCLUDED IN THE
EXAMINATION QUESTIONS IN THIS VOLUME.

It will be noticed that the Keys have been given the same section numbers as the Examination Questions to which they refer. All article references refer to the Instruction Paper bearing the same section number as the Key in which they occur, unless the title of some other Instruction Paper is given in connection with the references.

LEVELING

(1) $h = 6.879 - 6.817 = .062$, $e_n = 2$, and $d = 360$; applying formula **2**, Art. **22**,

$$n_s = \frac{.062}{2 \times 360 \times \tan 1''} = 17.8'', \text{ nearly. Ans.}$$

(2) Applying formula **4**, $m = \dfrac{1.20}{.04} = 30$. Ans.

(3) (*a*) See Art. **24**.
(*b*) See Art. **25**.

(4) (*a*) See Art. **38**.
(*b*) See Art. **39**.
(*c*) See Art. **40**.

(5) (*a*) The height of instrument is equal to the elevation of the given point plus the backsight on that point, or $61.84 + 11.81 = 73.65$ ft. By subtracting the foresight on the turning point, the elevation of the turning point is found to be $73.65 - 0.49 = 73.16$ ft. Ans.

(*b*) The new height of instrument is equal to the elevation of the turning point plus the backsight reading on this point, or $73.16 + 9.57 = 82.73$ ft. Ans.

(*c*) The elevation of the station is equal to the height of instrument minus the foresight reading, or $82.73 - 4.3 = 78.43$ or 78.4 ft., nearly. Ans.

(6) (*a*) See Art. **57**.

(*b*) Substituting the value 25 for M in the equation for item 9, Art. **59**, we have $E_f = .1\sqrt{25} = .5$ ft. Ans.

(*c*) Substituting the value 50 for M in item 6 of Art. **59**, we have $E_f = .042\sqrt{50} = .297$ ft. Ans.

(7) (*a*) See Art **60**.

(*b*) and (*c*) See Art. **63**.

(8) (*a*) The distance between Sta. 66 and 93 is 2,700 ft. and the rate of grade is +1.25 per cent. According to Rule II, Art. **63**, the total rise in the given distance is equal to $\dfrac{1.25 \times 2{,}700}{100} = 33.75$ ft., and the elevation of the grade at Sta. 93 is $126.50 + 33.75 = 160.25$ ft. Ans.

(*b*) The distance between Sta. 66 and 95 + 64 is 2,964 ft. The total rise in this distance is $\frac{1.25 \times 2,964}{100} = 37.05$ ft., and the elevation of the grade at Sta. 95 + 64 is, therefore, 126.50 + 37.05 = 163.55 ft. Ans.

(*c*) The distance between Sta. 66 and 99 + 32 is 3,332 ft. The total rise in this distance is $\frac{1.25 \times 3,332}{100} = 41.65$ ft., and the elevation of the grade at Sta. 99 + 32 is, therefore, 126.50 + 41.65 = 168.15 ft. Ans.

(9) The distance from the instrument to the center of the spire is 100 + 15 = 115 ft., = d, the angle $A = 45° 20'$, and tan 45° 20 = 1.01170; applying formula **12**,

$$h = 115 \tan 45° 20' = 115 \times 1.01170 = 116.3455 = 116.35 \text{ ft.}$$

The height of the instrument above the level of the base is 5 ft.; hence, 116.35 + 5 = 121.35 ft., the height of the spire. Ans.

(10) (*a*) The elevations of the accompanying level notes are worked out as follows: The first elevation recorded in the column of elevations

Station	*Rod Reading*	*Height of Instrument*	*Elevation*	*Grade*	*Remarks*
B. M.	+ *5.53*	*166.95*	*161.42*		*B. M. on poplar tree 70 ft. to left of Sta. 40*
40	− *6.4*		*160.6*	*162.0*	
41	− *7.2*		*159.8*	*160.485*	
41 + 60	− *10.9*		*156.1*		
42	− *8.6*		*158.4*	*158.97*	
43	− *8.8*		*158.2*	*157.455*	
T. P.	− *8.66*		*158.29*		
	+ *2.22*	*160.51*			
44	− *4.8*		*155.7*	*155.94*	
45	− *6.3*		*154.2*	*154.425*	
46	− *8.8*		*151.7*	*152.91*	
47	− *9.9*		*150.6*	*151.395*	
48	− *11.1*		*149.4*	*149.88*	
T. P.	− *11.24*		*149.27*		
	+ *3.30*	*152.57*			
49	− *4.7*		*147.9*	*148.365*	
50	− *7.1*		*145.5*	*146.85*	
51	− *8.7*		*143.9*	*145.335*	
52	− *9.8*		*142.8*	*143.82*	
53	− *10.9*		*141.7*	*142.305*	
T. P.	− *11.62*		*140.95*		

is that of the bench mark, abbreviated to B. M. This elevation is 161.42 ft. The first rod reading, 5.53 ft., is the backsight on the B. M., and is added to the elevation of the bench mark, in order to determine the height of instrument. Thus, 161.42 + 5.53 = 166.95 ft. = H. I. The next rod reading, which is a foresight on Sta. 40, is 6.4 ft. This rod reading indicates that the surface of the ground at Sta. 40 is 6.4 ft. below the line of sight. The elevation of that surface is, therefore, the difference between the height of instrument and the rod reading, or 166.95 − 6.4 = 160.55 ft. Since the elevations of the intermediate points are given to the nearest tenth of a foot, the elevation of the ground at Sta. 40 is taken as 160.6 ft. The rod reading at Sta. 41 is 7.2, which subtracted from 166.95 ft., gives for that station an elevation of 159.8 ft. The elevations of the remaining stations up to and including Sta. 43 are determined by subtracting the rod readings from the same height of instrument; viz., 166.95 ft. The elevation of the turning point is determined by subtracting the foresight rod reading on this point from the preceding height of instrument. This gives for the elevation of the T. P., 166.95 − 8.66 = 158.29 ft., which is recorded in the column of elevations. After the instrument has been moved forwards and set up in a new position, the new height of instrument is found by adding the backsight rod reading to the elevation of the turning point. Thus, 158.29 + 2.22 = 160.51 ft., the next H. I. The next rod reading, 4.8, is at Sta. 44, and the elevation at that station is the difference between the preceding H. I., 160.51, and that rod reading, giving an elevation of 160.51 − 4.8 = 155.7 ft., which is recorded in the column of elevations opposite Sta. 44. The remaining elevations are determined in a similar manner. The elevation of the last turning point is checked by the method explained in Art. **53**. The sum of the backsight or plus readings is equal to 11.05 and the sum of the foresight or minus readings on turning points is equal to 31.52. Adding the sum of the backsights to the elevation of the bench mark, we have 161.42 + 11.05 = 172.47, and subtracting the sum of the foresights, we obtain 172.47 − 31.52 = 140.95.

+ Readings	− Readings	
5.53 ft.	8.66 ft.	161.42 ft.
2.22 ft.	11.24 ft.	11.05 ft.
3.30 ft.	11.62 ft.	172.47 ft.
11.05 ft.	31.52 ft.	31.52 ft.
		140.95 ft.

This agrees with the elevation of the T. P. following Sta. 53, which is the last one determined. A check-mark √ is placed opposite the elevations checked, to show that the figures have been verified.

(*b*) In the accompanying diagram is shown a section of profile paper on which the level notes are platted, and on which the given grade line

is drawn. The spaces between the vertical lines represent 100 ft., and those between the horizontal lines, 1 ft. Assume the elevation of the sixth heavy line from the bottom to be 150 ft. The first heavy vertical line at the left represents Sta. 40, and the next heavy vertical line, ten spaces to the right of this line, represents Sta. 50. In the margin at the bottom of the page, the numbers 40 and 50 are written under the lines that represent Sta. 40 and 50, respectively. The elevation of Sta. 40, as recorded in the notes, is 160.6 ft. The elevations of the surface line are platted in the profile in accordance with the

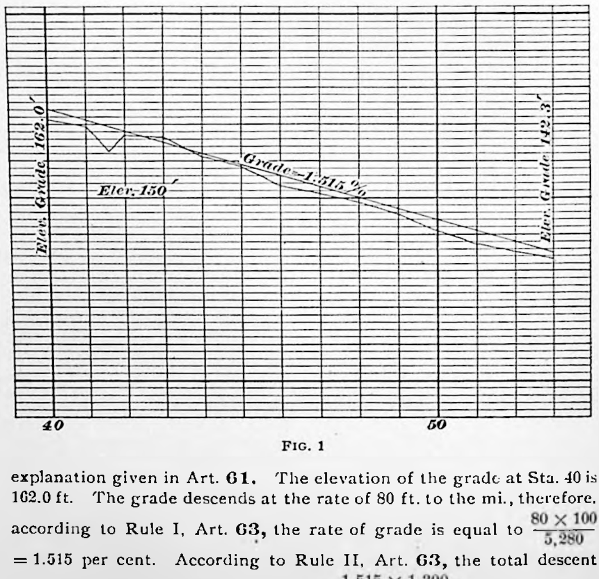

FIG. 1

explanation given in Art. **61.** The elevation of the grade at Sta. 40 is 162.0 ft. The grade descends at the rate of 80 ft. to the mi., therefore, according to Rule I, Art. **63,** the rate of grade is equal to $\frac{80 \times 100}{5,280}$ = 1.515 per cent. According to Rule II, Art. **63,** the total descent between Sta. 40 and Sta. 53 is equal to $\frac{1.515 \times 1,300}{100} = 19.695$ ft. The elevation of the grade at Sta. 53 will, therefore, be $162.0 - 19.695 = 142.305$ ft. Plat the elevation of the grade at Sta. 53 in the profile and join the grade point at Sta. 40 with that at Sta. 53 by a straight line, which will be the grade line required. Upon the grade line, mark the rate of grade − 1.515 per cent.

(*c*) The elevation of the grade at Sta. 40 is fixed at 162.0 ft. As the

grade descends from Sta. 40 at the rate of 1.515 per cent. or 1.515 ft. per station, the elevation of the grade at Sta. 41 is found by subtracting 1.515 ft. from 162.0 ft., which gives 160.485 ft., and the elevation of the grade for each succeeding station is found by subtracting the rate of grade from the elevation of the grade at the immediately preceding station. The elevation of the grade at each station is recorded in the column headed *Grade*.

(11) Applying formula **17**, $s = 60{,}384.3$ (log 29.4 − log 26.95) $\left(1 + \frac{74 + 58 - 64}{900}\right) = 2{,}454$ ft. Ans.

(12) (*a*) To apply formula **13**, we have the distance $m = 200$ ft., the angle $a = 13° 15'$, and the angle $c = 18° 10'$. By substituting these values in formula **13**, the height of the tower above the instrument is found to be

$$h = \frac{200}{\cot 13° 15' - \cot 18° 10'} = \frac{200}{4.24685 - 3.04749}$$
$$= \frac{200}{1.19936} = 166.76 \text{ ft.} \quad \text{Ans.}$$

(*b*) Substituting the values of h and cot c in the equation for n in Art. **69**, we have

$$n = 166.76 \times \cot 18° 10' = 508.20 \text{ ft.} \quad \text{Ans.}$$

(13) To apply formula **14**, we have $y = 3$, $m = 500$, $c = 40° 55'$, and $a = 20° 18'$. Substituting these values in formula **14** and giving y the + sign, since the point C is lower than the point A, we have

$$h = \frac{500 + 3 \times \cot 40° 55'}{\cot 20° 18' - \cot 40° 55'} = \frac{500 + 3 \times 1.15375}{2.70335 - 1.15375} = 324.90 \text{ ft.}$$

The height of the summit above the point C is equal to $h + y = 324.90 + 3 = 327.90$ ft. Ans.

(14) In this case, y is given the − sign as the point C is higher than the point A. Substituting known values in formula **14**, we obtain

$$h = \frac{500 - 3 \times \cot 40° 55'}{\cot 20° 18' - \cot 40° 55'} = \frac{500 - 3 \times 1.15375}{2.70335 - 1.15375} = 320.43 \text{ ft.} \quad \text{Ans.}$$

(15) (*a*) Substituting known values in formula **10**, we have

$$e = (0.5 - .0719)\frac{450^2}{20{,}890{,}590} = \frac{.4281 \times 450^2}{20{,}890{,}590} = .004 \text{ ft.} \quad \text{Ans.}$$

(*b*) Substituting known values in formula **10**, we have

$$e = (0.5 - .0719)\frac{850^2}{20{,}890{,}590} = \frac{.4281 \times 850^2}{20{,}890{,}590} = .015 \text{ ft.} \quad \text{Ans.}$$

(*c*) Substituting known values in formula **10**, we have

$$e = (0.5 - .0719)\frac{1{,}250^2}{20{,}890{,}590} = \frac{.4281 \times 1{,}250^2}{20{,}890{,}590} = .032 \text{ ft.} \quad \text{Ans.}$$

(16) (*a*) The elevation of the turning point is found by subtracting the foresight on it from the height of instrument. This gives 126.37 − 5.36 = 121.01 ft. as the elevation of the turning point. The next height of instrument is equal to the elevation of the turning point plus the backsight, or 121.01 + 6.27 = 127.28 ft. The required elevation of the first station is equal to the height of instrument minus the foresight reading, or 127.28 − 4.6 = 122.68 = 122.7 ft. Ans.

(*b*) The elevation of the second station is equal to 127.28 − 9.3 = 117.98 = 118.0 ft. Ans.

(*c*) The elevation of the third station is equal to 127.28 − 11.8 = 115.48 = 115.5 ft. Ans.

(17) This example is similar to the one explained in Art. **71** of the text. Referring to Fig. 21 of the same article, the angles measured on the horizontal circle are $B\,A\,D = 77°\ 36'$ and $D\,B\,A = 67°\ 45'$ and the angle measured on the vertical circle is $C\,A\,D = 28°\ 16'$. The angle $A\,D\,B = 180° - (77°\ 36' + 67°\ 45') = 34°\ 39'$. From trigonometry, we have $A\,D = \dfrac{A\,B \sin D\,B\,A}{\sin A\,D\,B} = \dfrac{500 \sin 67°\ 45'}{\sin 34°\ 39'}$. Hence, using logarithms, $A\,D = 813.94$ ft. The height of the mountain is then equal to $A\,D \tan 28°\ 16' = 813.94 \times \tan 28°\ 16' = 437.65$ ft. Ans.

(18) (*a*) The elevations of the surface and grade lines are shown in the accompanying notes. To the elevation of the bench mark, the backsight on the same point is added in order to determine the height of instrument. This gives 111.42 + 5.52 = 116.94 ft. for the first height of instrument.

The foresight on Sta. 40 deducted from the height of instrument gives 110.3 ft. for the elevation of this station. The elevation of Sta. 41 is 116.94 − 10.5 = 106.4 ft. In like manner, we find the elevation of the first turning point to be 116.94 − 11.28 = 105.66 ft. Adding to this elevation the backsight on the turning point, the second height of instrument is found to be 105.66 + 7.83 = 113.49 ft. The elevation of Sta. 42 is found by subtracting the foresight from this height of instrument. Its elevation is equal to 113.49 − 8.5 = 105.00 ft. The elevations of the surface line at the succeeding stations are found in the same manner.

+ Readings	− Readings	
5.52	11.28	
7.83	10.87	
0.49	11.54	111.42
0.56	10.65	29.01
3.24	0.98	140.43
11.37	11.72	57.04
29.01	57.04	83.39

Station	Backsight	Height of Instrument	Foresight	Elevation of Surface	Elevation of Grade	Cut or Fill	Remarks
B. M.	5.52	116.94		111.42			On poplar tree 60 ft. to left of Sta. 40
40			6.6	110.3	112.0	F 1.7	
41			10.5	106.4	110.5	F 4.1	
T. P.	7.83		11.28	105.66			
42		113.49	8.5	105.0	109.0	F 4.0	
43			0.9	112.6	107.5	C 5.1	
44			4.1	109.4	106.0	C 3.4	
45			5.8	107.7	104.5	C 3.2	
T. P.	0.49		10.87	102.62			
46		103.11	3.1	100.0	103.0	F 3.0	
47			6.2	96.9	101.5	F 4.6	
48			7.0	96.1	100.0	F 3.9	
49			10.1	93.0	98.5	F 5.5	
50			8.2	94.9	97.0	F 2.1	
51			5.7	97.4	95.5	C 1.9	
52			6.6	96.5	94.0	C 2.5	
T. P.	0.56		11.54	91.57			
53		92.13	2.0	90.1	92.5	F 2.4	
53 + 60			0.6	91.5			
54			4.5	87.6	91.7	F 4.1	
T. P.	3.24		10.65	81.48			
54 + 75		84.72	10.9	73.8			Mill Creek
55			4.7	80.0	90.9	F10.9	
55 + 40			1.8	82.9			
56			1.2	83.5	90.1	F 6.6	
57			2.7	82.0	89.3	F 7.3	
58			1.8	82.9	88.5	F 5.6	
T. P.	11.37		0.98	83.74			
59		95.11	8.9	86.2	87.7	F 1.5	
60			1.0	94.1	86.9	C 7.2	
61			5.5	89.6	86.1	C 3.5	
62			6.0	89.1	85.3	C 3.8	
63			11.4	83.7	84.5	F 0.8	
T. P.			11.72	83.39			

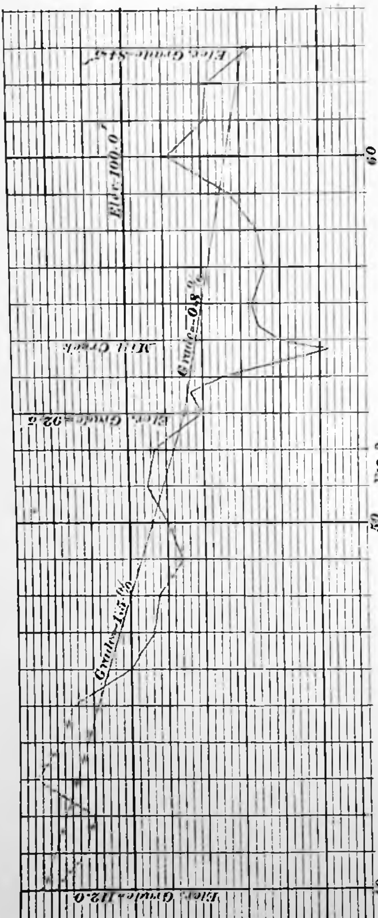

FIG. 2

To check the elevation of the last turning point, we employ the method explained in Art. **53.** The sum of the backsights is equal to 29.01 and the sum of the foresights on turning points is equal to 57.04. Adding the sum of the backsights to the elevation of the bench mark, we have 111.42 + 29.01 = 140.43, and subtracting the sum of the foresights on turning points, we obtain 140.43 − 57.04 = 83.39 for the elevation of the last turning point. Since this result agrees with the elevation of the turning point, 83.39 ft., as found before, the notes are probably correct.

(*b*) The profile of the notes is shown in Fig. 2. The elevation of the seventh heavy line from the bottom is assumed to be 100 ft., and the first vertical line at the left represents Sta. 40. The profile is constructed according to the instruction given in Art. **61.** The total descent between Sta. 40 and 53 is, according to Rule II, Art. **63,** equal to $\frac{1{,}300 \times 1.5}{100} = 19.5$ ft. The elevation of the grade at Sta. 53 is, therefore, 112.0 − 19.5 = 92.5 ft. At this station,

the rate of grade changes to $-$ .8 per cent. Since the grade descends from Sta. 53 at the rate of 0.8 ft. per station, the elevation of the grade at Sta. 54 is equal to $92.5 - 0.8 = 91.7$ ft. The total descent between Sta. 53 and 63 is equal to $\frac{1{,}000 \times 0.8}{100} = 8.0$ ft. The elevation of the grade at Sta. 63 is, therefore, $92.5 - 8.0 = 84.5$ ft. The grade point at Sta. 40 is connected by a straight line with the grade point at Sta. 53, where the rate of grade changes, and this point is connected by a straight line with the grade point at Sta. 63.

(19) (*a*) The grade line that is to have an elevation of 112.0 ft. at Sta. 40, is to have a grade of $-$ 1.5 per cent. from Sta. 40 to Sta. 53. The elevation of the grade at Sta. 41 is found by subtracting 1.5 ft. from 112.0 ft., which gives 110.5 ft., and the elevation of the grade at each succeeding station between Sta. 40 and 53 is found by subtracting 1.5 ft. from the elevation of the grade at the preceding station. Since the rate of grade between Sta. 53 and 63 is $-$ .8 per cent., the elevation of the grade at Sta. 54 and at each succeeding station is found by subtracting .8 ft. from the elevation of the grade at the preceding station. The elevation of the grade at each station is recorded in the column headed *Elevation of Grade*.

(*b*) See Art. **65**. Since at Sta. 40 the elevation of the grade is 112.0 ft. and the elevation of the surface is 110.3 ft., the depth of filling required at this station is $112.0 - 110.3 = 1.7$ ft. The depth of filling required at Sta. 41 and 42 is determined in the same manner.

At Sta. 43, the elevation of the surface exceeds the elevation of the grade, therefore, the grading will consist of cutting instead of filling. The amount of cutting at Sta. 43 is, therefore, $112.6 - 107.5 = 5.1$ ft. The amount of cutting or filling at the remaining stations is determined in the same manner and recorded in the notes under the column headed *Cut or Fill*, the letters *C* and *F* being used to designate cuts and fills respectively.

CIRCULAR CURVES

(1) The tangent distance T is found by applying formula **13**, Art. **34**, $T = R \tan \frac{1}{2}I$. From the Table of Radii and Deflections, the radius of a 6° 15′ curve is found to be 917.19 ft.; $\frac{1}{2}I = \frac{35° \; 10'}{2}$ $= 17° \; 35'$; $\tan 17° \; 35' = .31690$. Substituting these values in the formula, we have $T = 917.19 \times .31690 = 290.66$ ft. Ans.

(2) The angle of intersection 30° 45′ reduced to the decimal form is equal to 30.75°. The degree of curve 5° 15′ reduced to the decimal form is equal to 5.25°. Dividing the intersection angle by the degree of curve (see Rule, Art. **36**), the required length of the curve, in stations of 100 ft. each, is equal to $\frac{30.75°}{5.25°} = 5.8571$ full sta., or 585.71 ft. Ans.

(3) (*a*) In order to determine the P. C. of the curve, we must know the tangent distance, which, subtracted from the number of the station of the intersection point, will give the P. C. The tangent distance T is found by applying formula **13**, $T = R \tan \frac{1}{2}I$. From the Table of Radii and Deflections, the radius of a 5° curve is found to be 1,146.28 ft.; $\frac{1}{2}I = \frac{33° \; 06'}{2}$ $= 16° \; 33'$; $\tan 16° \; 33' = .29716$. Substituting these values in the formula, we have $T = 1{,}146.28 \times .29716 = 340.63$ ft. In Fig. 1, let AB and CD be the tangents, which intersect in the point E, forming an angle $DEF = 33° \; 06'$.

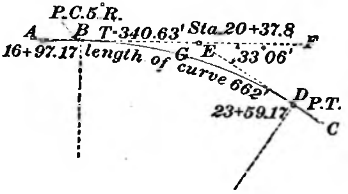

Fig. 1

The line of survey is being run in the direction AB, and the line is measured in regular order up to the intersection point E, the number

of which is 20 + 37.8. Subtracting the tangent distance BE, = 340.63 ft., from Sta. 20 + 37.8, we have 16 + 97.17, the station of the P. C. at B. Ans.

(*b*) The intersection angle 33° 06' reduced to decimal form is equal to 33.1°. Dividing the intersection angle by the degree of curve (see Rule, Art. **36**), the length BGD of the curve in stations of 100 ft. each is found to be $\frac{33.1}{5}$ = 6.62 sta., or 662 ft. Ans.

(*c*) The length of the curve, 662 ft., added to the station of the P. C., viz., 16 + 97.17, gives 23 + 59.17, the station of the P. T. at D. Ans.

(4) (*a*) From formula **13**, we have

$$T = R \tan \frac{1}{2} I, \text{ whence } R = \frac{T}{\tan \frac{1}{2} I}$$

$I = 20° \ 10'$ and $\frac{1}{2} I = 10° \ 05'$; $T = 291.16$. By substituting these values in the formula, we obtain

$$R = \frac{291.16}{\tan 10° \ 05'} = 1{,}637.3 \text{ ft. Ans.}$$

(*b*) From formula **10**, we have $R = \frac{50}{\sin D_{100}}$, or $\sin D_{100} = \frac{50}{R}$. Substituting in this equation, the value of R obtained above, we have

$$\sin D_{100} = \frac{50}{1{,}637.3}; \ D_{100} = 1° \ 45'$$

D_{100} is the deflection angle for a chord of 100 ft., and as stated in Art. **29**, the deflection angle for a chord of 100 ft. is equal to one-half the degree of curve. The degree of curve is, therefore, 2 × 1° 45' = 3° 30'. Ans.

(5) 1° 41.22' = 101.22'. The deflection angle for a chord of 100 ft. is equal to $\frac{101.22}{48.2} \times 100 = 210' = 3° \ 30'$. The degree of curve is, therefore, 2 × 3° 30' = 7°. (See Art. **29**.) From the Table of Radii and Deflections, the radius of a 7° curve is found to be 819.02 ft. Ans.

(6) The deflection angle for a chord of 100 ft. is $\frac{6° \ 15'}{2} = 3° \ 07\frac{1}{2}'$ = 187.5' (see Art. **29**), and the deflection angle for a chord of 1 ft. is $\frac{187.5'}{100} = 1.875'$. The deflection angle for a chord of 72.7 ft. is, therefore, 1.875' × 72.7 = 136.31' = 2° 16.3'. Ans.

(7) From the Table of Radii and Deflections, the radius of a 5° 30′ curve is found to be 1,042.14 ft. Substituting known values in formula **14**, we have for the tangent deflection,

$$f = \frac{50^2}{2 \times 1{,}042.14} = 1.199 \text{ ft. Ans.}$$

(8) The radius of a 5° curve as given in the Table of Radii and Deflections is 1,146.28 ft. According to formula **14**, the tangent deflection for a chord of 47.8 ft. is

$$f = \frac{47.8^2}{2 \times 1{,}146.28} = 0.997 \text{ ft.}$$

By applying formula **16**, the chord deflection is

$$d_s = 0.997\left(1 + \frac{100}{47.8}\right) = 3.083 \text{ ft. Ans.}$$

(9) Applying formula **22**,

$$D_c = \frac{55 \times .63}{12 \times .60 \times .60} = 8.02°. \text{ Ans.}$$

(10) The radius of a 5° curve, as given in the Table of Radii and Deflections, is 1,146.28 ft. According to formula **20**, Art. **42**, the middle ordinate to a chord of 60 ft., in a 5° curve, is

$$m = 1{,}146.28 - \sqrt{1{,}146.28^2 - \frac{60^2}{4}} = .393 \text{ ft. Ans.}$$

(11) The radius of a 6° curve is 955.37, which is $OA = R_1$; the radius OE is therefore $955.37 - 80 = 875.37 = R_2$. The length of each of the chords AB, BC, CD is 100 ft. $= c_1$. By substituting these in formula **25**, the length of each of the chords EF, FG, and GH is found to be

$$c_2 = 100 \times \frac{875.37}{955.37} = 91.63 \text{ ft. Ans.}$$

(12) (*a*) The deflection angle for a chord of 100 ft. is equal to one-half the degree of curve, or 3° 30′. The deflection angle $V'BE$ is then equal to $3° 30' \times 3 = 10° 30'$. Ans. (See Art. **32**.)

(*b*) The radius of a 7° curve, as given in the Table of Radii and Deflections, is 819.02 ft. By applying formula **12**, Art. **32**, we have for the length of the chord BE,

$$c = 2 \times 819.02 \times \sin 10° 30' = 298.52 \text{ ft. Ans.}$$

(13) (*a*) Substituting known values in the equation for MC' in Art. **52**, we have

$$MC' = 819.02 \times \sin 3° 30' = 50.00 \text{ ft. Ans.}$$

The distance $M D'$ is found in the same manner and in this case is the same.

(*b*) The distance $B M$ is equal to one-half the length of the chord $B E$, or 149.26 ft. Therefore, $B C' = B M - M C' = 149.26 - 50.00 = 99.26$ ft. Ans.

(*c*) $C' D' = 50.00 \times 2 = 100.00$ ft., therefore, $B D' = B C' + C' D' = 99.26 + 100.00 = 199.26$ ft. Ans.

(*d*) Substituting known values in the equation for $C' C$ in Art. **52**, we have

$$\begin{aligned} C' C &= 819.02 \times (\cos 3° 30' - \cos 10° 30') \\ &= 819.02 \times (.99813 - .98325) = 819.02 \times .01488 \\ &= 12.19 \text{ ft. Ans.} \end{aligned}$$

The ordinate $D' D$ is found in the same manner and in this case is the same.

STADIA AND PLANE-TABLE SURVEYING

(1) Substituting the given values in formula **2**, Art. **4**,

$$d = 100 \times 3.47 + 1.00 = 348.0 \text{ ft. Ans.}$$

(2) (*a*) Substituting the given values in the formula of Art. **6**,

$$d = (102 \times 6.48 \cos 20° 45' + 1.25) \cos 20° 45'$$
$$d = (102 \times 6.48 \times .93514 + 1.25)\ .93514$$
$$d = 579.2 \text{ ft. Ans.}$$

(*b*) Substituting the given values in the formula of Art. **7**,

$$v = \tfrac{1}{2} \times 102 \times 6.48 \sin 41° 30' + 1.25 \sin 20° 45'$$
$$v = \tfrac{1}{2} \times 102 \times 6.48 \times .66262 + 1.25 \times .35429$$
$$v = 219.4 \text{ ft. Ans.}$$

(3) Taking 50′ for the value of d_1 and 100 ft., 200 ft., etc. successively for the values of d_2 and applying formulas **1** and **2**, Art. **10**, we find, for d_1 = 50 ft. and d_2 = 100 ft.,

$$s = \frac{100 - 50}{.971 - .480} = 101.833$$
$$i = \frac{50 \times .971 - 100 \times .480}{.971 - .480} = 1.120$$

for d_1 = 50 and d_2 = 200,

$$s = \frac{200 - 50}{1.951 - .480} = 101.971$$
$$i = \frac{50 \times 1.951 - 200 \times .480}{1.951 - .480} = 1.054$$

for d_1 = 50 and d_2 = 250,

$$s = \frac{250 - 50}{2.441 - .480} = 101.989$$
$$i = \frac{50 \times 2.441 - 250 \times .480}{2.441 - .480} = 1.045$$

for d_1 = 50 and d_2 = 350,

$$s = \frac{350 - 50}{3.422 - .480} = 101.971$$
$$i = \frac{50 \times 3.422 - 350 \times .480}{3.422 - .480} = 1.054$$

Tabulating these results and taking a mean, we find $s = 101.941$ and $i = 1.068$. Ans.

s	i
101.833	1.120
101.971	1.054
101.989	1.045
101.971	1.054
4) 407.764	4) 4.273
101.941	1.068

(4) The difference in elevation between the stadia point and the instrument point is determined by substituting the given values in the formula of Art. **7**.

$$v = \frac{1}{2} \times 100 \times 6.00 \sin 16° 52' + .75 \sin 8° 26'$$

$$v = \frac{1}{2} \times 100 \times 6.00 \times .29015 + .75 \times .14666$$

$$v = 87.05 + .11 = 87.2 \text{ ft.}$$

Since the elevation of the instrument point above datum is 506.3 ft. and the vertical angle is an angle of elevation, the elevation of the stadia point above datum is $506.3 + 87.2 = 593.5$ ft. Ans.

(5) (*a*) The tabular horizontal distance for 15° 24′ is 92.95 and the correction for $i = 1.00$ is .96; hence, the horizontal distance is equal to $5 \times 92.95 + .96 = 465.7$ ft. Ans. (See Art. **22.**)

(*b*) The tabular difference in elevation is equal to 25.60 and the correction for $i = 1.00$ is .27; hence, the difference in elevation is equal to $5 \times 25.60 + .27 = 128.3$ ft. Ans.

(*c*) The difference in elevation between the stadia point and the instrument point is 128.3 ft. as obtained in (*b*). Since the elevation of the instrument point is 125.4 ft., the elevation of the stadia point is equal to $125.4 + 128.3 = 253.7$ ft. Ans.

(6) From formula **2**, Art. **4**, we have

$$s = \frac{d - i}{R}$$

Substituting the given values in this equation,

$$s = \frac{300 - 1}{3.03} = 98.68 \quad \text{Ans.}$$

From formula **2**, Art. **4**, we have, also,

$$R = \frac{d - i}{s}$$

Substituting, successively, the given values of d, we have

for $d = 101$ ft., $\quad R = \dfrac{101 - 1}{98.68} = 1.01$ Ans.

for $d = 201$ ft., $\quad R = \dfrac{201 - 1}{98.68} = 2.03$ Ans.

for $d = 401$ ft., $R = \frac{401 - 1}{98.68} = 4.05$. Ans.

for $d = 501$ ft., $R = \frac{501 - 1}{98.68} = 5.07$. Ans.

(7) (*a*) The stadia reading is equal to $4.9 \times 2 = 9.8$. Since the vertical angle is zero, the line of sight is horizontal. Substituting known values in formula **2**, Art. **4**,

$$d = 101 \times 9.8 + .92 = 990.7 \text{ ft.} \quad \text{Ans.}$$

(*b*) Substituting known values in the formula of Art. **6**,

$$d = (101 \times 9.8 \cos 10° 36' + .92) \cos 10° 36'$$
$$d = (101 \times 9.8 \times .98294 + .92) \times .98294$$
$$d = 957.2 \text{ ft.} \quad \text{Ans.}$$

(8) (*a*) Substituting the given values in the formula of Art. **7**,

$$v = \frac{1}{2} \times 101 \times 7.3 \sin 21° 12' + 1 \times \sin 10° 36'$$

$$v = \frac{1}{2} \times 101 \times 7.3 \times .36162 + .18395 = 133.5 \text{ ft.} \quad \text{Ans.}$$

(*b*) Substituting the given values in the formula of Art. **7**,

$$v = \frac{1}{2} \times 101 \times 7.3 \sin 42° 24' + 1 \times \sin 21° 12'$$

$$v = \frac{1}{2} \times 101 \times 7.3 \times .67430 + .36162$$

$$v = 248.9 \text{ ft.} \quad \text{Ans.}$$

(9) (*a*) Here, $e = 101 - 100 = 1$ and $R = 7.3$. Substituting these values in formula **1**, Art. **23**, and also the values of v_1 and I_v for an angle of 10° 36′ from the Stadia Reduction Table,

$$v = \left(18.08 + 18.08 \times \frac{1}{100}\right) 7.3 + .18$$
$$v = 133.5 \text{ ft.} \quad \text{Ans.}$$

(*b*) In a similar manner, for an angle of 21° 12′,

$$v = \left(33.72 + 33.72 \times \frac{1}{100}\right) 7.3 + .37$$
$$v = 249.0 \text{ ft.} \quad \text{Ans.}$$

(10) The difference of elevation is found from formula **2**, Art. **23**, as follows:

$$v_1 \text{ for an angle of } 20° 19' = 32.56, \text{ and } I_v = .44$$

Substituting in the formula,

$$v = 32.56 \times 6.42 + .44 = 209.5 \text{ ft.}$$

Since the vertical angle is an angle of depression, the stadia point is 209.5 ft. below the instrument point. The elevation of the instrument point is 275.0 ft.; therefore, the elevation of the stadia point is 275.0 − 209.5, or 65.5 ft. Ans.

(11) (*a*) According to Art. **13**, the length of the divisions on the stadia rod will be $\frac{100}{98.75}$, or 1.01 ft. Ans.

(*b*) In this case the length of the divisions will be $\frac{100}{101.38}$, or .99 ft. Ans.

Since questions 12 and 13 are to be solved on assumptions made by the student, no solutions can be given here.

TOPOGRAPHIC SURVEYING

(1) See Arts. **2**, **3**, **4**, and **5**.

(2) The tangent of the slope angle A, Fig. 1, is $\frac{1}{1.5}$; whence the slope angle is 33° 41′. In the triangle $A\,B\,C$, $A\,B$ = 50 ft., and the angle A = 33° 41′. Then, $A\,C = 50 \cos 33° 41' = 41.6$ ft. Ans.

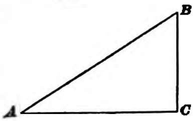

FIG. 1

(3) (*a*) In Fig. 1, $B\,C = 50 \sin 33° 41' = 27.7$ ft. Ans.

(*b*) The slope angle as determined above is 33° 41′. Ans.

(4) (*a*) and (*b*) Read Arts. **7** and **8**.

(5) See Art. **9**.

(6) The principles involved in the answer to this question are explained in Arts. **8** and **13**.

(7) See Art. **13**.

(8) Read Arts. **16**, **17**, and **18**.

(9) Read Art. **22**.

(10) Read Art. **28**.

(11) No, because the leveling rod is not long enough.

HYDROGRAPHIC SURVEYING

(1) In triangle *1*, the sum of the angles is $(67° 29' + 50° 17' + 62° 13') = 179° 59'$. $180° - 179° 59' = 1'$; therefore, the angles must be corrected by adding one-third of 1′, or 20″, to each angle. The corrected angles are 67° 29′ 20″, 62° 13′ 20″, and 50° 17′ 20″.

In triangle *2*, the sum of the angles is $(54° 19' + 76° 39' + 49° 3') = 180° 1'$. This sum is 1′ greater than 180°; therefore, the angles must be corrected by subtracting one-third of 1′, or 20″, from each angle. The corrected angles are 54° 18′ 40″, 76° 38′ 40″, and 49° 2′ 40″.

In triangle *3*, the sum of the angles is $(59° 47' + 58° 41' + 61° 32') = 180°$. It is not necessary to correct the angles in this triangle.

In triangle *4*, the sum of the angles is $(53° 10' + 58° 22' + 68° 29') = 180° 1'$. Since the sum is greater than 180°, the angles must be corrected by subtracting one-third of 1′, or 20″, from each angle. The corrected angles are 53° 9′ 40″, 58° 21′ 40″, and 68° 28′ 40″.

In triangle *5*, the sum of the angles is $(57° 9' + 64° 53' + 57° 59') = 180° 1'$. The angles are corrected by subtracting one-third of 1′, or 20″, from each angle. The corrected angles are 57° 8′ 40″, 57° 58′ 40″, and 64° 52′ 40″.

In triangle *1*, $AB = 500$ ft.

$$AC = \frac{500 \sin 62° 13' 20''}{\sin 67° 29' 20''} = 478.87 \text{ ft. Ans.}$$

$$CB = \frac{500 \sin 50° 17' 20''}{\sin 67° 29' 20''} = 416.36 \text{ ft. Ans.}$$

In triangle *2*,

$$CD = \frac{416.36 \sin 49° 2' 40''}{\sin 76° 38' 40''} = 323.18 \text{ ft. Ans.}$$

$$DB = \frac{416.36 \sin 54° 18' 40''}{\sin 76° 38' 40''} = 347.57 \text{ ft. Ans.}$$

In triangle *3*,

$$BE = \frac{347.57 \sin 59° 47'}{\sin 58° 41'} = 351.57 \text{ ft. Ans.}$$

$$DE = \frac{347.57 \sin 61° 32'}{\sin 58° 41'} = 357.66 \text{ ft. Ans.}$$

In triangle *4*,

$$DF = \frac{357.66 \sin 68° 28' 40''}{\sin 58° 21' 40''} = 390.81 \text{ ft.} \quad \text{Ans.}$$

$$FE = \frac{357.66 \sin 53° 9' 40''}{\sin 58° 21' 40''} = 336.22 \text{ ft.} \quad \text{Ans.}$$

In triangle *5*,

$$EG = \frac{336.22 \sin 57° 8' 40''}{\sin 57° 58' 40''} = 333.12 \text{ ft.} \quad \text{Ans.}$$

$$FG = \frac{336.22 \sin 64° 52' 40''}{\sin 57° 58' 40''} = 359.04 \text{ ft.} \quad \text{Ans.}$$

(2) Read Art. **38.**

(3) Read Arts. **30** and **51.**

(4) (*a*) See Art. **39.**
(*b*) See Art. **43.**

(5) No. It is not practicable to row the boat so that the speed will be absolutely uniform.

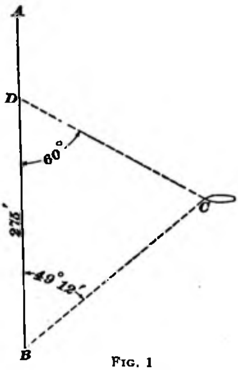

FIG. 1

(6) (*a*) In Fig. 1, *A B* is the base line; *B*, the position of the instrument; and *C*, the position of the boat. The angle *C* = 180° − (60° + 49° 12′) = 70° 48′.

$$BC = \frac{275 \sin 60°}{\sin 70° 48'} = 252.2 \text{ ft.} \quad \text{Ans.}$$

(*b*) $$DC = \frac{275 \sin 49° 12'}{\sin 70° 48'} = 220.4 \text{ ft.} \quad \text{Ans.}$$

(7) The sketch is shown in Fig. 2. Substituting the given values in equation (*b*), Art. **41**,

$$S = 360° - (159° 30' + 39° 13' + 23° 8') = 138° 9'$$

Substituting in the formula of Art. **41**, noticing that sin 138° 9′ = sin (180° − 138° 9′) = sin 41° 51′, and cot 138° 9′ = − cot (180° − 138° 9′) = − cot 41° 51′,

$$\cot X = \frac{300 \sin 23° 8'}{500 \sin 41° 51' \sin 39° 13'} - \cot 41° 51'$$

whence, $$X = 119° 9'$$

$$Y = 138° 9' - 119° 9' = 19° 00'$$

In the triangle *F A B*, the angle *F B A* = 180° − (39° 13′ + 119° 9′) = 21° 38′.

$$FA = \frac{300 \sin 21° 38'}{\sin 39° 13'} = 174.9 \text{ ft.} \quad \text{Ans.}$$

$$FB = \frac{300 \sin 119° 9'}{\sin 39° 13'} = 414.4 \text{ ft.} \quad \text{Ans.}$$

In the triangle FCB, the angle $FBC = 180° - (23° 8' + 19° 00')$ $= 137° 52'$.

$$FC = \frac{500 \sin 137° 52'}{\sin 23° 8'} = 853.8 \text{ ft.} \quad \text{Ans.}$$

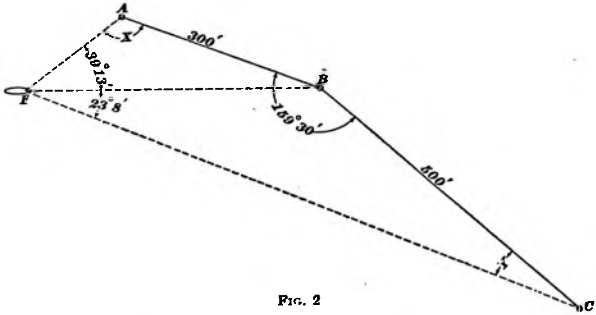

FIG. 2

(8) To apply formula of Art. 59, we have $h = \frac{5}{3}$ yd.; $A_0 = 11{,}500$, $A_n = 615$, and $\Sigma A_m = 8{,}740 + 5{,}170 + 2{,}650 = 16{,}560$.

$$V = \frac{5}{3}\left(\frac{11{,}500}{2} + 16{,}560 + \frac{615}{2}\right) = 37{,}696 \text{ cu. yd.} \quad \text{Ans.}$$

(9) To apply formula of Art. 60, we have $h = \frac{5}{3}$, $A_0 = 11{,}500$, $4\Sigma A_1 = 4(8{,}740 + 2{,}650) = 45{,}560$, $2\Sigma A_2 = 2 \times 5{,}170 = 10{,}340$, and $A_n = 615$.

$$V = \frac{5}{3 \times 3}(11{,}500 + 45{,}560 + 10{,}340 + 615) = 37{,}786 \text{ cu. yd.} \quad \text{Ans.}$$

(10) Fig. 3 represents the cross-section on the line $A_1 A_1$. The area is equal to the sum of the areas of the two triangles $A_1 B_1 B$ and

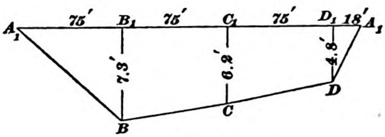

FIG. 3

$A_1 D_1 D$ and of the trapezoids $B_1 C_1 C B$ and $C_1 D_1 D C$. The area of the cross-section on $A_1 A_1$ is, therefore,

$$\frac{75 \times 7.3}{2} + \frac{7.3 + 6.2}{2} \times 75 + \frac{6.2 + 4.8}{2} \times 75 + \frac{18 \times 4.8}{2} = 1{,}235.7 \text{ sq. ft.}$$

Ans.

In a similar manner, the area of the cross-section on A_2A_2 is

$$\frac{75 \times 8.2}{2} + \frac{8.2 + 10.3}{2} \times 75 + \frac{10.3 + 9.6}{2} \times 75 + \frac{9.6 + 3.1}{2} \times 75 + \frac{3.1 \times 9}{2}$$
$$= 2{,}237.7 \text{ sq. ft. Ans.}$$

Area of cross-section on A_3A_3 is

$$\frac{150 \times 9.3}{2} + \frac{9.3 + 19.8}{2} \times 150 + \frac{19.8 + 16.4}{2} \times 150 + \frac{16.4 + 8.5}{2} \times 150$$
$$+ \frac{8.5 \times 75}{2} = 7{,}781.3 \text{ sq. ft. Ans.}$$

Area of cross-section on A_4A_4 is

$$\frac{150 \times 8.4}{2} + \frac{8.4 + 14.9}{2} \times 150 + \frac{14.9 + 7.3}{2} \times 150 + \frac{7.3 \times 102}{2}$$
$$= 4{,}414.8 \text{ sq. ft. Ans.}$$

Area of cross-section on A_5A_5 is

$$\frac{100 \times 5.7}{2} + \frac{5.7 + 6.1}{2} \times 100 + \frac{6.1 + 5.2}{2} \times 100 + \frac{5.2 + 4.7}{2} \times 100 + \frac{43 \times 4.7}{2}$$
$$= 2{,}036.1 \text{ sq. ft. Ans.}$$

(11) Substituting the values A_1, A_2, etc. just found, and those of h_{0-1}, h_{1-2}, etc. given on Fig. III, in formula of Art. 63, we have

$$V = \tfrac{1}{2}[1{,}235.7 \times 20 + (1{,}235.7 + 2{,}237.7)\,50 + (2{,}237.7 + 7{,}781.3)\,95$$
$$+ (7{,}781.3 + 4{,}414.8)\,80 + (4{,}414.8 + 2{,}036.1)\,75 + 2{,}036.1 \times 25]$$
$$= 1{,}330{,}298.5 \text{ cu. ft., or } \frac{1{,}330{,}298.5}{27} \text{ cu. yd., or } 49{,}270 \text{ cu. yd. Ans.}$$

(12) See Fig. 39.

UNITED STATES LAND SURVEYS

(PART 1)

(1) (*a*) The two principal classes of land surveys are original surveys and resurveys. See Art. **1**.

(*b*) See Art. **2**.
(*c*) See Art. **3**.

(2) (*a*) See Art. **4**.
(*b*) See Art. **5**.
(*c*) See Art. **4**.
(*d*) See Art. **6**.

(3) (*a*) and (*b*) See Art. **6**.

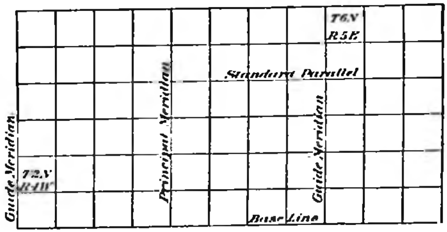

FIG. 1

(*c*) The method of numbering the sections in a township is shown in Fig. 2, Art. **7** of the text.

(*d*) See Art. **7**.

(4) See Art. **8**.

(5) (*a*) and (*b*). See Art. **8**.

(6) (*a*), (*b*), and (*c*). See Art. **5**.

(7) (*a*), (*b*), and (*c*). See Art. **16**.

(8) The correction for latitude 37° to be applied to the bearing of the range line is found by reference to Table I. Applying rule I of

Art. **17**, the bearings of the meridional section lines are found to be (*a*) N 0° 1′ W, (*b*) N 0° 1′ W, (*c*) N 0° 2′ W, (*d*) N 0° 3′ W, and (*e*) N 0° 3′ W. Ans.

(9) (*a*) See Art. **20**.
(*b*) See Art. **21**.

(10) See Art. **23**.

(11) (*a*) log 480 = 2.68124, log cos 43° = $\bar{1}$.86413, and log cos 43° 5.2′ = $\bar{1}$.86352. By substituting these values in formula **2**, Art. **25**, the logarithm of the length of the north boundary is log d_n = 2.68124 + $\bar{1}$.86352 − $\bar{1}$.86413 = 2.68063, which is the logarithm of 479.32 ch. Ans. See Arts. **25** and **26**.

(*b*) The natural tangent of 43° is .93252. Substituting known values in formula **2**, Art. **24**, we have

$$c' = .8675 \times 6 \times .93252 = 4.85' = 4'\ 51''. \quad \text{Ans.}$$

(*c*) The random for any line must close within 50 li. per mi. of line run; therefore, the random for the township line must close within 50 × 6 = 300 li. or 3 ch. The least length permitted for the length of this random line is therefore 479.32 − 3 = 476.32 ch., and the greatest length permitted is 479.32 + 3 = 482.32 ch. Ans. See Art. **23**.

(*d*) The difference between the northern and southern boundaries of this township is 480 − 479.32 = .68 ch. Section 18 is in the western tier of sections and the deficiency is placed in the western half of this section, which is midway between the northern and the southern boundary of the township. Therefore, the theoretical length of the southern boundary of Section 18 is 80 − ½ × .68 = 79.66 ch. Ans.

(*e*) The least length permitted for this line is 79.66 − .50 = 79.16 ch., and the greatest length permitted is 79.66 + .5 = 80.16 ch. Ans.

(12) (*a*) See Art. **31**.
(*b*) See Art. **29**.

(13) (*a*), (*b*), (*c*), (*d*), and (*e*). See Art. **33**.

(14) (*a*), (*b*), and (*c*). See Art. **35**.

UNITED STATES LAND SURVEYS

(PART 2)

(1) See Art. **2.**

(2) See Art. **3.**

(3) See Art. **4.**

(4) See Art. **5.**

(5) See Art. **7.**

(6) The north quarter-section corner was laid off to be 40 ch., original measure, west from the section corner between Sections 5 and 6. Since the returned length of the section line is 77.32 ch. and the actual length is 81.22 ch., the returned length 77.32 ch. is to the returned length of the quarter section line 40 ch. as the actual length 81.22 ch. is to the actual length x of the quarter-section line; that is, $77.32 : 40 = 81.22 : x$

whence $x = 42.02$

Therefore, the length of the line from the section corner between Sections 5 and 6 to the north quarter section corner for Section 6 is 42.02 ch. Ans.

(7) (*a*), (*b*), and (*c*) See Art. **9.**

(8) (*a*) and (*b*) See Art. **10.**

(9) See Art. **13.**

(10) See Arts. **14, 15, 16, 17, 18,** and **19.**

(11) (*a*), (*b*), and (*c*) See Art. **23.**

(12) (*a*) and (*b*) See Art. **23.**

(13) (*a*) See Art. **23.**
(*b*) and (*c*) See Art. **24.**

(14) In Fig. 1, NS represents the direction of the needle in its original position, and $N'S'$, the direction of the needle in its present position. The original bearing of the line AB was N 48° 52′ E. The present bearing of the line is equal to the angle $N'AB$, or $48° 52' + 4° = 52° 52'$. Therefore, the bearing of the line AB is N 52° 52′ E.

(15) The original bearing of the line *A B* was N 1° 30′ E (the angle *N A B*, Fig. 2) and the present bearing of the line is N 4° 45′ W (angle *N′ A B*). The variation in the needle is, therefore, equal to *N A B* + *N′ A B*, or 1° 30′ + 4° 45′ = 6° 15′, east. *N S* represents the direction of the needle in its original position and *N′ S′* represents the direction of the needle in its present position.

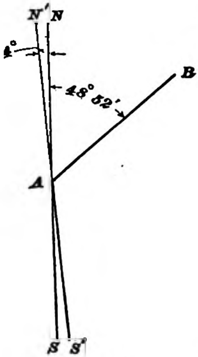

FIG. 1

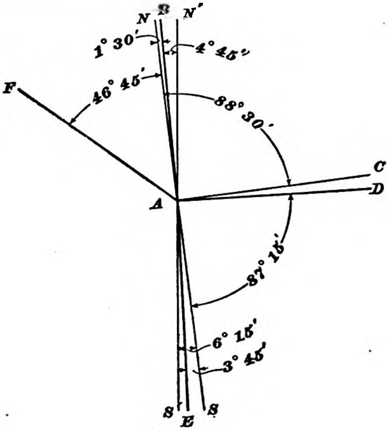

FIG. 2

(*a*) From an inspection of the figure, it is seen that the present bearing of the line *A C* (angle *N′ A C*) is N 88° 30′ − 6° 15′ E = N 82° 15′ E. Ans.

(*b*) The present bearing of the line *A D* (angle *S′ A D*) = S 87° 15′ + 6° 15′ E = S 93° 30′ E. Since bearings are not expressed by angles greater than 90°, the bearing of this line is N 86° 30′ E. Ans.

(*c*) The present bearing of the line *A E* is equal to the angle *S′ A E* or 6° 15′ − 3° 45′ = 2° 30′. Therefore, the bearing of *A E* = S 2° 30′ E. Ans.

(*d*) The present bearing of the line *A F* (angle *N′ A F*) = N 46° 45′ + 6° 15′ W = N 53° 00′ W. Ans.

MAPPING

(PART 1)

(1) See Art. **2**. The scale of the map is $\frac{1}{12} \div 250 = \frac{1}{3,000}$. Ans.

(2) There are 792 in. in 1 ch. Let x represent number of chains to the inch; then, $\frac{1}{792} \div x = \frac{1}{3,960}$, whence $x = 5$. Ans.

(3) Read Art. **8**.

(4) Read Arts. **9** and **11**.

(5) Read Art. **12**.

(6) An azimuth of 296° 10′, counted from the north toward the east, may be laid off by laying off to the west an angle equal to $360° - 296° 10' = 63° 50'$.

MAPPING

(PART 2) .

(1) See Art. **3.** Read article in *Topographic Surveying* describing the method of performing the field work.

(2) See Art. **29.**

(3) In making a profile drawing, the vertical scale is usually made larger than the horizontal in order to exaggerate the slope.

(4) The profile, which is constructed as explained in Art. **23,** is shown in Fig. 1. The distances between successive stations are determined by subtracting the number of each station from the one

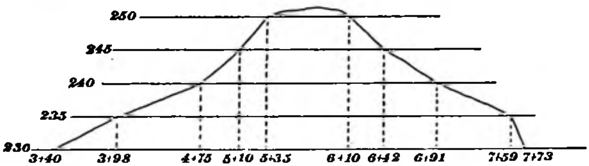

FIG. 1

immediately following. These distances are laid off on a horizontal line, which represents the elevation of the lowest contour, or contour 230. Horizontal lines are drawn at intervals of 5 ft., and each station point is projected on the horizontal line that represents the contour at that station. The points of intersection of the horizontal and vertical lines are joined by a continuous line drawn freehand. This line represents the profile.

(5) Read Art. **24.**

(6) The latitudes and longitudes are first calculated from a meridian through *A*. The latitude of the most northerly point is +1,229.7 and that of the most southerly point is −25.3. The longitude of the most easterly point is 1,444.2. The most westerly point is *A*, the point from which latitudes and longitudes are reckoned. The scale must be

such that $1,229.7 + 25.3$, or 1,255.0 ft. can be laid off in a north-and-south direction, and 1,444.2 ft. in an east-and-west direction. Take the meridian parallel to the shorter edge. The traverse is not to come within $\frac{1}{2}$ in. of the border. In order that the map may come within the limits of the border in an east-and-west direction, the scale must be about $\frac{1,444.2}{18-1} = 85$ ft. to the in. And, in order that the map may come within the border in a north-and-south direction, the scale must be $\frac{1,255.0}{10-1} = 140$ ft. to the in. The latter scale is chosen. The point A should be $\frac{1}{2}$ in. from the left-hand border, and $\frac{25.3}{140} + \frac{1}{2} = .7$ in., nearly, from the lower border of the map.

PRACTICAL ASTRONOMY
(PART 1)

(1) (*a*) All great circles passing through the north and south poles. See Art. **17**.

(*b*) The north and south poles.

(*c*) All circles parallel to the equator. See Art. **21**.

(*d*) The east and west points.

(2) (*a*) See Art. **18**.

(*b*) The poles of the celestial sphere are the poles of the equator; since the meridian passes through the poles, it is secondary to the equator; therefore, the angle between the meridian and the equator is 90°. See Art. **17**.

(*c*) The east and west points are the poles of the meridian, and since the prime vertical passes through these points, it is secondary to the meridian. The angle between the prime vertical and the meridian is therefore 90°. See Art. **17**.

(3) (*a*) Since the vernal equinox is on the equator, its polar distance is 90°. See Arts. **39** and **45**.

(*b*) The declination of the south pole is $-90°$. See Art. **42**.

(*c*) Since the zenith is on the meridian, the hour angle of the zenith is 0. See Art. **64**.

(*d*) The azimuth of the north pole measured from the south toward the west is 180°; from the north toward the east, 0°. See Art. **61**.

(4) (*a*) West point, zenith, east point, and nadir. See Art. **58**.

(*b*) South point of horizon, south point of equator, zenith, north pole, north point of horizon, nadir, and south pole.

(*c*) Along the horizon from the south point toward the west, or from the north point toward the east. See Art. **61**.

(*d*) Along the equator from the south point toward the west. See Art. **64**.

(*e*) Along the equator from the vernal equinox toward the east. See Art. **41**.

(5) (*a*) The sidereal time is the hour angle of the vernal equinox; therefore, the hour angle of the equinoctial colure is 22 hr.

(*b*) $18^h\ 10^m\ 20^s$. See Art. **70**.

(6) (*a*) See Art. **79**.

(*b*) Since the hour angle of the sun is $-3^h\ 30^m$, the apparent solar time is $8^h\ 30^m$ A. M. The local mean time is $8^h\ 30^m + (-3^m\ 12.4^s)$ $= 8^h\ 26^m\ 47.6^s$ (see Art. **80**). The difference in time between the 105th meridian and the given place is $7^h - 6^h\ 44^m\ 10.7^s = 15^m\ 49.3^s$. The standard time at the given place is, then, $8^h\ 26^m\ 47.6^s - 15^m\ 49.3^s$ $= 8^h\ 10^m\ 58.3^s$ Ans.

(*c*) By rule I, Art. **78**, the astronomical date is Aug. 16, 22 hr.

(7) Applying the formula of Art. **71**, the sidereal time is equal to $13^h\ 2^m\ 40.1^s + 6^h = 19^h\ 2^m\ 40.1^s$. The corresponding mean solar time is determined by rule I, Art. **87**. The sidereal time of mean noon is found as explained in Art. **86**. The sidereal time of Washington mean noon, January 5, 1903, is (Table IV) $18^h\ 56^m\ 27.8^s$. The correction for $+1^h\ 7^m$ is 11.006. The correction for $+3.5^s$ is .009. Therefore, the correction for longitude $+1^h\ 7^m\ 3.5^s$ is 11.02^s, and the sidereal time of local mean noon is $18^h\ 56^m\ 27.8^s + 11.02^s = 18^h\ 56^m\ 38.82^s$. Subtracting this time from the given sidereal time, the sidereal interval of time elapsed since local mean noon is $19^h\ 2^m\ 40.1^s - 18^h\ 56^m\ 38.82^s$ $= 6^m\ 1.28^s$. This interval is changed into its equivalent solar interval, as explained in Art. **85**. The correction for $6^m\ 1.28^s$ is found, from Table II, to be $.99^s$. Subtracting this correction, the required local mean solar time is $6^m\ .29^s$. Ans.

(8) (*a*) The right ascension and declination of the sun are found by the rule stated in Art. **90**. The local time is 11 A. M., January 2, 1903. The corresponding Washington time is $3^h\ 58^m\ 55.4^s$ later, or $2^h\ 58^m\ 55.4^s$ P. M., January 2, 1903, = January 2, 2.982^h. The right ascension of the sun at Washington mean noon, January 2, was $18^h\ 48^m\ 30.52^s$ (Table IV). The hourly motion was 11.037. Increase of right ascension during 2.982 hr. is $11.037 \times 2.982 = 32.91^s$. Hence, the required right ascension is $18^h\ 48^m\ 30.52^s + 32.91^s = 18^h\ 49^m\ 3.43^s$. Ans.

(*b*) The declination is found in a similar manner. The declination of the sun at Washington mean noon, January 2, was $-22°\ 58'\ 51.0''$. Change in declination during 2.982 hr. is $12.78 \times 2.982 = 38.1''$. Therefore, the required declination is $-22°\ 58'\ 51.00'' + 38.1'' = -22°$ $58'\ 12.9''$. Ans.

(*c*) 2.982 hr. = .124 da. From Table VI, the right ascension of Vega corresponding to January 2, .124 is found to be $18^h\ 33^m\ 37.86^s$ $+ 1.124 \times \frac{.11}{11} = 18^h\ 33^m\ 37.87^s$. Ans.

(*d*) The declination of Vega is equal to $38°\ 41'\ 43.5'' - 1.124$ $\times \frac{3.1}{11} = 38°\ 41'\ 43.2''$. Ans.

(*e*) The equation of time for noon, January 2, was $+3^m\ 52.48^s$, and that for noon, January 3, was $+4^m\ 20.63^s$. The change in the equation of time for one day is, therefore, $+28.15^s$. Total change since Washington noon is $+28.15 \times .124 = +3.49^s$. The required equation of time is, therefore, $+3^m\ 52.48^s + 3.49^s = +3^m\ 55.97^s$. Ans.

(*f*) The semi-diameter of the sun at noon, January 2 and 3, is 16′ 17.83″. Ans. See Table IV.

(9) As the mercury horizon was used, no correction for index error is required. See Art. **100**. The correction for refraction is −2′ 33.6″. See Art. **105**. The correction for parallax, taken from Table VIII, is +8″. The correction for semi-diameter, taken from Table IV, is +16′ 17.8″. The total correction is, therefore, +13′ 52.2″, and the corrected altitude is 20° 12′ 10″ + 13′ 52.2″ = 20° 26′ 2.2″. Ans.

(10) As the altitude of both the upper and lower edges was observed, the mean of the two altitudes will be the altitude of the sun's center, and no correction for semi-diameter is required. As the mercury horizon was used, no correction for index error is required. The corrections are applied to the measurements on the upper and lower edges separately, because the refraction is slightly different at the two altitudes. The true altitude of the sun's center is then obtained by taking the mean of the two corrected altitudes. The circle reading on the upper edge of the sun direct is 30° 12′ 10″. The circle reading on the upper edge of the sun reflected from the mercury is 329° 48′ 00″. The measured double altitude is, therefore, 30° 12′ 10″ + (360° − 329° 48′ 00″) = 60° 24″ 10″. The measured altitude is then $\frac{60^\circ\ 24'\ 10''}{2}$ = 30° 12′ 05″. The correction for refraction is −1′ 37.6″, and that for parallax, +8″. The corrected altitude of the upper edge is 30° 12′ 05″ + 8″ − 1′ 37.6″ = 30° 10′ 35.4″. The circle reading direct on the lower edge of the sun is 29° 40′ 50″. The circle reading on the lower edge of the sun reflected from the mercury is 330° 19′ 30″. The measured double altitude is, therefore, 29° 40′ 50″ + (360° − 330° 19′ 30″) = 59° 21′ 20″. The measured altitude is $\frac{59^\circ\ 21'\ 20''}{2}$ = 29° 40′ 40″. The correction for refraction is −1′ 39.4″, and that for parallax is +8″. The corrected altitude of the lower edge is 29° 40′ 40″ + 8″ − 1′ 39.4″ = 29° 39′ 8.6″. The true altitude of the sun's center is,

$$\frac{30^\circ\ 10'\ 35.4 + 29^\circ\ 39'\ 8.6''}{2} = 29^\circ\ 54'\ 52''.\ \text{Ans.}$$

(11) The mean solar time is changed into sidereal time by applying rule II, Art. **87**. The mean solar interval since preceding mean noon is $2^h\ 10^m\ 8^s$. The correction to reduce this interval to sidereal interval is found, from Table II, to be 21.378^s. The sidereal interval since

preceding mean noon = $2^h\ 10^m\ 29.378^s$. The sidereal time of local mean noon is found as explained in Art. 86. From Table IV, the sidereal time of mean noon at Washington, January 4, 1903, is found to be $18^h\ 52^m\ 31.24^s$. The correction for longitude $+2^h\ 6^m$ is $+20.699^s$. Hence, the sidereal time of mean noon is $18^h\ 52^m\ 31.24^s + 20.699^s = 18^h\ 52^m\ 51.939^s$. The sidereal time corresponding to the given mean solar time is, therefore, $18^h\ 52^m\ 51.939^s + 2^h\ 10^m\ 29.378^s = 21^h\ 3^m\ 21.317^s$. Ans.

(12) Mean time is equal to the hour angle of the mean sun (Art. 77). Sidereal time is equal to the hour angle of the vernal equinox (Art. 69). Therefore, when the two clocks indicate the same time, the hour angle of the mean sun must be equal to the hour angle of the vernal equinox. The mean sun is therefore at the vernal equinox, and the right ascension of the mean sun is $0^h\ 0^m\ 0^s$. Ans.

(13) The longitude of the place is 85°, or $5^h\ 40^m$. The difference in longitude between this place and the 90th meridian is $5^h\ 40^m - 6^h = -20^m$. Subtracting this difference algebraically, the local time is $10^h\ 32^m + 20^m = 10^h\ 52^m$. Ans.

(14) Correcting the observed altitude for index error, we have $60°\ 40'\ 30'' - 1'\ 30'' = 60°\ 39'$. The correction for refraction, from Table VII, is $-32''$; therefore, the true altitude is $60°\ 39' - 32'' = 60°\ 38'\ 28''$. Ans.

PRACTICAL ASTRONOMY
(PART 2)

(1) The mean of the first two readings is 41° 11′ 27.5″. The mean of the next two readings is 41° 11′ 42.5″. The measured altitude is equal to $\frac{41^\circ\ 11'\ 27.5'' + 41^\circ\ 11'\ 42.5''}{2} = 41^\circ\ 11'\ 35''$. The correction for refraction (Table VII, *Practical Astronomy*, Part 1) is −1′ 5″; the true altitude is, therefore, 41° 10′ 30″. The zenith distance is 90° − 41° 10′ 30″ = 48° 49′ 30″. The declination of Polaris is found, from Table VI, *Practical Astronomy*, Part 1, to be +88° 47′ 42.2″. The latitude is equal to the declination − the zenith distance = 39° 58′ 12.2″. Ans. See Art. 2.

(2) Since 24° 21′ 30″ is the greatest measured altitude, the corresponding time ($11^h\ 16^m\ 50^s$) is the time of meridian passage. The difference between this time and each of the recorded times is the interval from time of meridian passage in the second column of the table given below. In column three are written the squares corresponding to these intervals, taken from Table II. The declination of Sirius is

Number	Interval	Square	Product	Corrected Meridian Altitude
1	$6^m\ 36^s$	43.6	58.9″	24° 21′ 28.9″
2	4 20	18.8	25.4	24 20 55.4
3	2 20	5.4	7.3	24 21 7.3
4	0 0	0.0	0.0	24 21 30.0
5	2 0	4.0	5.4	24 20 55.4
6	4 4	16.5	22.3	24 20 52.3

−16° 35′ 13.4″ (Table VI, *Practical Astronomy*, Part 1). The declination is of a different name (or sign) from the latitude. Looking in Table I, the number corresponding to a latitude of 49° and a declination of −16° 35′ is 1.35″. The numbers in column four are obtained by multiplying the squares by 1.35″. The corrected meridian altitudes are found by adding the products to the corresponding measured altitudes.

The mean of the six meridian altitudes is 24° 21′ 8.2″. The correction for index error is +1′ 30″, and that for refraction −2′ 6″ (Table VII, *Practical Astronomy*, Part 1). The true altitude is therefore 24° 20′ 32.2″. The zenith distance is 90° − 24° 20′ 32.2″ = 65° 39′ 27.8″. Declination of Sirius is −16° 35′ 13.4″. Latitude = zenith distance + declination = 65° 39′ 27.8″ − 16° 35′ 13.4″ = 49° 4′ 14.4″. Ans. See Art. **10.**

(3) Local apparent solar time is $12^h\ 0^m\ 0^s$. From Table IV, *Practical Astronomy*, Part 1, the equation of time is found to be 5^m 42.67 + .0856 × 26.44 = $5^m\ 44.9^s$. The local mean time of apparent noon is therefore $12^h\ 5^m\ 44.9^s$. The watch is $1^m\ 31.1^s$ slow; therefore, the watch time of apparent noon is $12^h\ 5^m\ 44.9^s - 1^m\ 31.1^s = 12^h\ 4^m\ 13.8^s$. Ans.

(4) Local apparent solar time	$12^h\ 0^m\ \ 0^s$
Equation of time for January 6, 1903 . . .	+5 42.67
Local mean time of apparent noon	$12^h\ 5^m\ 42.67^s$

Washington is $8^m\ 15.75^s$ west of the standard 5-hr. meridian; therefore, the standard time of apparent noon at Washington is $12^h\ 5^m\ 42.67^s + 8^m\ 15.75^s = 12^h\ 13^m\ 58.42^s$. The watch is $9^m\ 11^s$ fast; therefore, the watch time of apparent noon is $12^h\ 13^m\ 58.42^s + 9^m\ 11^s = 12^h\ 23^m\ 9.42^s$ P. M. Ans.

(5) No corrections for semi-diameter and index error are required. The correction for refraction is −3′ 42.7″; the correction for parallax is +8.5″; therefore, the true altitude is 14° 10′ 58.2″ − 3′ 42.7″ + 8.5″ = 14° 7′ 24″. The zenith distance is 90° − 14° 7′ 24″ = 75° 52′ 36″. The declination of the sun and the equation of time are found as explained in *Practical Astronomy*, Part 1.

Local time	Jan. 7 $9^h\ 9^m\ 13^s$ A. M.
Longitude	−7 37
Washington time	$9^h\ 1^m\ 36^s$ A. M.

21.027^h since noon of January 6 = .8761 da. since noon of January 6.

Declination of sun at Washington noon, January 6, 1903 (Table IV, *Practical Astronomy*, Part 1)	−22° 34′ 46″
Increase in declination is 17.3 × 21.027	+6 3.8
Declination	−22° 28′ 42.2″
Equation of time at Washington noon, January 6, 1903 (Table IV, *Practical Astronomy*, Part 1)	$+5^m\ 42.67^s$
Increase in value since preceding noon is .8761 × 26.44	+23.16
Equation of time	$+6^m\ 5.83^s$

To apply the formula in Art. **24**, we have z = 75° 52′ 36″, δ = −22° 28′ 42.2″, m = 39° 58′ 2″ + 22° 28′ 42.2″ = 62° 26′ 44.2″. $\frac{1}{2}(z + m)$ = 69° 9′ 40″; $\frac{1}{2}(z - m)$ = 6° 42′ 56″.

Substituting in the formula,

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\sin 69^\circ\, 9'\, 40'' \sin 6^\circ\, 42'\, 56''}{\cos 39^\circ\, 58'\, 2'' \cos 22^\circ\, 28'\, 42''}}$$

$\frac{1}{2} t = -23^\circ\, 7'\, 54''$; $t = -46^\circ\, 15'\, 48''$; $-46^\circ\, 15'\, 48'' = -3^h\, 5^m\, 3.2^s$.

The apparent time is therefore $12 - 3^h\, 5^m\, 3.2^s = 8^h\, 54^m\, 56.8^s$. Adding the equation of time, the local mean solar time is $9^h\, 1^m\, 2.63^s$. Ans. The mean of the watch times is $9^h\, 9^m\, 13.2^s$. Therefore, the watch is $9^h\, 9^m\, 13.2^s - 9^h\, 1^m\, 2.63^s = 8^m\, 10.6^s$ fast. Ans.

(6) Mean of the vertical circle readings is $29^\circ\, 3'\, 5''$. The correction for refraction is $-1'\, 42''$; for parallax, $+8''$; for semi-diameter, $-16'\, 27''$; for index error, $-3'\, 40''$. The total correction is, therefore, $-21'\, 41''$, and the true altitude is $28^\circ\, 41'\, 24''$. Zenith distance is $90^\circ - 28^\circ\, 41'\, 24'' = 61^\circ\, 18'\, 36''$. The mean of the horizontal readings is $25^\circ\, 49'\, 15''$. The correction for semi-diameter is $16'\, 27'' \div \cos 28^\circ\, 41'\, 24'' = +18'\, 45''$ (see Art. 37). The corrected horizontal reading is $26^\circ\, 8'\, 00''$. $z = 61^\circ\, 18'\, 36''$; $\varphi = 38^\circ\, 53'\, 18''$; $\delta = +13^\circ\, 55'\, 33''$. $\frac{1}{2}(z + \varphi + \delta) = \frac{1}{2}(61^\circ\, 18'\, 36'' + 38^\circ\, 53'\, 18'' + 13^\circ\, 55'\, 33'') = 57^\circ\, 3'\, 43.5''$; $\frac{1}{2}(z + \varphi - \delta) = \frac{1}{2}(61^\circ\, 18'\, 36'' + 38^\circ\, 53'\, 18'' - 13^\circ\, 55'\, 33'') = 43^\circ\, 8'\, 10.5''$.
Substituting known values in the formula in Art. 38,

$$\sin \tfrac{1}{2} a = \sqrt{\frac{\cos 57^\circ\, 3'\, 44'' \sin 43^\circ\, 8'\, 11''}{\sin 61^\circ\, 18'\, 36'' \cos 38^\circ\, 53'\, 18''}}$$

$$\tfrac{1}{2} a = 132^\circ\, 26'\, 55'';\ a = 264^\circ\, 53'\, 50''$$

The obtuse angle is taken as the value of $\frac{1}{2} a$ since the observations are made in the afternoon. The azimuth of the line is equal to the azimuth of the sun minus the horizontal reading, or

$$264^\circ\, 53'\, 50'' - 26^\circ\, 8' = 238^\circ\, 45'\, 50''.\quad \text{Ans.}$$

(7) The mean of the vertical circle readings is $43^\circ\, 58'\, 23''$. The correction for refraction is $-0'\, 59''$; and for parallax, $+6''$. The true altitude of the sun's center is, therefore, $43^\circ\, 57'\, 30''$. The zenith distance is $90^\circ - 43^\circ\, 57'\, 30'' = 46^\circ\, 2'\, 30''$.

$$\tfrac{1}{2}(z + \varphi + \delta) = \tfrac{1}{2}(46^\circ\, 2'\, 30'' + 40^\circ\, 36'\, 27'' + 19^\circ\, 54'\, 5'') = 53^\circ\, 16'\, 31''$$
$$\tfrac{1}{2}(z + \varphi - \delta) = \tfrac{1}{2}(46^\circ\, 2'\, 30'' + 40^\circ\, 36'\, 27'' - 19^\circ\, 54'\, 5'') = 33^\circ\, 22'\, 26''$$

Substituting in the formula in Art. 38,

$$\sin \tfrac{1}{2} a = \sqrt{\frac{\cos 53^\circ\, 16'\, 31'' \sin 33^\circ\, 22'\, 26''}{\sin 46^\circ\, 2'\, 30'' \cos 40^\circ\, 36'\, 27''}}$$

$$\tfrac{1}{2} a = 50^\circ\, 52'\, 54'';\ a = 101^\circ\, 45'\, 48''$$

Since the observations were made in the morning, the angle $\frac{1}{2} a$ is an acute angle. The mean of the horizontal readings is $65^\circ\, 1'\, 30''$. The azimuth of the line is equal to $101^\circ\, 45'\, 48'' - 65^\circ\, 1'\, 30'' = 36^\circ\, 44'\, 18''$. Ans.

(8) The following table is formed in the same manner as in the example after Art. 46:

Time P. M.	Declination of Sun	Correction for Refraction	Vertical Circle Setting
3^h 0^m	$-23°$ 2′ 34.1″	+2′ 38″	$-22°$ 59′ 56.1″
3 10	−23 2 32.2	+3 4	−22 59 28.2
3 20	−23 2 30.2	+3 30	−22 59 00.2
3 30	−23 2 28.3	+3 56	−22 58 32.3
3 40	−23 2 26.3	+4 22	−22 58 4.3
3 50	−23 2 24.4	+4 48	−22 57 36.4
4 0	−23 2 22.5	+5 14	−22 57 8.5

The declinations of the sun for the hours of 3 P. M. and 4 P. M., San Francisco mean time, are first found. Washington mean time corresponding to 3 P. M. is 3 hr. + longitude = 6^h 1^m 27^s P. M. Declination of sun at Washington noon, January1, is $-23°$ 3′ 44.2″. Increase of declination between noon and 6^h 1^m 27^s P. M. is 11.64″ × 6.024 = +1′ 10.1″. Declination January 1, 3 P. M. San Francisco mean time, is $-23°$ 2′ 34.1″. Adding the hourly motion to this, the declination at 4 P. M. is found to be $-23°$ 2′ 22.5″. These two declinations are written in the second column, and the intermediate declinations are found by simply adding one-sixth of 11.64″ = 1.94″ to each of the declinations, beginning with 3 P. M.

From Table IV at the end of the text, the correction for refraction corresponding to 3 hr. and a latitude of 36° 27′, is +2′ 14.3″ for a declination of $-20°$, and +2′ 41.3″ for a declination of $-23°$ 27′. The difference is 27″. The correction corresponding to the given declination of $-23°$ 2′ will be $2'\,14'' + \left(\frac{3°\ 2'}{3°\ 27'} \times 27\right) = 2'\,38''$. Similarly, the correction for 4 P. M. is found to be 5′ 14″. The other numbers of the third column are found by adding successively one-sixth of the difference between these two corrections; that is, $\frac{1}{6}(5'\,14'' - 2'\,38'') = 26''$. Finally, each correction is added to the corresponding declination to obtain the settings in the last column.

(9) Subtracting each reading on the star from the corresponding reading on the mark, the following differences are obtained: 100° 1′ 10″, 100° 1′ 20″, 100° 1′ 20″, 100° 1′ 30″. The mean of the differences is 100° 1′ 20″. Declination of Polaris is 88° 47′ 42″, and the latitude of Washington is 38° 53′ 20″. The azimuth of Polaris is found by applying formula 2, Art. 49,

$$\sin a = \frac{\cos 88°\ 47'\ 42''}{\cos 38°\ 53'\ 20''}$$

whence, $a = 1° 32' 53''$. Since the star is at eastern elongation, its azimuth is added to the mean of the readings. The azimuth of the line is, then, $100° 1' 20'' + 1° 32' 53'' = 101° 34' 13''$. Ans.

(10) The clock times are first corrected by applying the errors and rates given.

First time	$8^h\ 48^m\ 02^s$
Error	-10
O_1	$8^h\ 47^m\ 52^s$
Second time	$18^h\ 13^m\ 36^s$
Error	$+5.8$
O_2	$18^h\ 13^m\ 41.8^s$
Third time	$9^h\ 56^m\ 24^s$
$-\left(10 + \frac{25.14 \times 1.2}{24}\right)$	-11.3
O_3	$9^h\ 56^m\ 12.7^s$

To apply the formula in Art. 26, we have, $C_2 - O_2 = 3^h\ 0^m\ 48.2^s$; $C_1 - O_1 = 8^m\ 32^s$; $C_3 - O_3 = 8^m\ 33.3^s$; $C_3 - C_1 = 12^h\ 18^m\ 6^s = 12.3^h$; $O_3 - O_1 = 25^h\ 8^m\ 20.7^s = 25.14^h$.

Substituting in the formula,

$$d = 3^h\ 0^m\ 48.2^s - 8^m\ 32^s - (8^m\ 33.3^s - 8^m\ 32^s)\frac{12.3}{25.14}$$

$$d = 2^h\ 52^m\ 15.6^s. \quad \text{Ans.}$$

(11) The watch is $2^m\ 4.8^s$ slow on local time, but is $8^m\ 42.8^s$ fast on standard 75th-meridian time; therefore, the difference in time between the meridian and the place is $2^m\ 4.8^s + 8^m\ 42.8^s = 10^m\ 47.6^s$, and since standard time is slower than local time, the place is $10^m\ 47.6^s$ east of the 75th- or 5^h-meridian, and the longitude west of Greenwich is $5^h - 10^m\ 47.6^s = 4^h\ 49^m\ 12.4^s$. Ans.

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NOTE.—All items in this index refer first to the section (see the Preface) and then to the page of the section. Thus, "Angle book, §20, p44," means that angle book will be found on page 44 of section 20.

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